Harold is a new pupil in your class. He has just been transferred to your school. You were annoyed by his poor performances in the mathematics tests. However, his past records showed that he was an average pupil in mathematics from his former school. Jane is also another new pupil in your class. Based on her previous results from her former school, she was only a mediocre student but now, you find that she performed very well in your tests. You may be puzzled by these two incidents.

One possible explanation to these extreme cases was that there was no standardisation in the tests administered at the various schools. For example, the tests administered by Harold's previous school was too easy. He was a mediocre in his previous school. However when he came to your school, he could only match the lower abilities of your class pupils. Likewise, Jane could have come from a better school where her former teacher might gear the test to a higher level than your school.

Distribution of Questions According to Bloom's Taxonomy of Educational Objectives

According to Bloom's Taxonomy, pupils' mathematics cognitive abilities could be classified into five categories of Educational Objectives viz. knowledge (K), skills (S), comprehension (C), application (A) and higher abilities (H). In his definition, higher abilities include analysis and synthesis. In mathematics, educationists often assess their students by preparing their test questions based on these five categories of cognitive objectives. The two incidents described earlier could be due to the various ways of designing the tests according to the distribution of the types of questions classified in the five categories of Bloom's Educational
Objectives. The following graphs illustrate the four different ways in which a mathematics test/examination could have been constructed:

Figure 1 – Distribution of Types of Questions

(A) No. of Questions

<table>
<thead>
<tr>
<th>No. of Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
</tr>
</tbody>
</table>

(B) No. of Questions

<table>
<thead>
<tr>
<th>No. of Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
</tr>
</tbody>
</table>

(C) No. of Questions

<table>
<thead>
<tr>
<th>No. of Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
</tr>
</tbody>
</table>

(D) No. of Questions

<table>
<thead>
<tr>
<th>No. of Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
</tr>
</tbody>
</table>

* K – Knowledge
  S – Skills
  C – Comprehension
  A – Application
  H – Higher Abilities

Figure 1 (A) shows that the given test emphasizes too many questions at the lower abilities such as the knowledge or the skills types of questions. The situation is reversed in Figure 1(B) where the test was pitched at a higher level. Pupils would find this type of test difficult as compared to the type shown in Figure 1(A). In Figure 1(C), emphasis is placed on the two extreme types of questions, namely, the knowledge and the higher abilities. Questions set are 'normally' distributed in Figure 1(D). It is desirable to have this form of distribution of questions as it is hypothesized that this model in preparing a test would usually conform to the overall abilities of the pupils.
Table of Specifications

In view of the importance of preparing a test to reflect a whole range of questions according to Bloom's Taxonomy of Educational Objectives in the cognitive domain, a Table of Specifications could be used as a general guideline to design a mathematics achievement test. The table below shows an example of a Table of Specifications relating each topic with the different categories of questions classified as knowledge, skills, comprehension, application and higher abilities. At the primary level, a teacher may leave out the higher abilities category as questions of this type may not be suitable or may be too difficult for most pupils.

<table>
<thead>
<tr>
<th>Category</th>
<th>Knowledge</th>
<th>Skill</th>
<th>Comprehension</th>
<th>Application</th>
<th>Higher Abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Once you have drawn up the Table of Specifications, the next step is to decide how many questions should be set in each category of the cognitive domain. Preferably, your distribution of questions conforms to the normal distribution (as in Figure 1(D)). For example you might have allotted 3, 8, 10, 5 and 2 questions to knowledge, skills, comprehension, application and higher abilities categories respectively. Altogether, there are 28 questions. There is no standard formula for this distribution of questions. A teacher has to make his
own decision depending on the topic, the extent and nature of the topic, and the pupil’s level.

After the total number of questions to be set has been determined, the next stage is to distribute the number of questions to each topic under each category. For example, for the comprehension category, you may decide to distribute the total 10 questions evenly for each topic. In the skill category, you may set 2 questions each for topics B, C and D and 1 question each for topics A and E. At the end of the second stage, you will have the following table as a detailed guideline to prepare the number of questions in each topic and category of cognitive objective.

**Figure 3 – Table of Specifications completed with the number of questions in each cell**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Knowledge</th>
<th>Skill</th>
<th>Comprehension</th>
<th>Application</th>
<th>Higher Ability</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>28</td>
</tr>
</tbody>
</table>

In this hypothetical example, notice that the number of questions are quite evenly distributed. The number of questions with respect to the educational objectives is ‘normally’ distributed.
Constructing Test Items

You have seen that a mathematics test item could be categorised into one of the five cognitive objectives: knowledge, skills, comprehension, application and higher abilities. An item is said to be classified as knowledge when it requires pupils to recall facts, definitions, notions, concepts, technical terms, formulae or relationships. In this category, no understanding of the knowledge is required. Here are two examples:

Knowledge

(1) The place value of the digit 4 in 236.42 is ________.
(2) The area of a rectangle of length l cm and breadth b cm is __________.

Item 1, is used to test the knowledge or definition of place value. Testees are required to state the place value of 4 through recalling or recognising its position. Hence it is a 'knowledge' question. In item 2, the testees are required to recall the formula of the area of rectangle.

Test items would be classified as the skill type when testees' tasks are tested on computation, manipulation of expressions, simplification or performance of algorithms. In the process of performing the skills, rules may be recalled. The following are some items for testing the skills in simplifying and computing areas of composite figure:

Skill

(3) Simplify \( \frac{24}{56} \)
(4) Find the area of the shape

\[ \begin{array}{c}
3 \\
\hline
4 \\
\hline
5 \\
\end{array} \]

4 cm
To simplify the fraction in item 3, a common factor is used to divide both the numerator and the denominator. This algorithm has to be recalled in order to proceed with the simplification. Similarly, in item 4, the formula of the rectangle is required to work out the problem.

One objective in testing is to determine pupils' understanding of the concept itself and the concepts leading to procedural skills in mathematics. This type of test items is classified as *comprehension*. In this category, pupils' behaviours could be further classified as translation, interpretation and extrapolation in solving mathematics problem when they show the ability to understand, interpret and extend mathematics concepts. For example, a mathematics statement may be changed from verbal to symbolic form. Similarly, a mathematics symbol may be transformed into another mathematics statement. The intellectual processes in carrying out these procedures require some understanding of and the linking of mathematics relationship. The following are some examples illustrating the definition above:

**Translation**

(5) Change 35% into fractional form.

(6) \(8 \times 7\) is the same as \((4 \times \_\_\_) + 10\)

(7) I earned a monthly income of \(m\) for a year. I spent \(n\). How much money did I have?

In item 5, testees are expected to apply a procedural skill to transform the percentage (\%) into another symbolic form (fraction). In item 6, they have to carry out some form of operation and understanding before a parallel form of symbolic numeral is obtained. Hence, items 5 and 6 are classified as translation. Notice that the objective of item 7 is to test the testees' abilities to translate the verbal statements to the parallel symbolic expressions. It involves some form of understanding and construction of mathematics
structures in operation before the mathematics expression \((12m - n)\) could be expressed.

Solving mathematics problems also involves identification and abstraction of the major ideas and the understanding of the interrelationships between these related variables. This type of comprehension is classified as *interpretation*. Items 8 and 9 below illustrate this category of mathematics objective:

**Interpretation**

(8) What is the fraction between \(\frac{5}{6}\) and \(\frac{7}{8}\)?

(9) Which type of ice cream is the favourite of the children?

![Bar chart]

Item 8 is used to test testees' understanding of the order of fraction by interpreting the fraction. Here testees should also have grasped the concept of relationships between the two fractions. In the process of working out the problem, the testees would use translation and procedural skills. In item 9, testees must have the ability to interpret the representation of the graph. Hence, both items are classified as interpretation questions.

In mathematics testing, some problems could involve pupils' abilities to extend the data, facts or concepts beyond what is given. However, before this behavioural skill of extension is made possible,
pupils would have already interpreted the interrelationships between some concepts. The following are two items classified as the *extrapolation* type of problems.

### Extrapolation

(10) Find the missing number of

\[1, 4, 9, 16, 25, \ldots, 49\ldots\ldots\]

(11) Given that

\[\frac{3}{4} > \frac{1}{4}\]

\[\frac{4}{5} > \frac{3}{4}\]

\[\frac{5}{6} > \frac{4}{5}\]

Find the fraction which is greater than \(\frac{5}{6}\).

Pupils are required to abstract the relevant mathematics sequences before they could extrapolate and provide the answer in Item 10. They should have conceptualised and understood the 'square' series before they are able to find the value in the space as 36. Hence conceptualisation and understanding of the number series are required before pupils can work on this type of problem. Similarly in item 11, pupils are required to see how the following inequality is developed from the preceeding step. They must first know interreleationship through perception. Then only they are able to extrapolate and obtain the next greater than \(\frac{5}{6}\).

Problems are said to be classified as *application* when the testees are involved in applying general ideas, principles or methods to new situations. For this category of problems, pupils are required to retrieve some familiar concepts so that they are able to use them to solve the problems in the unfamiliar situations. The item below is used to illustrate the concept of application.
(12) The area of a rectangle is 56 cm². Its breadth is 7 cm. What is the ratio of its length to its perimeter?

In this problem, the topic tested is ratio. However, testees are required to be familiar with the previous topic of the area of rectangle. Hence, they are required to recall the formulae for finding the area and the perimeter of the rectangle. This question is thus classified as application because testees are required to recall the formulae of area and perimeter of a rectangle and to apply them in a new situation i.e. to find the ratio of its length to its perimeter.

At the primary level, questions related to analysis and synthesis are unusual. This aspect of cognitive domain is not discussed in this article.

An Example of Achievement Test on Percentage

Once the table of specifications has been drawn up, items can be constructed based on the Bloom’s Taxonomy of instructional objectives. The following is an achievement test on percentage at P5 level showing the table of specifications and the different types of test items classified as knowledge, skills, comprehension and application:

(1) Table of Specifications

<table>
<thead>
<tr>
<th>Topic</th>
<th>K</th>
<th>S</th>
<th>C</th>
<th>A</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>
Knowledge

1) Which of the following is a percentage \( \frac{3}{4} \), 0.59, 10%?

(2) What is the equivalent fraction of 10%?

Skill

(1) Express \( \frac{61}{100} \) as a decimal.

(2) Simplify \( \frac{35}{100} \).

(3) 72% of 600 = ________

Comprehension

(1) What % of the square is shaded?

(2) Charlie spent 75% of his pocket money. If his pocket money was $150, how much did he spend?

(3) Change 88% into fractional form.

(4) Fill in the correct sign <, > or = in the box.

\[ \frac{3}{4} \quad \square \quad 70\% \]

Application

(1) There are 440 ml of oil in a bottle. 0.367 l is poured out. What percentage of the liquid is left?

(2) Mr Lee sold \( \frac{3}{4} \) of the buns he had in his shop. What percentage of the buns did he sell?
(3) A man had 1200 m$^2$ of land. He used 15% of it to set up a garden. How much land did he use to set up the garden?

**Conclusion**

To prepare a mathematics achievement test is not a difficult task. It is possible to use Bloom's Taxonomy of Educational Objectives as a model in planning the overall structure of the test. The following steps may be followed in designing the achievement test:

1. Decide on the number of questions to be set in a test based on a topic or a combination of topics.

2. Distribute the total number of questions across the categories of the cognitive domain of Bloom's Taxonomy of Educational Objectives. (Knowledge, Skills, Comprehension, Application).

3. Distribute questions from each category across the different topics.

4. Construct the test items according to the definition of each category of question.

5. Test items are then compiled as an achievement test.
References


