Title	How to teach the mathematical concept of variation in secondary mathematics
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HOW TO TEACH THE MATHEMATICAL CONCEPT OF VARIATION IN SECONDARY MATHEMATICS

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One of the biggest problems in the teaching of mathematics is the teaching of problem solving. This problem was one of the findings of the National Assessment Educational Progress (NAEP) conducted in 1977–1978 in U.S.A. In particular, word problems that require the understanding of inhibited mathematical concepts create great difficulties for many students. Besides the NAEP survey, a series of studies was conducted in Australia using mainly Newman's hierarchy of errors model. These studies were carried out by Clements (1982), Watson (1980) and Clarkson (1983). The following table summarizes the results of the studies by Newman, Watson, Clements and Clarkson on the categories of errors.

PERCENTAGE OF ERRORS CLASSIFIED INTO ERROR CATEGORIES OF ERROR CRITERION

INVESTIGATORS	ERROR CATEGORIES							
	A	B	C	D	E	F	G	H
Newman	13	22	12	26	2	22	3	0
Watson	15	26	7	47	3	2	0	0
Clements	5	8	25	32	2	28	0	0
Clarkson	12	21	23	31	1	12	0	0

A — Reading ability
B — Comprehension
C — Transformation
D — Process skills
E — Encoding ability
F — Careless error
G — Motivation
H — Question form

The results indicate that, in each study, approximately 50–60% of the errors made by the students was due to transformation and process skills. Apparently, this group of pupils who had undergone these studies lacked understanding of mathematical concepts.

According to Krulic (1975), the secondary school student may well be at the concrete operational period of cognitive development. The mathematics teacher may have to work in an activity oriented setting or laboratory setting to provide for the student who is still at the concrete operational stage. This article presents some techniques of teaching mathematical concept on the topic on 'Variation' using the activity-oriented method.

The following are two lessons which can be used for the teaching of

(1) direct variation and

(2) inverse variation.

Direct Variation

Activity

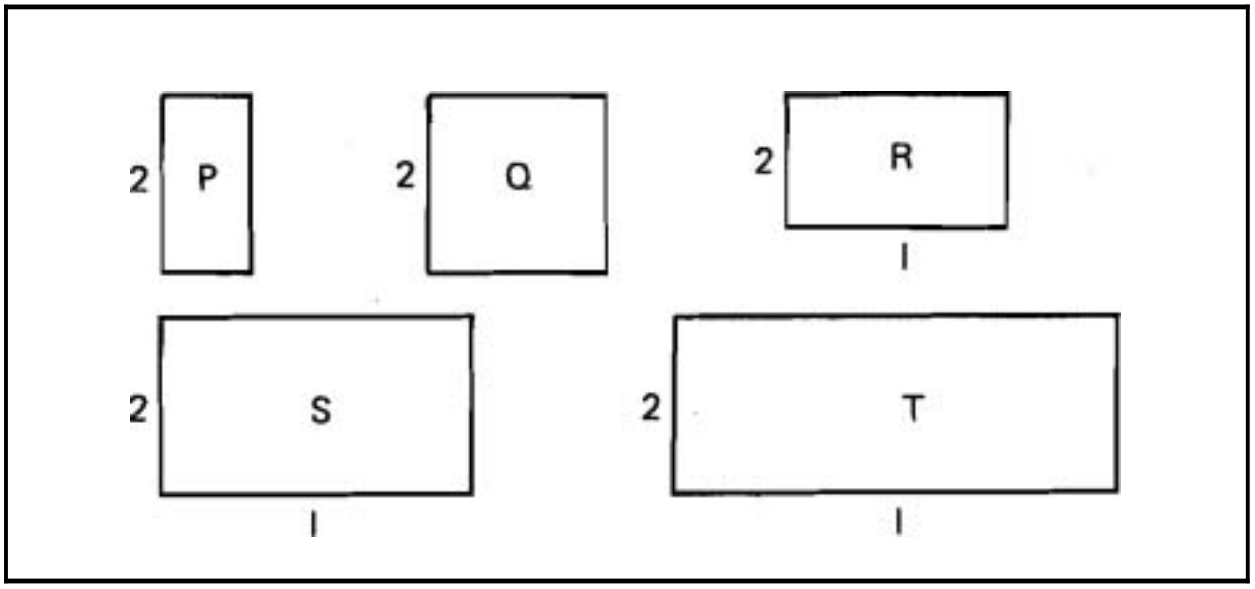

Figure 1

(a) On a graph paper, draw 5 different rectangles with a constant breadth of 2 units as in figure 1.

(b) Count the number of small squares inside each rectangle.

(c) Measure the length of each rectangle.
Record them as shown in the table below.

Rectangle	P	Q	R	S	T
No. of small squares (A)	2	4			
Length of rectangle (l)	1	2			
$\frac{A}{l}$	$\frac{2}{1} = 2$				

For each entry in this table, you should have found that

$$\frac{A}{l} = 2$$

or $$A = 2l$$

Using l, the length of the rectangle, and A, the number of small squares as coordinates of the ordered pairs, we have

(1,2), (2,4) (3,6), (4,8), and (5,10)

Draw these points on a square graph paper, a straight line passing through the origin is obtained (figure 2).

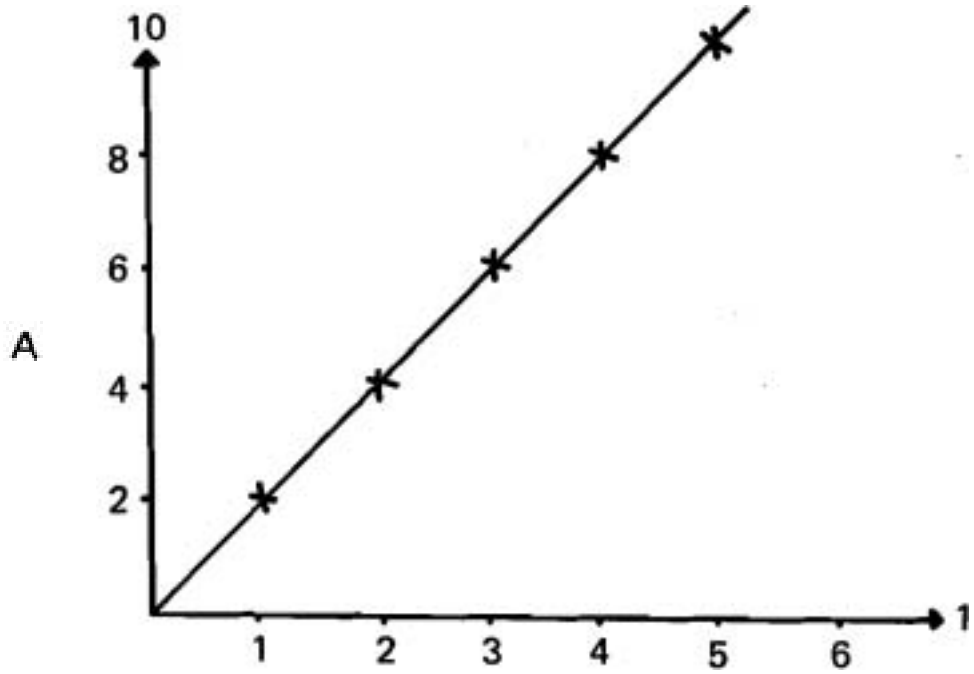

Figure 2

From the table and the graph, notice that as the values of l increase, the values of A also increase.

$$l \propto A$$

and in each case $\frac{A}{l} = 2$

We say that A varies directly as **l.** We write A a l.

In this case, the symbol a means 'varies directly as'.

Hence, if A a l, then we have

$$A = kl$$

where k is a constant of the variation.

In the example discussed above, the constant of variation is 2.

Now look at the cuboids below (figure 3). All measurements are in cm.

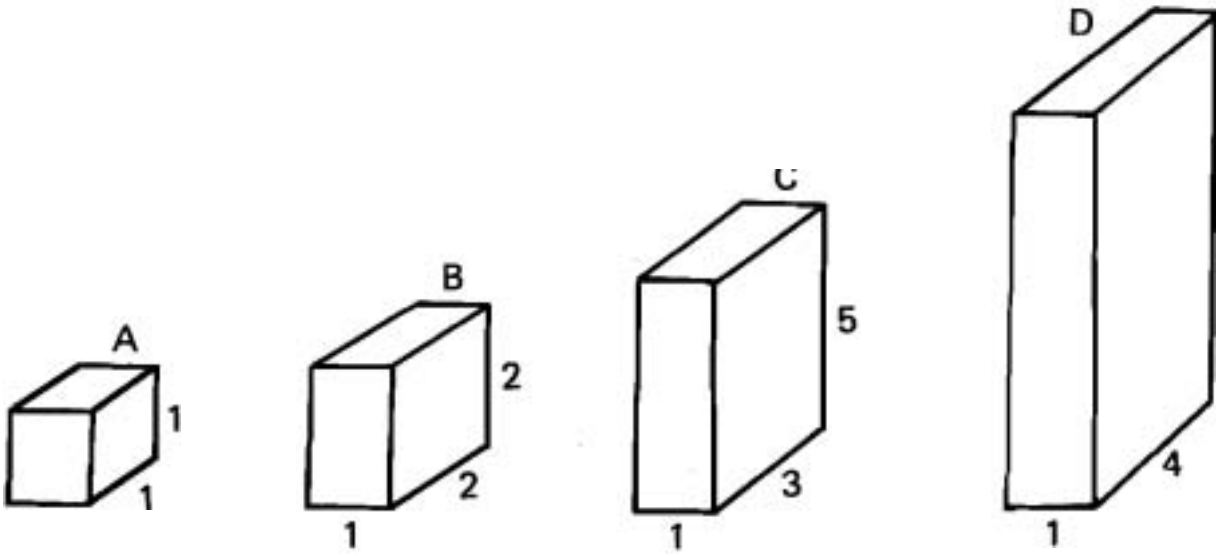

Figure 3

Notice that as the height increases its volume increases.

Can you say that the volume of the cuboid varies directly to its height?

The volume of each cuboid is calculated. and the following table is constructed using the data obtained.

Cuboid	A	B	C	**D**
Volume (V cm^3)	**1**	**4**	15	**44**
Height (h cm)	**1**	2	5	**11**
$\frac{V}{h}$	**1**	**2**	**3**	**4**

Notice that the ratio $\frac{V}{h}$ is not a constant. $\frac{V}{h} \neq K$

$\Rightarrow$ that V does not vary directly as h.

You will also notice that the base area is not fixed in each cuboid. Therefore the volume does not increase in the same

proportion as that of the height although visually it looks as though V varies directly as h..

Inverse Variation

Figure 4

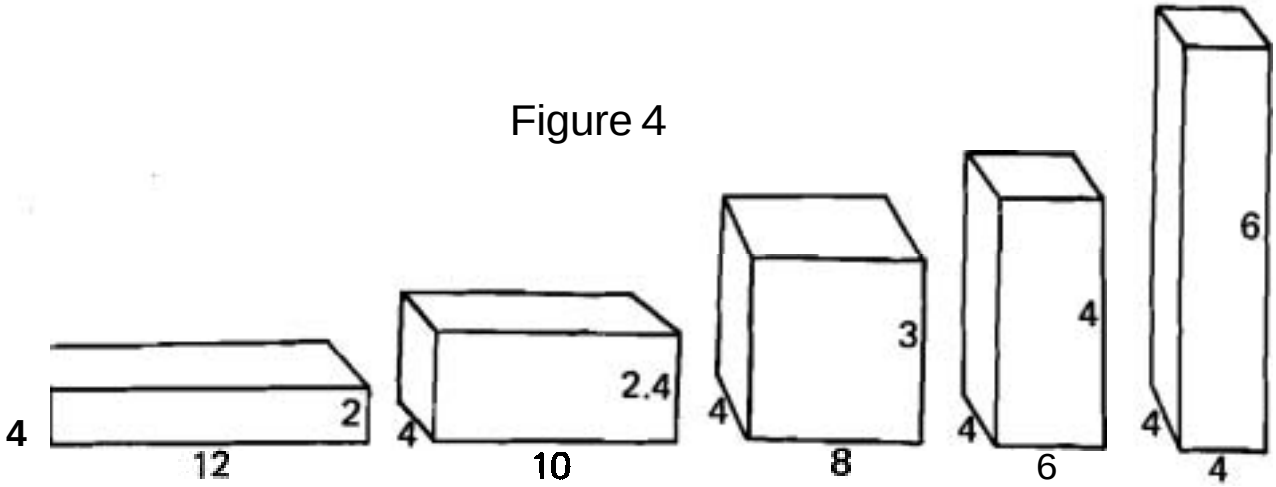

Figure 4 are some cuboids with the given dimensions. All measurements are in cm.

Notice that the breadths of the cuboids above are fixed at 4 cm each while their lengths and heights vary.

Calculate the volume of each cuboid. Copy and complete the table of values below.

Cuboid	A	B	C	D	E
Breadth (b cm)	4	4	4	4	4
Length (l cm)	12	10	8	6	4
Height (h cm)	2	2.41	3	4	6
Volume (V cm^3)	96				
l h					
l x h					

From the values calculated in your table, can you say that the length of the cuboid varies directly to its height?

Notice that $\frac{l}{h}$ = constant

l does not vary directly as h.

However, from the table

$$l\,h = 24$$

$$l = \frac{24}{h} = k\left(\frac{1}{h}\right)$$

where k is called the constant of variation.

l varies directly as $\frac{1}{h}$

or l varies inversely as h.

we write $\boxed{l \;\alpha\; \frac{1}{h}}$

Using h, the height of the cuboid and l, the length of the cuboid as coordinates of the ordered pairs (h, l), we have

(2,12), (2.4,10), (3,8), (4,6), (6,4)

Plot these ordered pairs on a graph paper.

Do you still get a straight line?

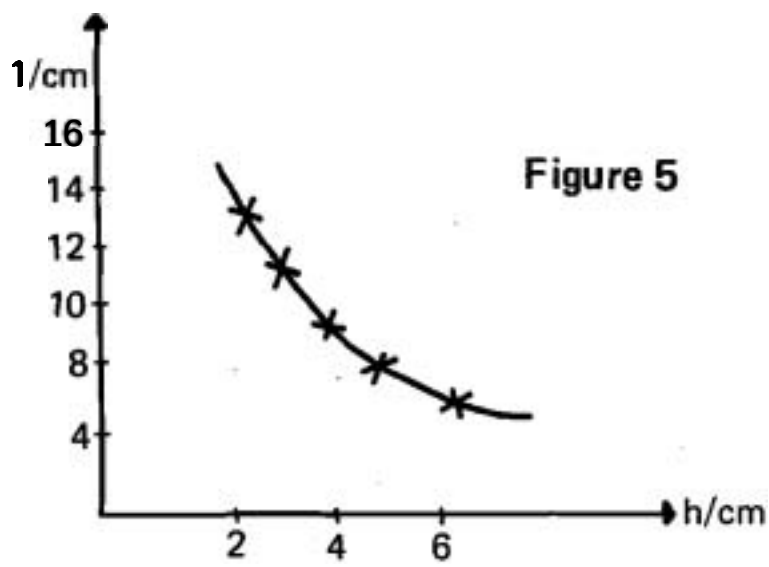

Figure 5

Notice that a curve is obtained (figure 5) as h increases, I decreases.

$$h \uparrow, \ I \downarrow$$

The above two lessons have shown how we can introduce an abstract lesson, like variation, using some concrete materials for pupils to visualise the concept of variation. With the use of these concrete materials, we hope that they will find it more meaningful to learn mathematics.

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