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A FRAMEWORK TO EXAMINE THE MATHEMATICS IN LESSONS OF COMPETENT MATHEMATICS TEACHERS IN SINGAPORE

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This paper outlines an analytical framework that was developed, to examine the mathematics in mathematics lessons of competent teachers in Singapore secondary schools. The framework is guided by Schoenfeld's Teaching for Robust Understanding (TRU) framework and also the field notes of the project – A study of the enacted school mathematics curriculum which is presently underway in Singapore. The framework was trialled and the indicators were suitable but may not be comprehensive. Therefore more trials and also more codes on how the teacher aided students in developing mathematical knowledge and student engagement with mathematical ideas are needed. In addition student perspectives of the lesson are also necessary to make any valid claims related to the quality of the lessons.

TEACHING OF MATHEMATICS IN SINGAPORE SCHOOLS

A few studies done so far provide us with glimpses about how teachers teach mathematics in Singapore schools (See Kaur & Yap, 1997; Chang, Kaur, Koay & Lee, 2001; Kaur, 2009; Hogan et al., 2013). In this paper we briefly elaborate the findings of two studies, the CORE 2 research led by Professor Hogan (Hogan et al., 2013) and Learner's Perspective Study (LPS) in Singapore led by Professor Kaur (Kaur & Low, 2009). As part of the CORE 2 research the quality of the enacted curriculum in Secondary 3 (grade 9) mathematics lessons was assessed using criteria and standards identified by Hattie in Visible Learning (2012). Thirty-one mathematics teachers, sampled randomly, teaching secondary three mathematics in Singapore participated in the study. Sequences of lessons of the teachers in the study were video-recorded. Two main findings from the study were that: i) teachers focused more on procedural knowledge than conceptual knowledge and engaged students in domain-specific knowledge practice in about a third of the phases of a typical lesson. Of the domain-specific knowledge practices, knowledge representation was emphasized. Also, procedural learning support was evident as teachers often helped with the "how to do" steps; ii) students were engaged in doing performative tasks (77.3%) more often than knowledge building tasks (22.7%). A performative task mainly entails the use of lower order thinking skills such as recall, comprehension and application of knowledge while a knowledge building task calls for higher order thinking skills such as synthesis, evaluation and creation of knowledge.

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Kaur (2009) in her study of grade eight competent mathematics teachers found that lessons were deemed good by students and teachers when they had the following characteristics: i) whole-class demonstration (exposition) where the teacher explained clearly the concepts and steps of procedures; made complex knowledge easily assimilated through demonstrations, use of manipulatives, real life examples and introduced new knowledge, ii) seatwork and out of class assignments where teacher gave clear instructions related to mathematical activities for in class and after class work; provided interesting activities for students to work on individually or in small groups; provided sufficient practice tasks for preparation towards examinations, and iii) review and feedback – where teacher reviewed past knowledge, and used student work or group presentations to give feedback to individuals or the whole class.

The findings of both Hogan et al. (2013) and Kaur (2009) indicate that there appears to be an apparent focus on the development of skills in mathematics lessons in Singapore schools. These findings certainly do not explain the stellar performance of Singapore students in PISA 2012 and 2015 that required students to complete tasks that were of the knowledge building type (OECD, 2014, 2016). As noted by Fan and Bokhove (2014), perhaps mathematical algorithms lead to proficiency of skills thereby stimulating thoughts about the conceptual aspects of the mathematics explored.

At present a study of the enacted school mathematics curriculum (secondary schools) is underway in Singapore. It attempts to document the practices of 30 competent mathematics teachers. The study aims to examine i) pedagogies adopted by competent teachers when enacting the curriculum, and ii) competent teachers' use of instructional materials for the enactment of the curriculum. Amongst others, one of the research questions explored is "How does the pedagogy of the teachers compare with that of mathematically powerful classrooms advocated by Schoenfeld (2011)?" To explore this question an appropriate analytical framework, comprising five parts, is being developed by the researchers of the study. This paper is based on one part of the framework which is used to examine the mathematics in the mathematics lessons of competent teachers in the study.

MATHEMATICALLY POWERFUL CLASSROOMS

The three decades of extensive research by Schoenfeld in the US on mathematical problem solving and mathematics instruction (2011) affirms that people's moment to moment decision making in teaching can be modelled as a function of their i) resources (esp. knowledge); orientations (esp. beliefs) and goals. He advocates that the five dimensions of mathematically powerful classrooms are: i)The mathematics context; ii) Cognitive demand; iii)Access to mathematical content; iv)Agency, Authority and Identity; and v)Uses of assessment. The Teaching for Robust Understanding framework proposed by Schoenfeld, Floden, and the Algebra teaching Study and Mathematics Assessment Project (2014) provides a tool for teacher learning and growth, according to the five dimensions of mathematically powerful

classrooms, with regards to student learning of mathematics. Figure 1, provides a general top-level description of the Teaching for Robust Understanding (TRU) framework Schoenfeld, 2016, p. 10). In our study reported in this paper, we use the TRU framework instead to examine two dimensions, namely the mathematics and cognitive demand, in mathematics lessons of two competent teachers in Singapore.

The Five Dimensions of Mathematically Powerful Classrooms	
The Mathematics	The extent to which the mathematics discussed is focussed and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained.
Cognitive Demand	The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development.
Access to Mathematical Content	The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class.
Agency, Authority, and Identity	The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency and authority resulting in positive identities as doers of mathematics.
Formative Assessment	The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Figure 1. The five dimensions of mathematically powerful classrooms

METHODOLOGY

The analytical framework we created for the dimension: The Mathematics was guided by i) the respective prompts for teacher thought and discussion in the TRU guide (Schoenfeld, Floden, and the Algebra Teaching Study and Mathematics Assessment Project, 2014), and also the field notes from the project – A study of the enacted school mathematics curriculum.

Four researchers involved in the study of the enacted school mathematics curriculum, contributed towards the crafting of the indicators guided by the prompts from the TRU framework. Figure 2 shows the analytical lens that was created to examine lessons of competent mathematics teachers in Singapore for the dimension – The Mathematics. The analytical lens crafted was used to examine the lessons of two teachers. These teachers were "experienced and competent", where experience is a measure of the number of years they have taught mathematics in secondary schools and competency is a composite measure of their students' performance at examinations and their performance in class in the eyes of their students. The teachers were nominated by their respective school leaders and the research team followed up on the nominations and interviewed the teachers. A strict requirement for participation in the study was that the teacher had to teach the way she / he did all the time, i.e. no special preparation was allowed. The lessons of these two teachers were selected, as they are both lead teachers and they also taught the same topic. Teacher 1

[T1] is a male who has taught mathematics for the last 20 years and Teacher 2 [T2] is a female teacher who has also taught mathematics for the last 20 years. For both teachers sequences of their lessons were recorded according to the protocol developed for the Learner's Perspective Study in Singapore (Kaur, 2009).

Dimension 1 – The Mathematics	What we looked out for in the lessons?
Aspect	Indicators
Were the mathematical goals of the lesson apparent?	Did the teacher articulate the goal/s of the lesson? Did the teacher articulate the goal/s of the mathematics students worked on during the lesson? Did the teacher articulate the goal/s of the mathematics students were assigned to do after the lesson during out of class time?
Were important ideas in the lesson connected with those in past and future lessons?	Did the teacher connect the important idea/s in the lesson to what students already know? Did the teacher relate concepts to each other — not just in a single lesson, but also across lessons and units in past and future?
How were math procedures in the lesson justified and connected with important ideas?	How did the teacher develop mathematical knowledge in the class? (Telling and showing / developing concepts through student activities / through systematic logical steps) Did the teacher identify the important ideas behind concepts and procedures? Did the teacher highlight connections between skills and concepts?
Were students engaged with mathematical ideas during lessons?	Did the teacher get the students to participate in meaningful math learning, so that they could make sense of concepts and ideas for themselves during lessons? Did the teacher get the students to participate in meaningful math learning, so that they could make sense of concepts and ideas for themselves as part of their out of class work after lessons? Did the teacher engage the students in authentic performances of important disciplinary practices (e.g., reasoning from evidence, communicating one's thinking, clarifying doubts, etc.) Did the teacher invite the students to explain things, or just give answers?

Figure 2. Analytical lens for the dimension – The Mathematics

Altogether two, the first in the sequence of lessons of the two teachers were coded. Both teachers were teaching the same topic – Vectors and they covered the same content during their first lesson. As part of the science curriculum, students had knowledge of vectors as this topic had been taught to them by their Science teachers during Physics lessons. The mathematical ability of students in the class of T1 was slightly below average as they were from the 40th percentile of their cohort and those in the class of T2 were from the 50th percentile of their cohort.

The coding was done in the following manner. Two researchers viewed the video-records of the lessons. They first segmented a lesson into episodes. An episode was

delineated by the beginning and end of an activity, for e.g. it may comprise the teacher beginning the lesson and telling the class about the day's lesson, or the beginning of an activity that had a specific goal such as engaging students in recall of past knowledge. Next they scanned one episode at a time for indicators of the dimension and recorded its presence. The inter-rater agreement was 83%. When a disagreement arose, the two researchers discussed their differences and arrived at consensus, either agreeing on the presence of the indicator or dismissing it.

The following show a few of the indicators with sample excerpts from the lessons.

Were the mathematical goals of the lesson apparent?

Did the teacher articulate the goal/s of the lesson?

T2 – Episode 1: (2:50) we will see what are vectors, how do we represent vectors on a diagram, how do we find magnitude, add/subtract vectors, and the use of vectors.

Did the teacher articulate the goal/s of the mathematics students worked on during the lesson?

T1 – Episode 11: 20:18) I'd like to test your understanding now….(25:16) The reason why I'm giving you this task is …

Did the teacher articulate the goal/s of the mathematics students were assigned to do after the lesson during out of class time?

T2- Episode 11: 52:38) I want you to do some thinking on your own. You need to understand what you've just learnt.(53:33) Why do I give you part a and part b? Are they the same? (54:05) You will show me the answers tomorrow. And then tomorrow we will do addition of vectors.

Were important ideas in the lesson connected with those in past and future lessons?

Did the teacher connect the important idea/s in the lesson to what students already know?

T2-Episode 2: (05:05) How do you represent your vectors when you do Science?

Did the teacher relate concepts to each other — not just in a single lesson, but also across lessons and units in past and future?

T1-Episode 2: 42:21) Many quantities have only magnitude… you are all familiar with that in the primary school. When you come to secondary school, you started learning in physics, … These are the various quantities that you are familiar with.

How were math procedures in the lesson justified and connected with important ideas?

How did the teacher develop mathematical knowledge in the class?

T1-Episode 7: (08:37) Now, what did you observe about these four vectors? How are they different and how are they the same? (09:14) What other observations did you observe? (10:08) What do you notice about OA and OC?

T2-Epiosde 4: (15:00) If your vector is not represented by a column vector, then how do you find the magnitude? … And you will use all kinds of knowledge that you have to find length. (16:49) Look at the diagram and ask yourself, what do you know? What are the concepts, what are the skills your already have? What can you use to find …

Table 1, shows the number of episodes in which the respective indicators were present in the lessons of the two teachers.

Dimension 1 – The Mathematics	Teacher 1 (18 episodes) (68 minutes)	Teacher 2 (12 episodes) (52 minutes)
Did the teacher articulate the goal/s of the lesson? Did the teacher articulate the goal/s of the mathematics students worked on during the lesson? Did the teacher articulate the goal/s of the mathematics students were assigned to do after the lesson during out of class time?	8 (2+7+1)*	6 (1+3+2)
Did the teacher connect the important idea/s in the lesson to what students already know? Did the teacher relate concepts to each other — not just in a single lesson, but also across lessons and units in past and future?	5 (3+4)	5 (3+7)
How did the teacher develop mathematical knowledge in the class? (Telling and showing / developing concepts through student activities / through systematic logical steps) Did the teacher identify the important ideas behind concepts and procedures? Did the teacher highlight connections between skills and concepts?	6 (10+7+5)	5 (8+5+4)
Did the teacher get the students to participate in meaningful math learning, so that they could make sense of concepts and ideas for themselves during lessons? Did the teacher get the students to participate in meaningful math learning, so that they could make sense of concepts and ideas for themselves as part of their out of class work after lessons? Did the teacher engage the students in authentic performances of important disciplinary practices (e.g., reasoning from evidence, communicating one's thinking, clarifying doubts, etc.)? Did the teacher invite the students to explain things, or just give answers?	5 (5+1)**	7 (7+1)

Note: in some episodes, more than one aspect was present. Also in some episodes more than one indicators of an aspect was present. * (?+?+?) shows the number of times the respective indicators in an aspect were present. ** (?+?) represents the number of episodes for the first two indicators.

Table 1: Number of episodes where the respective indicators were apparent

FINDINGS AND CHALLENGES

From Table 1, it is apparent that for all the four aspects of the dimension – the Mathematics the indicators crafted by the researchers were apparent in the episodes of the lessons of the two teachers, though with varying density. We found the indicators suitable but may not be comprehensive as they were only trialled with two

lessons. Therefore they have to be trialled more extensively. In addition, we also found that specifically for the indicators:

- How did the teacher develop mathematical knowledge in the class?
- Did the teacher engage the students in authentic performances of important disciplinary practices (e.g., reasoning from evidence, communicating one's thinking, clarifying doubts, etc.)
- Did the teacher invite the students to explain things, or just give answers?

we needed sub-codes to capture the range of approaches used by the teachers. Some of these approaches may be unique to the pedagogy of mathematics learning in Singapore. Furthermore in trying to rate the lessons according to the rubric shown in Figure 3 taken from Schoenfeld (2011) we felt that the level of both lessons may be rated as high but a more fine grained rubric may be needed to differentiate between lessons at this level for our research project.

Dimension	Level	
The Mathematics How accurate, coherent, and well justified is the mathematical content?	Low	Classroom activities are unfocussed or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.
	Medium	Classroom activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices such as reasoning and problem solving.
	High	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices such as reasoning and problem solving.

Figure 3. Summary Rubric of Dimension 1 – The Mathematics

Also, to make any valid and rigorous claims, we feel that we have to interrogate our data from the perspective of the students and answer the following questions which were presented by Schoenfeld (2016) during his plenary lecture at PME 40 in Szeged. The questions are:

Dimension 1 – The Mathematics

- What's the big idea in this lesson?
- How does it connect to what I already know?

Dimension 2 – Cognitive Demand

- How long am I given to think, and to make sense of things?
- What happens when I am stuck?
- Am I invited to explain things, or just give answers?

References

Chang, A.S.C., Kaur, B., Koay, P.L. & Lee, N.H. (2001). An exploratory analysis of current pedagogical practices in primary mathematics classrooms. The NIE Researcher, 1(2), 7-8.

Hogan, D., Towndrow, P., Chan, M., Kwek. D. & Rahim, R.A. (2013). CRPP Core 2 research program: Core 2 interim final report. Singapore: National Institute of Education.

Fan, L. & Bokhove, C. (2014). Rethinking the role of algorithms in school mathematics: A conceptual model with focus on cognitive development. ZDM – Mathematics Education, 46, 481-492.

Kaur, B. (2009). Characteristics of good mathematics teaching in Singapore grade 8 classrooms: A juxtaposition of teachers' practice and students' perception. ZDM Mathematics Education, 41, 333-347.

Kaur, B. & Yap, S.F. (1997). Kassel project (NIE-Exeter Joint Study). Second Phase (October 95-June 96). Singapore: National Institute of Education.

Kaur, B. & Low, H.K. (2009). Student perspective on effective mathematics pedagogy: Stimulated recall approach study. Singapore: National Institute of Education.

Hattie, J. (2012). Visible learning for teachers: Maximising impact on learning. London: Routledge.

OECD. (2014). PISA 2012 Results: What students know and can do (Volume 1 – Revised Edition). Paris: OECD Publishing.

OECD. (2016). PISA 2015 Results (Volume 1): Excellence and equity in education. Paris: OECD Publishing.

Schoenfeld, A.H. (2011). How we think. Routledge.

Schoenfeld. A.H. (2016). Solving the problem of powerful instruction. In C. Csikos, A. Rausch, & J. Szitanyi (Eds.), Proceedings of the 40th Conference of the international Group for the Psychology of Mathematics Education, Vol. 1, pp. 3-18. Szeged, Hungary: PME.

Schoenfeld, A.H., Floden, R.E., & the Algebra Teaching Study and Mathematics Assessment Project (2014). An introduction to the TRU Math Dimensions. Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California, Berkeley & College of Education, Michigan State University. Retrieved from: http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/pd.php.