A Constructivism Model for Helping Children Solve Mathematical Problems

Dr Fong Ho Kheong
Dr Victoria Y. Hsui
School of Science

Many teachers have difficulties handling pupils who cannot solve mathematical problems. There are many reasons why pupils are unable to solve mathematical problems. Newman's study (1977) on the difficulties of solving mathematical problems showed that the main factors that contribute to this problem are reading, comprehension, process skills and transformation. There are many other psychological reasons such as forgetfulness and lack of interest that may play a part in pupils' ability to solve mathematical problems. One hypothesis is that these pupils are not able to grasp the main concepts and procedural skills for solving certain categorical problems. Mayer (1983) showed that a good problem solver is one who is able to recognise a problem and apply the relevant strategy to solve it. Hence, if we wish to help children solve mathematical problems, then Mayer's advice is very useful.

A programme based on the concept of constructivism is developed here to determine if a below average pupil can be taught to solve problems which he/she could not do so before. A four-stage model (see Figure 1), which involved learners constructing problem-solving processes, was developed for the study. The four stages of this mode1 are: (1) Evaluation; (2) Transmission; (3) Construction; and (4) Reinforcement.

When knowledge is transmitted to a learner and he/she attempts to construct his/her own meaning on the subject matter, mental activities such as recalling, relating and reflecting are activated. A person is said to have accommodated to the imparted knowledge and integrated it to his/her existing knowledge when some kind of meaning has been formed in the person's mind. This is the constructivism stage at which the person is attempting to find some form of meaning, from which the acquired knowledge is stabilised.

The model developed for this study views the construction of knowledge as central to the learning of specific problem-solving processes (or strategies) in mathematics. It consists of a four-stage guide, with steps 2 and 3 being of pivotal importance to the learning of problem-solving strategies and the construction of mathematical knowledge by learners.

The four-stage model consists of the following steps (refer to Figure 1):

(1) *Evaluation* of performance;
(2) *Transmission* of problem-solving processes by explanation by the teacher (This stage simultaneously involves learners' construction of specific problem solving processes through interaction with the teacher);
(3) Construction of mathematical knowledge by self (This stage involves self-construction and reconstruction of mathematical knowledge through recalling, relating and reflecting); and
(4) Reinforcement of problem-solving processes which have been constructed.

The Evaluation stage is the diagnostic stage where learners are assessed to see if they have already mastered the target problem-solving process(es). Specific diagnostic mathematical problems are assigned for this purpose. If they have not mastered the processes, the learners will move to the second stage.

At the *Transmission* stage, the learners are taught the process(es) and procedures for solving a specific categorical problem. Thorough explanation is given at this stage. The teacher explains how the problem can be solved and a clear solution is given; in this regard, the learners are expected to have already mastered the pre-requisites and basic skills for solving the problem. The learners, in interaction with the teacher, attempt to construct the target problem-solving process by tackling the same problem again without reference to the given solution. At this stage, learners have the opportunity to clarify their errors or misconceptions in interaction with the teacher,

Next, during the Construction stage, learners are asked to solve the same problem again without looking at the given solution and without help from the teacher or peer. At this stage they have the opportunity to construct the concepts which may not have been fully understood in the transmission stage by recalling, relating and reflecting on what they have learned in the Transmission stage. In the the construction of knowledge, the learners may reorganise their ideas and accommodate some of the new ideas into their existing concepts in solving the problem.

Whether the learners are able to exit or not exit from the Construction stage to the final Reinforcement stage hinges on the following factors:

a The learners are able to solve the problem independently, or
b The learners have difficulty in or make errors in solving the problem independently.

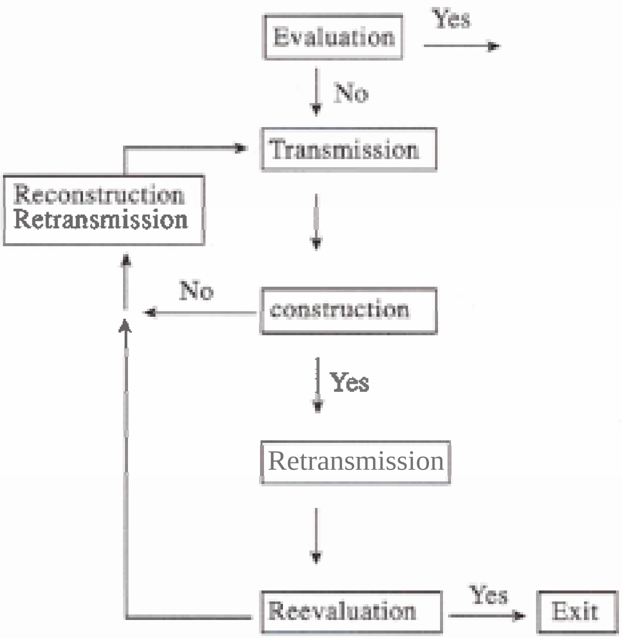
If (a), the learners will exit from the Construction stage and will be given tasks for the fourth stage, Reinforcement (to be discussed later). If (b), the learners will undergo another round – retransmission and reconstruction – to ensure mastery in the construction of the target problem-solving processes. If some learners are found to be in (b) – i.e. they still have difficulty or are unable to produce an acceptable and correct solution - the teacher will explain the whole procedure again until they have fully understood it (i.e. retransmission). They are then required to recall, relate and reflect on the procedures by working on the target mathematical problem individually (i.e. reconstruction). The process of undergoing a second round in the construction of knowledge is shown in the loop of the flow chart, Figure 1. According to the constructivism explanation, these learners have not found a similar meaning (i.e., complete solution) or arranged the segmented knowledge in an appropriate form. The errors they make are indications of their inadequacies or incomplete knowledge in constructing the problem-solving process(es). This state of their knowledge, therefore, justifies time expended on retransmission and reconstruction.

Learners will exit from the Construction stage if they are found to have mastered the intended knowledge or target problem-solving procedures by succeeding in solving the target problem independently. They will enter the Reinforcement stage, during which a number of problems that are similar and/or parallel are assigned. The objective of assigning more problems that are similar is to reinforce the concepts that have been learned.

This process model was implemented in two different pedagogical settings to test its viability. One setting was an individualised tutorial situation involving a 12-year-old student of below-average mathematical ability. The second was a whole-class situation involving a group of 22 pre-service teachers pursuing a Diploma in Education program at NIE. In both these settings, the results showed that the four-stage model provides an effective guide for helping learners understand with mathematical concepts and problems that they had initially not been able to handle. The results also showed that the constructivism activity had enabled the subjects to improve their problem-solving skills.

In the teaching of mathematics, the authors have observed that teachers do not generally optimise the benefits that the construction stage has to offer in order to help learners fully understand the mathematical problems that are presented to them. This study has shown that integrating constructivism into mathematics pedagogy strengthens learners' mental structure so that they can understand and retain the concepts more effectively.

Figure 1. A Four-Sue Constructivism Model for Problem-Salving



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