Square Root Two

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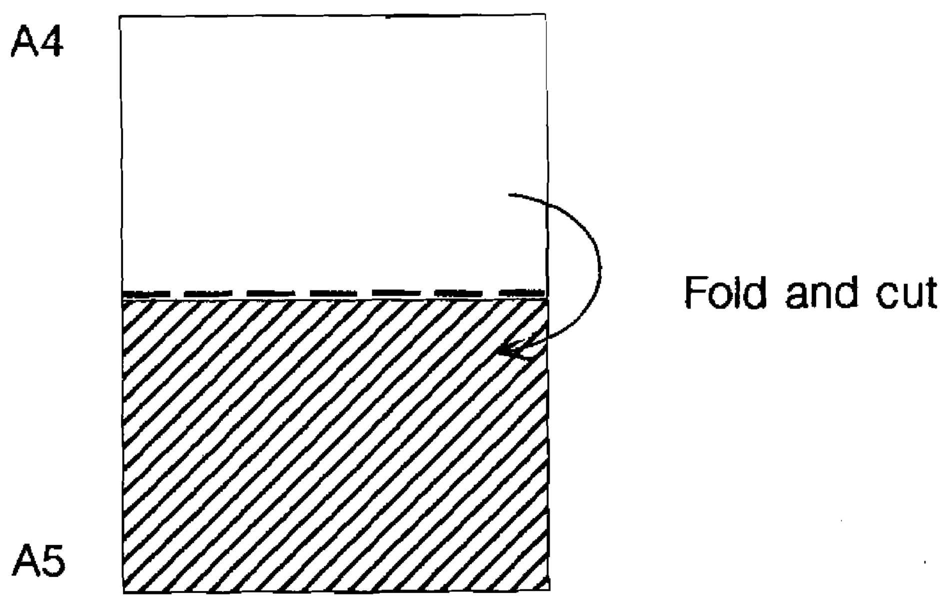
During a seminar on teaching, a colleague asked me to give an example of the application of square root two ($\sqrt{2}$) in real-life situation. The following activity should be of interest as a response to the above query.

An Activity with A4 Paper

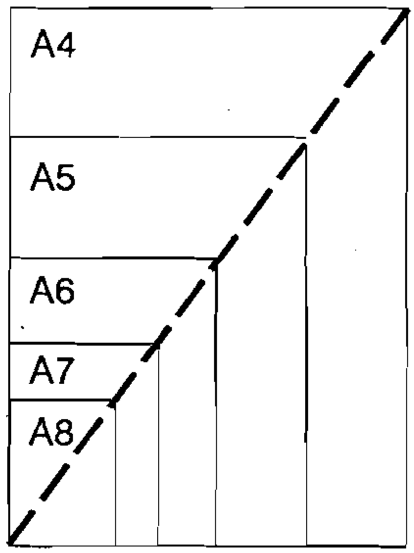
1. Take a sheet of A4 paper. Cut it into two equal halves lengthwise. Each of the resulting sheets is called an A5 paper.



2. Repeat step 1 with your A5 paper. You should get two sheets each of which is an A6 paper.

3. Repeat the above process until you get to A8.

4. Line up the sheets of paper from A4 to A8 as follows, with their lengths vertical.

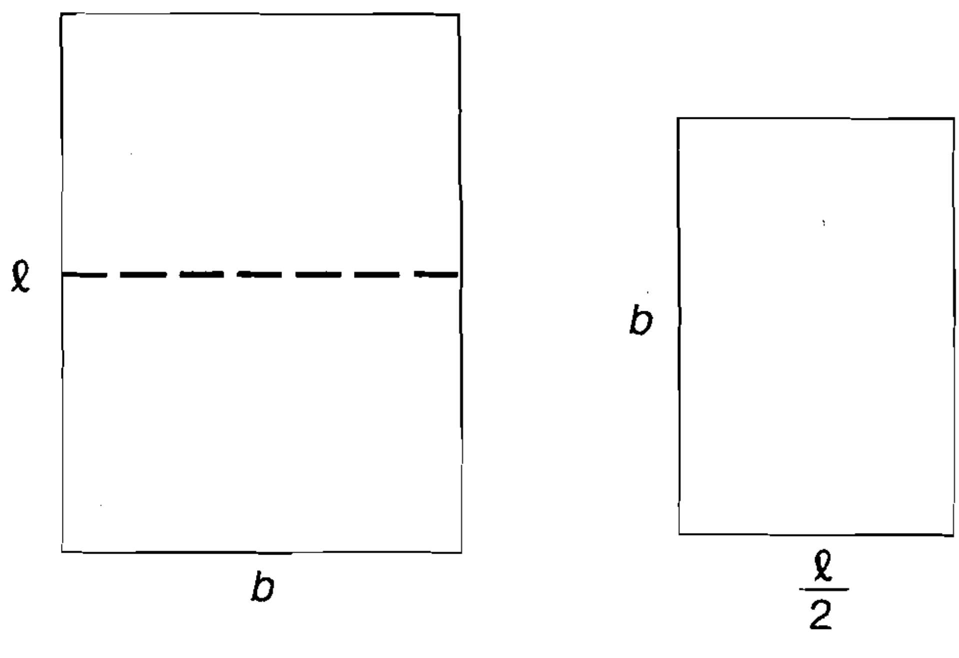


5. You should notice that a straight line can be drawn to pass through the vertices as shown. This property shows that all the sheets of paper are *similar* in shape. What is the scale factor for this set of rectangles?

Where does √2 appear?

The gradient of the straight line can be shown to be √2.

Try it before you read on!



Let the length and breadth of any of the rectangles be ℓ and b respectively. After the cut, the resulting rectangle has length b and breadth $\ell/2$ respectively. From similar rectangles, we get:

$$\frac{\ell}{b} = \frac{b}{(\ell/2)}, \qquad \ell^2 = 2b^2, \qquad \ell = \sqrt{2}\, b.$$

The better students can be challenged to find the proof themselves after they have carried out the activity. It is by no means an easy task. Four experienced teachers working together took nearly half an hour to find the proof. The difficult part is to translate the physical situation (folding and cutting paper, doing measurement) into a mathematics problem before one applies the rather elementary operations used in the proof. Most of our students are not trained in this process of linking up the real situations to abstract mathematical operations, since most of their mathematical experiences consist of applying well learned rules to worked examples in a mechanical manner. This activity provides the students an opportunity to have such an important but neglected mathematical experience.

Doing it by measurement

We can modify the above activity for students at a lower level. The size of an A4 paper is given as 297 *mm* and 210 *mm*. Check the values by measurement. How accurate can one measure? Are papers produced accurately? These questions can lead to the concepts of sampling and measurement errors, which can be explored.

By the above process of dividing by half lengthwise, we can deduce that the size of an A5 paper should be 210 *mm* by 148 *mm*. Ask the students to extrapolate in this manner to obtain the sizes of paper from A0 to A10 (the smallest size used). These values can be checked against the values obtained by measuring the actual paper.

This set of paper sizes is used internationally as the standard for paper used in writing and printing. Having obtained these values, the students can plot them on graph paper and notice the 'straight line' property. They can estimate the value of the gradient, which also gives the ratio of the length to breadth of each kind of paper.

In the above A series, the largest size A0 has the dimensions 1189 *mm* by 841 *mm*. By preserving the ratio, one can begin with different dimensions and thereby obtain different series. In fact, the B series begins with B0 of dimensions 1414 *mm by* 1000 *mm*. This B series is used for making posters and charts. Get your students to find other series in use, for example, in stamps, envelopes, plastic bags, calenders, and so on.

Exploring a Paper Microworld

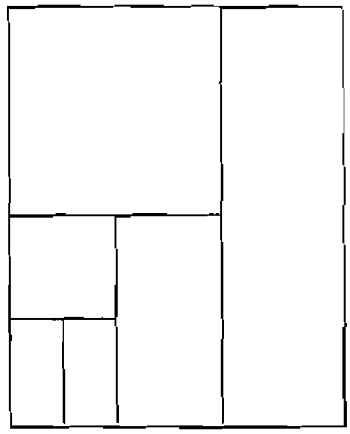
Among Logo enthusiasts, a microworld is an environment consisting of a well defined set of procedures that allow a person to explore the properties of certain objects in that environment. The following is a simple microworld for 'paper'.

Key in the following two procedures and begin by typing START at the? prompt.

```
TO START
TYPE [ENTER A VALUE FOR THE RATIO...]
MAKE "K RW
CS PU SETPOS [-50 -50] PD HT
MAKE "L 80
RECT :L*:K :L
RECT :L :L*:K/2
RECT :L*:K/2 :L/2
RECT :L/2 :L*:K/4
RECT :L*:K/4 :L/4
RECT :L/4 :L*:K/8

TO RECT :X :Y
REPEAT 2 [FD :X RT 90 FD :Y RT 90]
END
```

Choose several values for the ratio of length to breadth. You should find that a straight line is obtained only when the value is close to $\sqrt{2}$ (about 1.414). Other values do not give similar rectangles. The diagram for K=1 is shown below.



K =1

Conclusion

This article shows how students can explore the mathematics behind the paper they use so often in their work and yet are so ignorant about. It demonstrates an application of surd and quadratic equation in real situation. The various approaches (measurement, graphing, activity, algebra, computer-related) to this situation should help students see that in mathematical thinking, alternative lines of attack can lead to fruitful results. This widens their understanding of the nature of mathematics beyond just trying to get the only 'right' answer to each mathematical problem. The activity and exploration also reinforce basic measurement and algebraic skills. Teachers are encouraged to use similar activities to arouse the interests of their students in the study of mathematics.