Incorporating Partially Completed Worked Examples With Scaffolding Instructions in a Calculus Course to Facilitate Student Learning: To What Effect?

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The teaching of calculus in undergraduate courses has its share of curricular and pedagogical challenges. Suggestions on how to teach it well abound. To facilitate the learning of calculus of students enrolled on teacher education programmes, the authors devised an instructional strategy called guided-exercise in the form of a series of partially completed worked examples with instructional scaffoldings which the students used alongside the lectures, with assistance from the lecturers when necessary. This paper describes the rationale, design and use of these guided-exercises as an instructional strategy to facilitate and enrich student learning. It reports the students' perception on the effect the strategy has on their learning. Implications are discussed also.

Keywords: scaffolding, worked example, guided exercise, undergraduate mathematics, calculus

Introduction

How can we teach calculus better? The teaching of calculus in undergraduate courses continues to face numerous curricular and pedagogical challenges. In this paper, we report on an instructional strategy which we devised with the aim of helping students enrolled on teacher education programmes to learn undergraduate calculus. These prospective mathematics teachers have successfully completed and have passed Mathematics in the Singapore-Cambridge GCE A-Level Examinations.

Calculus II is one of the core modules for Year 2 student teachers (hence forth referred to as students) who are pursuing a Bachelor of Science (Education) or a Bachelor of Arts (Education) Programme to teach Mathematics in Singapore schools. It is a one-semester 12-week course comprising four topics : (I) Sequences, (II) Infinite Series, (III) Partial Derivatives and (IV) Multiple Integrals. Calculus II is delivered via two 1-hour lecture periods and one 1-hour tutorial period per week for 12 weeks. In Year 1, the students took Calculus I, which covered the topics: Functions and Graphs, Limits and Continuity, Differentiation and Its Applications, and Integrations and Its Applications.

Given that there is only one hour tutorial time for Calculus II per week, it was not always possible to carry out extensive scaffolding instructions to engage students in the conceptual discourse and help them consolidate and deepen their understanding of the concept taught in a typical tutorial session. A study by Lim, Ahuja and Lee (1999) in Singapore revealed that our pre-university students generally had a positive attitude towards learning mathematics, albeit a little too reliant on memorizing formulae or mathematical procedures in their approach to learning the subject. Their study also revealed the difficulties a number of our student teachers had in keeping up with the mathematical demands of the undergraduate mathematics courses. Indeed, over the years of teaching the same course, the authors also noticed an increasing number of Calculus II students showed difficulties and limited mathematical cognitive resources or motivation to solve tutorial problems. In view of these students' weakness in mathematics in general and in a topic like Calculus in particular, our latest pedagogical innovation is an instructional strategy which we called 'guided-exercise'. These guided-exercises consist of a few partially worked-out examples incorporating conceptual and heuristic scaffoldings with the aim of guiding students to solve the problems independently, thereby strengthening their conceptual and procedural knowledge that they have learnt during the lectures. Students were strongly advised to attempt these questions before they solved the weekly tutorial questions which the students are expected to present in class.

Literature Review

We have deliberately been eclectic in our search for ways to teach better. To this end, we have borrowed ideas from the field of educational psychology. Cognitive load theory, as reported in Sweller (1988, 2010) and van Gog, Paas and Sweller (2010) suggest that integrating worked examples in a learning environment or as an instructional technique reduces the cognitive load on the working memory of the learners (especially for the novices), and allows them to focus their limited available capacity on studying the solution process. As a result, effective learning takes place.

The use of fully worked-out examples to teach mathematical concepts and procedures has been the predominant instructional technique in mathematics classrooms. Worked examples are also ubiquitous in many mathematics textbooks. There is abundant empirical evidence showing that learning from worked examples is more effective than learning from solving problems alone (Witter & Renkl, 2010). However, there are empirical evidence to suggest that learning from complete worked examples may not be as effective as learning from incomplete worked examples in which "blanks" are inserted in the solution steps of the examples (Renkl, Atkinson, Maier, & Staley, 2002). These "blanks" would force the learners to determine the next solution step on their own, helping them to acquire cognitive skills. Atkinson and Renkl (2007) concluded that "interactive" elements in the form of gaps (or blanks), prompts, and help on demand (in the case of computer-based learning environment), incorporated in worked examples can foster learning, in particular for learners with weak prior knowledge.

Apart from using complete or incomplete worked examples, mathematics educators also use scaffolding instructions to facilitate and guide students' in their learning. The concept of scaffolding as a teaching strategy originates from Vygotsky's (1978) socio-cultural theory and his idea of the zone of proximal development (ZPD). Student learning is posited to occur within the ZPD when the learner aided or scaffold by a "more knowledgeable" person. There are studies advocating the positive effect of scaffolding and recommending it use in instruction at different academic levels. For example, Johnson and Koedinger (2005) concluded from their study that scaffolding conceptual, contextual and procedural knowledge are promising tools for improving student learning while Valkenburge (2010) opined that scaffolding when used during tutorial sessions is a powerful tool for helping students to actively engage in their work and in promoting self-sufficiency. Witter and Renkl (2010), in a meta-analytic review of 21 experimental studies on instructional explanations in example-based learning, found that there was corroborating evidence to conclude that the instructional explanations had pronounced impact on learning.

Studies on using scaffolding instruction in partially worked-out examples have not been reported in Singapore. Hilbert, Renkl, Kessler and Reiss (2008) described the use of "heuristic examples"—a specially crafted worked-out examples with instructional support in the form of self-explanation prompts—to teach proofs in geometry. Their study showed that learning with heuristic examples led to an improvement in students' ability to construct proofs and their understanding of proofs.

In our search for better instructional strategies to undergraduate calculus, we think it is valuable and novel to look at how scaffolding can be used with incomplete worked examples, which we called 'guided-exercises' to improve students' learning and to meet our instructional goal which is to equip our students with the most important concept(s) in each chapter and to enable them to link procedural knowledge with conceptual knowledge so that there is relational understanding (Skemp, 1976).

Design of the Guided-Exercise

In the building industry, scaffoldings are necessary and essential temporary physical structures that provide workers support to access and build another storey of a building. In the education setting, analogously speaking, the construction of knowledge and understanding by a learner requires cognitive scaffoldings to enable the learner to overcome hurdles to their understanding and knowledge acquisition. The idea of instructional scaffoldings has a similar function—for the teacher or knowledgeable others to provide support structures so that novice learners may advance to a higher level of learning.

According to Hartman (2002), the goal of scaffolding teaching strategies such as using models, cues, prompts, hints, partial solutions and think-aloud modelling is for students to become an independent and self-regulating learners and problem solvers. Holton and Clark (2006) argue that scaffolding students' learning does not require the educators to be face-to-face with the students in a classroom setting. It can be done in a form of specially prepared teaching instructions provided in written form and other means. They identify two types of scaffolding: one being the conceptual scaffolding which aims at promoting conceptual development of the learners and the other being the heuristic scaffolding which develops the skills of using heuristics for learning and problem solving.

Atkinson, Derry, Renkl, and Wortham, (2000) suggested that for worked examples to be instructionally effective, teachers have to look into three factors: (a) intra-example feature: how examples are designed and their solutions are presented (b) inter–example features: how multiple examples are sequenced and related to practice problems (in our case, tutorial questions) (c) how example-processing is done in the form of "self-explaining" by individual student.

Bearing in mind these factors, the need to scaffold students' learning in our intention of reducing students' cognitive load, and eliciting students' thoughts and explanations in order to achieve our ultimate goal of enabling students to acquire both the conceptual and procedural understanding, we designed the guided-exercise with the following considerations : (I) using instructional explanations; (II) crafting appropriate questions to engage students; and (III) incorporating controlled variability in the partial worked-examples. The focus of this study is not to test the cognitive load theory; rather it is to use the theory to inform the design of the guided exercises.

(I) Instructional explanations

Giving appropriate amount of instructional guidance and explanations is one aspect of the intra-example features that we need to deal with. We believe that students must involve or get inducted in solving the problem in order to make progress in their learning. Once a student is able to get started solving a problem, other scaffolding steps become operative which would help the student make further progress in working out the solution. In this respect, initial instructional explanations can help bridge the gaps in students' cognitive understanding and knowledge of the subject. Indeed, van Gog, Paas, and van Merrienboer (2004) argue that worked examples alone are not effective in supporting students' acquisition of meaningful knowledge if process-oriented information such as the rationale of the solution steps and heuristics are not provided. So a delicate balance between the extent of the solution steps to be given and how much for them to think and write about is one of our major considerations when we prepared the guided-exercise. For example, Figure 1 shows a problem in the first set of the guided-exercise, heuristic scaffoldings in the form of questions such as "can you find an easy way to sum the terms in the brackets?", "What is your next consideration?" Instructional explanations and cognitive scaffoldings such as "To use the theorem, you need to establish inequalities. How would you begin?" and conceptual scaffolding such as "Thus $\frac{1}{(n+1)^3} \geq \frac{1}{(n+r)^3} \geq \frac{1}{(2n)^3}$ for r = 1, 2, 3, …, n $\Rightarrow$ $\sum_{r=1}^{n}\frac{1}{(n+1)^3} \geq \sum_{r=1}^{n}\frac{1}{(n+r)^3} \geq \sum_{r=1}^{n}\frac{1}{(2n)^3}$ " is shown in Figure 1.

> 2. Find $\lim_{n\to\infty}\left(\frac{1}{(n+1)^3}+\frac{1}{(n+2)^3}+\dots+\frac{1}{(2n)^3}\right)$.
>
> Suggested solution
>
> Can you find an easy way to sum the series in the brackets ? __________ .
>
> If not, what is your next consideration ? ______________(One theorem you have learnt)
>
> > Can you find $\lim_{n\to\infty}\left(\frac{1}{(n+1)^3}+\frac{1}{(n+2)^3}\right)$? ________ .
> >
> > If there are infinite number of terms in the brackets and each term tends to zero, can we conclude that the sum of all these terms is still zero ? Why ? ______________
>
> To use the theorem, we need to establish inequalities. How would you begin ? __________
>
> Observe that $(n+1)^3 \leq (n+r)^3 \leq (n+n)^3$ if r = 1, 2, 3, …, n.
>
> Thus $\frac{1}{(n+1)^3} \geq \frac{1}{(n+r)^3} \geq \frac{1}{(2n)^3}$ for r = 1, 2, 3, …, n $\Rightarrow \sum_{r=1}^{n}\frac{1}{(n+1)^3} \geq \sum_{r=1}^{n}\frac{1}{(n+r)^3} \geq \sum_{r=1}^{n}\frac{1}{(2n)^3}$
>
> That is (please evaluate the summations) ______________________________
>
> [Note : summation is over r and NOT n. What does this mean ? ________________]
>
> Since $\lim_{n\to\infty}\frac{n}{(2n)^3} = \lim_{n\to\infty}\frac{1}{8n^2} = 0$ and $\lim_{n\to\infty}\frac{n}{(n+1)^3} =$ ____ [Why ? $\lim_{n\to\infty}\frac{1}{n^2+3n+3+\frac{1}{n}} =$ ____
>
> By __________ Theorem , $\lim_{n\to\infty}\left(\frac{1}{(n+1)^3}+\frac{1}{(n+2)^3}+\dots+\frac{1}{(2n)^3}\right) =$

Figure 1. An incomplete worked-example with instructional scaffoldings.

We also encourage students to do "self-explaining" and reflection by asking questions to elicit their reasons and explanations of what they are doing. As far as possible, "self-explaining" feature was incorporated into the guided-exercise to help students to monitor their procedural and conceptual thinking. From past year teaching experience, we noticed that students had a tendency to compute $\lim_{n\to\infty}\left(\frac{1}{(n+1)^3}+\frac{1}{(n+2)^3}+\dots+\frac{1}{(2n)^3}\right)$ by taking limit term by term to conclude that the answer is zero. So, we deliberately ask the students to find $\lim_{n\to\infty}\left(\frac{1}{(n+1)^3}+\frac{1}{(n+2)^3}\right)$ which consists of only 2 terms and both terms tend to zero when n tends to infinity. We direct their cognitive focus on the concept of taking limits of infinite number of terms—to make them think and explain why it does not make sense to add an infinite number of very small terms. So, in our guided-exercise worksheets, we insert blanks for students to fill in so as to channel students' cognitive domain into a reflection and self-explaining mode.

(II) Sequencing of questions

In order to design the guided-exercise, we need to understand the learning difficulties faced by students and hence scaffold their learning by way of a gradual increase in difficulty in sequencing of the questions. In addition, the choice of the partially worked-out examples in our guided-exercises should address the difficulties and misconceptions students may encounter and increase their mathematical self-efficacy in doing the weekly tutorial problems, independently without any form of guidance. In Calculus, students require both the mathematical procedural ability and conceptual understanding and knowledge to solve problems successfully. In Figure 2, we deliberately arranged the task of finding $\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^x$ to be the first problem for students.

1. It has been proved in the lecture that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$ or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$, find $\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^x$.

Possible solution: Observe that $1+\frac{1}{x}$ is actually $\frac{x+1}{x}$.

How would you make $\frac{x+2}{x+1}$ similar to the above, a constant plus a fractional term?

____________________.

$\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^x=\lim_{x\to\infty}\left(1+\frac{(\ )}{x+1}\right)^x$

$=\lim_{x\to\infty}\left\{\left(1+\frac{1}{x+1}\right)^{(\ \ )}\left(1+\frac{1}{x+1}\right)^{(\ )}\right\}$

$=$

$=e$ [Please show the necessary steps to arrive at this answer]

Is the indicial term same as the denominator ? ________

Can you apply the result $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$? Why ?

____________________________________.

What must you do ?

Let $y=x+1$ so as $x\to\infty$, $y\to(\ \ )$.

What do you observe now ? ________________

Figure 2. Solution steps with prompts in Guided Exercise 3.

This was to emphasise the importance of applying and highlighting the "beauty" of the result $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$ which the lecturer would have had explained and proved during the lecture. Instructional support such as "How would you make $\frac{x+2}{x+1}$ similar to the above?" was given so that students could apply the procedural knowledge of converting the fractional term into one that the given result could be applied. The sequencing of the solution steps and the questioning prompts for student self-explaining (e.g., "What do you observe now? ") have to be carefully crafted with reference to the anticipated students' learning difficulties.

(III) Adaptive variability

According to a study done by Pass and van Merrienboer (1994) on secondary school students learning Geometry, greater variations in the worked-examples given within a lesson supplemented with instructional guides benefited students more than those who worked on less-varied worked examples in terms of knowledge transferring. This is in agreement with Dienes' (1960) principle of variability in the learning of mathematics, which recommends varying the mathematical (also perceptual) details of the example while retaining some structural characteristics so that the learners may notice the essential similarity among the illustrative examples. This, to us is a form of scaffolding. As such, when we designed the guided-exercise, we deliberately created problems which looked slightly different. For example, in Figure 3, the region R in Question 1 is defined by a set of points based on the set builder notation whereas in Question 4, the region of the double integration is defined by the integration limits.

Not only did we craft different types of questions and sequence the problems according to the difficulty level as shown in Figure 3, we also encourage variability in solutions. For example, in the solutions to this problem : "Test the series $\sum_{n=1}^{\infty}\frac{1+\sin n}{n^2}$ for convergence" in the topic of Infinite Series, we start off by prompting the students to check the limit of the sequence $\left\{\frac{1+\sin n}{n^2}\right\}$ to ensure the students understand why they need to proceed with other tests. We produce two partially worked-out solutions, one using the Limit Comparison Test with $a_n = \frac{1+\sin n}{n^2}$ and $b_n = \frac{1}{n^2}$. Students are reminded to explain why it does not work. Next, we put in a heuristic scaffolding to guide the students choosing a suitable value of p in $b_n = \frac{1}{n^p}$ so that the Limit Comparison Test shows the series is convergent. In addition, we want the students to think of any alternative solution or shorter solution before we suggest using the inequality $-1 \le \sin n \le 1$ to arrive at the answer based on the Squeeze Theorem and the Comparison Test with the convergent series $\sum_{n=1}^{\infty}\frac{1}{n^2}$. This problem in fact engaged some students in thinking why the use of $b_n = \frac{1}{n^2}$ in the Limit Comparison Test and Comparison Test produces different outcomes.

1. Evaluate the integral $\iint_R x\cos xy \, dA$ where $R = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le \pi/2,\ 0 \le y \le 1\}$

Solution

First of all, you need to identify the region represented by the set R. It is a ____________.

So the given double integral $\iint_R x\cos xy \, dA$ is actually the same as $\int_{y=} \int_{x=}^{\frac{\pi}{2}} x\cos xy \, dxdy$.

[Note: write down the correct limits of integration - the inner integral is with respect to x]

That is $\iint_R x\cos xy \, dA =$ ______________________________

Take a look closely, $\iint_R x\cos xy \, dA = \int_{y=0}^{1} \int_{x=0}^{\frac{\pi}{2}} x\cos xy \, dxdy$ requires integration by parts.

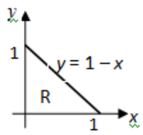
So we prefer $\iint_R x\cos xy \, dA =$

4. Use the transformations $u = x + y$ and $v = y - 2x$ to evaluate the integral $\int_0^1 \int_0^{1-x} (y-2x)^2 \sqrt{x+y} \, dydx$.

Solution: As it is, can you find the inner integral easily ? So what must you do ? _______

Yes, find out what the region of integration is. Integration limits for y is from 0 to _______ and limits for x is from 0 to ___.

Draw the region R which is a ______. (Pause here, draw the region first before you continue)



What must you do if you want to transform the region R onto a u-v plane ?

____

Let us solve x and y in terms of u and v (using $u = x + y$ and $v = y - 2x$)

$x = \frac{1}{3}(u - v)$ and $y =$ ________ .

x-y equation for region R	**u-v equation for new region**	**Simplified u-v equation**
$x + y = 1$	________________	$u = 1$
$y = 0$	$\frac{1}{3}(2u + v) = 0$	______
$x = 0$	________________	$u = v$

Next, draw the transformed region on the u-v plane.

Figure 3. Different variations of multiple integrals.

Results

As it was our first attempt at using incomplete worked-example with instructional scaffolding, we surveyed our students to find out their reactions to the guided-exercises. The end-of-course instructional survey was administered to the student teachers during one of the revision lectures two weeks before the semestral examination. The survey questionnaire captured the student teachers' demographic data and their perception of the various aspects of the Calculus II programme, one of which is the guided-exercise.

The participants of this survey were the 127 Year Two student teachers (84 female and 43 male students) of which 36 were enrolled on the Bachelor of Science (Primary Education), 79 on the Bachelor of Science (Secondary Education) and 11 on the Bachelor of Arts (Education). In the survey questionnaire, one of the areas we were looking at was whether this new initiative in our instructional programme was useful for students. The survey consists of 4-point Likert type questions (1 = Strongly Disagree and 4 = Strongly Agree).

Table 1 shows the responses from the 101 participants, which is 80% of the cohort. With regards to whether the guided exercises were useful for their learning, about 90%

of the students responded with Agree or Strongly Agree as indicated by items Q3A1 and Q3A2. In addition, it is not surprising to see that students who did better in the end-of-exam tended to respond favourably to the guided exercises as we found that there were significant positive correlation between the Calculus II results and the responses to Item Q3A1, Q3A2 and Q3A3 (p values are .029, .043 and .002 respectively). One undesirable observation was that the mean score for Item Q3A3 was quite low (mean 2.44) – due to only about 40% of the student teachers did the guided-exercises before their tutorial sessions. We believe that students' lacking self-discipline and poor time management may be the reasons contributing to this disappointing percentage despite the benefits perceived by the students. The implication is that our students need to be closely monitored and tutors may have to make the guided-exercise a compulsory task for our future batch of students.

Table 1: Means and Standard Deviations of Students' Survey Response

Item		1(SD)	2 (D)	3 (A)	4 (SA)	Mean (SD)
Q3A1	The guided exercise helps me to understand the topic better	1.0%	10.0%	52.5%	36.5%	3.25 (0.669)
Q3A2	The guided exercise helps me to reinforce the concepts	1.0%	7.9%	53.5%	37.6%	3.28 (0.650)
Q3A3	I usually do my guided exercises before the tutorials	14.9%	46.5%	24.7%	13.9%	2.44 (0.899)

The survey instrument included a free response section for students to comment on the guided exercise. Examples of their feedback on our guided-exercise are shown below.

- Useful to build foundation
- Useful for understanding concepts
- Very good
- Guided exercises gave solutions
- good guidance
- Guided exercises emphasized the important concepts required for the topics
- I practiced most of the questions in the guided exercises. It helped me to understand the concepts better

In particular, Student A commented that "guided exercises were good and the step-by-step promptings help me to go through the thinking process even though there were questions I didn't know how to do …. . It boosts confidence as the questions were more manageable compared to tutorials". Another student B said that "guided exercises were indeed useful in terms of helping me to consolidate my revisions". It is worth noting that Student A and B were among the weakest (in terms of Calculus I grade), these students showed great improvement in Calculus II result – an improvement of 4 grade points. That they were most positive about the guided exercise may indicate the usefulness of the exercise for them. It appears that the guided-exercises have benefitted at least two weaker students.

Conclusions

This paper describes our rationale, design and use of the guided-exercises as an instructional strategy to scaffold and enrich student learning. We believe that our guided-exercise worksheets in which incomplete worked-out examples were specially crafted with 'interactive blanks or prompts' would reduce students' cognitive load and misconception. It is our objective that after working through the guided-exercises, students would gain confidence and improve their conceptual and procedural understanding and mathematics self-efficacy which would better prepare them attempting the tutorial questions which are usually non-structured and non-guided. However, one limitation of our exploratory study is that we were not able to create controlled and experimental groups or conduct a pre and post test to investigate how effective this particular instructional initiative is. Nevertheless, the data from our preliminary study do give us an indication that the guided-exercise could supplement and enhance students' learning in the Calculus course. This study has given us enough positive students' feedback to encourage us adopting the guided-exercise for our next batch of students. It certainly gives us an impetus to carry out future research on how the guided-exercise approach can be further improved to enhance students' learning.

References

Atkinson, R. K. & Renkl, A. (2007). Interactive example-based learning environments: Using interactive elements to encourage effective processing of worked examples. *Educational Psychology Review*, *19*, 375–386.

Atkinson, R. K., Derry, S. J., Renkl, A., & Wortham, D. (2000). Learning from examples: Instructional principles from the worked examples research. *Review of Educational Research*, *70*(2), 181–214.

Dienes, Z. P. (1960). *Building up mathematics*. London: Hutchinson Educational Ltd.

Hartman, H. (2002). Scaffolding & cooperative learning. *Human Learning and Interaction.* 23–69, New York : City College of City University of New York.

Hilbert, T. S., Renkl, A., Kessler, S., & Reiss, K. (2008). Learning to prove in geometry: Learning from heuristic examples and how it can be supported. *Learning and Instruction*, *18*, 54–65.

Holton, D. & Clark, D. (2006). Scaffolding and metacognition. *International Journal of Mathematical Science and Technology*, *37(2)*, 127–143.

Johnson, R. B. & Koedinger, K. R. (2005). Designing knowledge scaffolds to support mathematical problem solving. *Cognition And Instruction*, *23*(3), 313–349.

Lim, S. K., Ahuja, O. P., & Lee, P. Y. (1999). Tertiary and junior college students' attitude towards mathematics. *The Mathematics Educator, 4*(1), 15–33.

Paas, F., & Van Merrienboer, J. (1994). Variability of worked examples and transfer of geometrical problem-solving skills: A cognitive-load approach. *Journal of Educational Psychology, 86*, 122–133.

Renkl, A., Atkinson, R. K., Maier, U. H., & Staley, R. (2002). From example study to problem solving : smooth transitions help learning. *Journal of Experimental Education*, *70*, 293–315.

Skemp, R.R. (1976). Relational understanding and instrumental understanding. *Mathematics Teacher*, *77*, 20–26.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12,* 257–285.

Sweller, J. (2010). Element interactivity and intrinsic, extraneous and germane cognitive load. *Educational Psychology Review*, *22*, 123–138.

van Gog, T., Paas, F., & van Merrienboer, J. J. G. (2004). Process-oriented worked examples: Improving transfer performance through enhanced understanding. *Instructional Science*, *32*, 83–98.

van Gog, T., Paas, F., & Sweller, J. (2010). Cognitive load theory: Advances in research on worked examples, animations, and cognitive load. *Educational Psychology Review*, *22*, 375–378.

Valkenburg, J. (2010). Scaffolding and tutoring mathematics. *Learning Assistance Review*, *15*(2), 34–41.

Vygotsky, L. S. (1978). *Mind and society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wittwer, J. & Renkl, A. (2010). How effective are instructional explanations in example-based learning? A meta-analytic review. *Educational Psychology Review*, *22*, 393–409.