Conceptual Understanding of Algebra in Singapore Classrooms Using Generative Activities: A GenSing Project

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ABSTRACT

This paper reports on a classroom-based design study into Generative Activities in Singapore (GenSing) which implemented an algebra reform curriculum based on the principals of generative design (Stroup, Ares, Hurford, & Lesh, 2007) and function-based algebra, using a classroom network. The generative function-based approach to teaching algebraic concepts has the potential to improve student understanding of the structural aspects of introductory algebraic concepts (Stroup, Carmona, & Davis, 2005). The research project is iteratively designing an Algebra pre/post-test to look at students' conceptual understanding of algebraic topics. A paired samples t-test showed a significant difference between pre-test and post-test results and the mean increased from the pre-test to the post-test on the multiple-choice assessment ($t = 6.41$, $df=280$, $p \leq 0.05$).

This paper reports on a continuing design study into Generative Activities in Singapore (GenSing). This classroom-based research to design and implement an algebra reform curriculum is based on the principals of generative design (Stroup, Ares, Hurford, & Lesh, 2007) and function-based algebra, facilitated by the use of a classroom network. Generative teaching as discussed by Wittrock, "is to teach relations among the concepts…by giving the structure and by getting students to generate associations and relations that elaborate that…structure, make it more precise, and extend it, always using their own words or images to show how…concepts are related to one another" (Wittrock, 1991, p. 182). Generative learning, in his model, takes places when students actively create their own artifact tying new ideas to existing knowledge structures. Function-based algebra uses the function as the organizing structure for the mathematics curriculum (Kaput, 1995). The generative function-based approach to teaching algebraic concepts has the potential to improve student understanding of the structural aspects of introductory algebraic concepts (Stroup, Carmona, & Davis, 2005). New networked technologies that harness the knowledge and ideas within the classroom, rather than outside the classroom, are becoming more widely available. These networks have the potential to increase student participation and engagement with the topics of instruction (Davis, 2002, 2003; Roschelle, Patton, & Tatar, 2007; Roschelle, Penuel, & Abrahamson, 2004). The combination of the three, generative design, function-based algebra and classroom networks allow for powerful new pedagogical practices to be created.

Generative Activities are activities where all of the data used for classroom discourse and instruction are generated by the students. These activities allow for a space to play with mathematics, increased participation and agency, and change dynamically as they are enacted in the classroom (Stroup, Ares, & Hurford, 2004; Stroup et al., 2002). Foundational to the work reported on in this paper is the theory that 80% of traditional algebra could be covered in a curriculum that focused on concepts of equivalence, equals and aspects of the linear function (Stroup et al., 2005).

Research Design

The multi-tiered design study (Kelly, Lesh, & Baek, 2008) focused on students' developing understanding of function-based algebra via generative activities in a networked classroom and the teachers' pedagogical practices needed to foster these activities. The data reported in this paper focuses on the students' data.

Participants

The data for this study was collected at an upper-performing secondary school in Singapore. The participants were all 2008 school year, Secondary 1 students at the research site. There were 281 students (12-13 years old). Data was collected from six sections of both normal and express-track classes. Class sizes ranged from 39-42 students.

Activities

In collaboration with the schools' curriculum specialist, the Sec 1 scheme of work (scope and sequence) was reorganized to gather all of the algebraic topics covered across the year into two cohesive groupings, one lasting eight weeks and one lasting four weeks. During the first eight-week segment, concepts of equivalence, rate, and linear functions were covered. During the second four-week segment, concepts of solving equations and inequalities were explored. Additionally, the topics of instruction were grouped into concepts (i.e. equivalence) and skills (i.e. factoring), where concepts were taught using generative activities and skills were taught via traditional instruction (Lesh & Doerr, 2003). Over the course of the twelve weeks of intervention, students experienced ten generative activities. Data for this paper was collected during the second year of a multi-year study.

In accordance with generative design principles, each activity in the intervention asked questions that opened up a space of possible correct answers. The class analysis and discussion of the set of answers generated by the students is the foundation of instruction. As an example, for the concept of equivalence, students explore different algebraic expressions that create the same graphical

representation. Given the expression 4X, students are asked to come up with three new expressions that create the same graph. The resulting display of over 100 equivalent expressions is then used to explore ideas of equivalence and the companion skills of simplification and expansion. For activities related to linear functions, we start with motion detectors. In exploring the richness of the messy graphs created by real motion, the students are able to separate the different elements of slope and intercept. This knowledge is then applied to the simple situation of the linear function. Finally, the activities on equals (and inequalities) work to tie the graphical concepts of above, below, and the same to greater-than, less-than and equals. In all of the activities, the questions and students' inputs are structured to allow for individual creativity and exploration of the mathematical topic.

Data sources

For the 2008 implementation, a pre- and post-test assessment was created. The pre-test was administered to all students in the week prior to the beginning of the eight-week intervention. The post-test was administered at the completion of the four-week implementation in the second semester. Finally, for each generative activity, two of the Sec 1 classes were video taped.

Assessment

The research project is iteratively designing an Algebra pre/post-test to look at students' conceptual understanding of algebraic topics, not just their computational ability to manipulate variables. In the first year of the project, there was no control group. For this reason, the assessment contained both project-created items and items from the National Assessment of Educational Progress from the United States, the Trends in International Mathematics and Science Study (TIMSS), and the 2005 Cambridge O-level exam. This gave us validated test items with which we could compare the Singapore results to an international sample of results. All of the Secondary 1 students at BPGHS took the pre- and post-assessment in GenSing1. Data was collected towards the end of the first term and the beginning of the fourth term of the school year from all classes for all activities.

A 20-item assessment was designed to assess student learning in the algebra activities. Ten multiple-choice items were taken from NAEP and TIMSS released items connected to the content covered in the activities. The assessment was administered in a pre- and post-test format to 281 students in seven sections. The remaining items were open-ended response questions. For the purposes of this analysis, we will analyze the multiple-choice (MC) and open-ended (OE) portions separately.

Results

For the 10 multiple-choice items, we analyzed the students' total scores giving them one-point per item answered correctly. The results of the MC assessment are summarized in the Table 1. A paired samples t-test showed a significant difference between pre-test and post-test results and the mean increased from the pre-test to the post-test on the multiple-choice assessment ($t = 6.41$, $df=280$, $p \leq 0.05$).

Table 1. Results for multiple choice pre-test and post-test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	MC Pre-test	7.19	281	2.287	.136
	MC Post-test	8.27	281	2.463	.147

A one-way ANOVA showed that there was a significant difference between sections (classes) on both the pre-test (F-value = 8.289, $df = 6$, $p \leq 0.05$) and the post-test (F-value = 15.569, $df=6$, $p \leq 0.05$).

For the open-ended questions, Students' performance was low on the pre-test and improved significantly on the post-test (questions 1 and 2 are exceptions as students' performance was initially high and had no

significant difference from pre to post). On the post-test, students showed some improvement in their ability to correctly graph functions and to explain their reasoning. Students also attempted the questions and were less likely to leave the open-ended questions blank.

Table 2: Analysis of Responses to Question 5

Code	Total Responses (Pre-test, N=279)	Total Responses (Post-test, N=281)	Example
Blank	238	168	
Colloquial (to work)	2	3	A function is the use of the thing.
Equation & algebra	10	22	A function is numbers and symbols which help form an equation.
Equation & graph	8	48	A function is an equation using the graph in form of X and Y.
Input/output Relationship	2	10	A function is a rule.
Line	2	2	A line that joins different points on the graph together.
Find next number in a pattern	1	1	To find the next number in a pattern.
Point	4	1	Function is a point where all the lines intersect through it.
Problem solving	2	2	Allows us to understand the problem better.
Process	2	0	Steps that make an answer.
Referred to answer to previous problem	6	2	It is to find the number of stages through the number of tiles.
Grand Total	279	281	

One of the open-ended questions without a quantitative component (question 5) asked students to define function and was not quantified. For the pre-test (N=279), the students largely either had no response (229 students) or responded incorrectly about the definition. For the post-test, the students could associate "function" with an equation, graphing, or with a relationship between x and y variables. In addition, fewer (168) students left the question blank in the pre-test.

Discussion

There is research that links lack of conceptual understanding with a tendency to focus on procedural aspects of functions (Stein, Baxter, & Leinhardt, 1990). This project is working to foster activities and pedagogical practices that focus on the larger concepts related to algebra, specifically equivalence, equals and the linear function. It is our belief that using Generative Activities and a classroom network facilitates the growth of conceptual understanding. Our early analysis of the 2008 school-year data shows some indication that the intervention is improving students' understanding of algebra and function. Further analysis will be required to analyze the fidelity of implementation and the resulting differences between classrooms. In addition, we will also need to examine more clearly the nature of students' function-based reasoning in algebra.

Davis, S. M. (2002). Research to industry: Four years of observations in classrooms using a network of handheld devices. In M. Milrad, H. U. Hoppe & Kinshuk (Eds.), *Proceedings of the IEEE International Workshop on Wireless and Mobile Technologies in Education* (pp. 31-38). Växjö, Sweden.

Davis, S. M. (2003). Observations in classrooms using a network of handheld devices. *Journal of Computer Assisted Learning, 19*(3), 298-307.

Kaput, J. J. (1995). *A research base supporting long term algebra reform?* Paper presented at the Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, OH.

Kelly, A. E., Lesh, R. A., & Baek, J. Y. (Eds.). (2008). *Handbook of Design Research Methods in Education*. New York: Routledge.

Lesh, R. R., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahway, NJ: Lawrence Erlbaum Associates.

Roschelle, J., Patton, C. C., & Tatar, D. (2007). Designing networked handheld devices to enhance school learning. In M. Zelkowitz (Ed.), *Advances in Computers* (Vol. 70, pp. 1-60).

Roschelle, J., Penuel, W. R., & Abrahamson, L. (2004, April 12-16, 2004). *Classroom response and communication systems: Research review and theory.* Paper presented at the American Educational Research Association, San Diego, CA.

Stein, M. K., Baxter, J. A., & Leinhardt, G. (1990). Subject-Matter Knowledge and Elementary Instruction: A Case From Functions and Graphing. *American Educational Research Journal, 27*(4), 639-663.

Stroup, W. M., Ares, N., & Hurford, A. C. (2004). *A taxonomy of generative activity design supported by next-generation classroom networks.* Paper presented at the PME-NA 26, Toronto, Canada.

Stroup, W. M., Ares, N., Hurford, A. C., & Lesh, R. (2007). Diversity by design: The what, why and how of generativity in next-generation classroom networks. In *Foundations for the Future in Mathematics Education*. Mahwah, NJ: Lawrence Erlbaum Publishing Company.

Stroup, W. M., Carmona, L., & Davis, S. M. (2005). Improving on Expectations: Preliminary Results from Using Network-Supported Function-Based Algebra. In S. Wilson (Ed.), *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Blacksburg, VA.

Stroup, W. M., Kaput, J. J., Ares, N., Wilensky, U., Hegedus, S., Roschelle, J., et al. (2002). The nature and future of classroom connectivity: The dialectics of mathematics in the social space. In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant & K. Nooney (Eds.), *Proceedings of the Twenty-Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 195-213). Athens, GA.

Wittrock, M. C. (1991). Generative Teaching of Comprehension. *The Elementary School Journal, 92*(2), 169-184.