PATTERN IN MATHEMATICS

ERIC PLANT

W. W. Sawyer in ***Prelude to Mathematics, 1961,*** page ***12*** states: "It is interesting to note that pure mathematicians, moved only by their sense of mathematical form, have often arrived at ideas which were later of the utmost importance to scientists. The Greeks studied the ellipse more than a millenium before Kepler used their ideas to predict planetary motion". Patterns in mathematics are often studied for their own intrinsic interest.

Contemporary discoveries and patterns in mathematics are so varied that mathematicians of the past would find it difficult to recognise them even as mathematics. Sawyer in ***Prelude to Mathematics*** suggested that "Mathematics is the classification and study of all possible patterns". Obviously this definition is not all embracing as Bertrand Russell found when he contended that mathematics is a "study of relationships". Both ideas, however, are worth exploring in the teaching and learning of mathematics. The following suggestions have proved effective in stimulating understanding of mathematical ideas in actual classroom situations. The reason is that, in nature and also in mathematical thinking,, the same pattern is found again and again in different contexts. The idea of recurring pattern is seen in mathematics with the frequent use of the word ***Isomorphic*** (isos meaning like, morphic meaning shape). Nothing delights a mathematician more than to discover that two things previously regarded as entirely distinct are mathematicially identical.

In mathematics, if a regular pattern occurs, it is sensible to ask, why does it occur? What does it signify?

Consider then, the following patterns in the learning of school mathematics.

The '100' Square

At all times we should try to provide a variety of experiences in our number work so that patterns and relationships are seen in many situations. We do not want children always to be thinking of number in terms of length or on a horizontal strip.

The '100' square can be a useful aid to children for observing the patterns which are made by numbers. They enjoy colouring certain numbers and seeing a pattern emerge. This all helps their understanding and gives a visual image of what could otherwise be most abstract to a young mind.

100 Square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

These squares can be duplicated and children asked to cross off or colour numbers in a particular sequence. Patterns and useful information can be obtained in this way.

Children can be asked 'What is the interval between numbers in the vertical columns or diagonal columns?'

'100' squares can be numbered in a different way and numbers omitted for children to fill in. There are many variations of this on the same theme.

100	90	80	70	60	50	40	30	20	10
	89	79	69	59	49	39	29	19	9
98		78	68	58	48	38	28	18	8
97	87		67	57	47	37	27	17	7
96	86	76		56	46	36	26	16	6
95	85	75	65	55	45	35	25	15	5
94	84	74	64	54		34	24	14	4
93	83	73	63	53	43		23	13	3
92	82	72	62	52	42	32		12	2
91	81	71	61	51	41	31	21		1

Other useful practical work can be obtained from:

Numbers on tickets

Pages of a book

Numbering a plan of seats for a concert

Another useful device for displaying patterns of numbers on the '100 square' is to make the square on a larger sheet of thin card (10 inches square is a useful size), and cut windows in other 10-inch squares of card so that for each card, when placed on the '100 square', a pattern of twos, or threes, etc. shows through. Interesting here is that when the cards for twos and threes are placed on the square, the pattern of sixes shows through, illustrating that 6 is a multiple of 2 and 3.

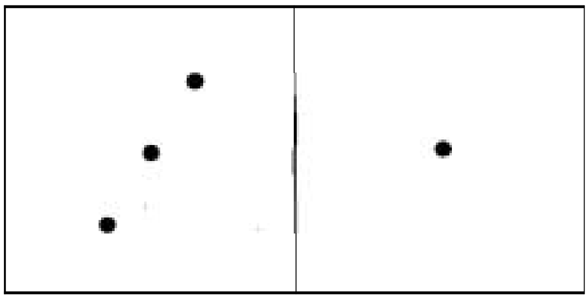
'100 Square' with 'window-cards'

'Window-card' for two:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

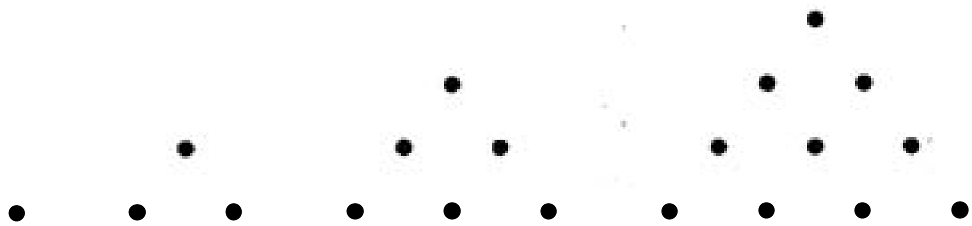
Dot patterns

The previous patterns were based primarily on examining patterns of squares on a "square grid." Arrays of dots present pattern in another visual form, as for example, the number of dots on a domino.



In this sense the display is seen as a ***Spatial Arrangement.*** There can then be arrangements which incorporate both number, sequence and geometry as in the following patterns:-

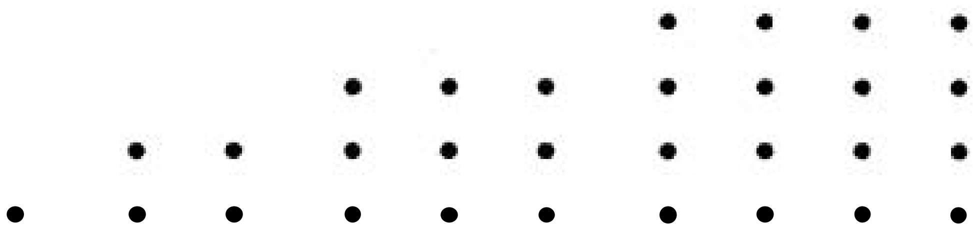
THESE ARE TRIANGULAR NUMBERS



etc.

1 3 6 **10**

THESE ARE SQUARE NUMBERS



etc.

1 4 9 **16**

Can you think of other ***Polygonal Numbers?***

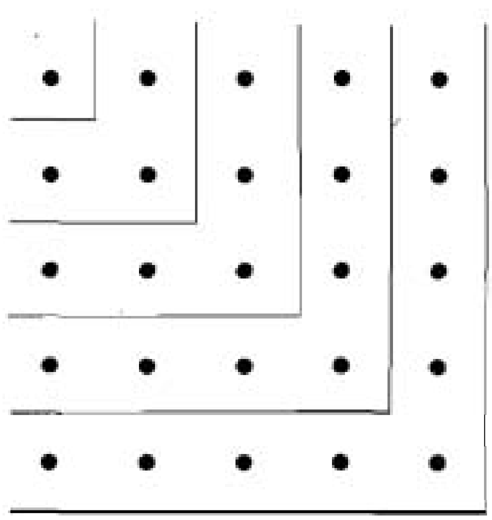
From the table square, we get interesting patterns. Look along the Diagonal 1 to 25.

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

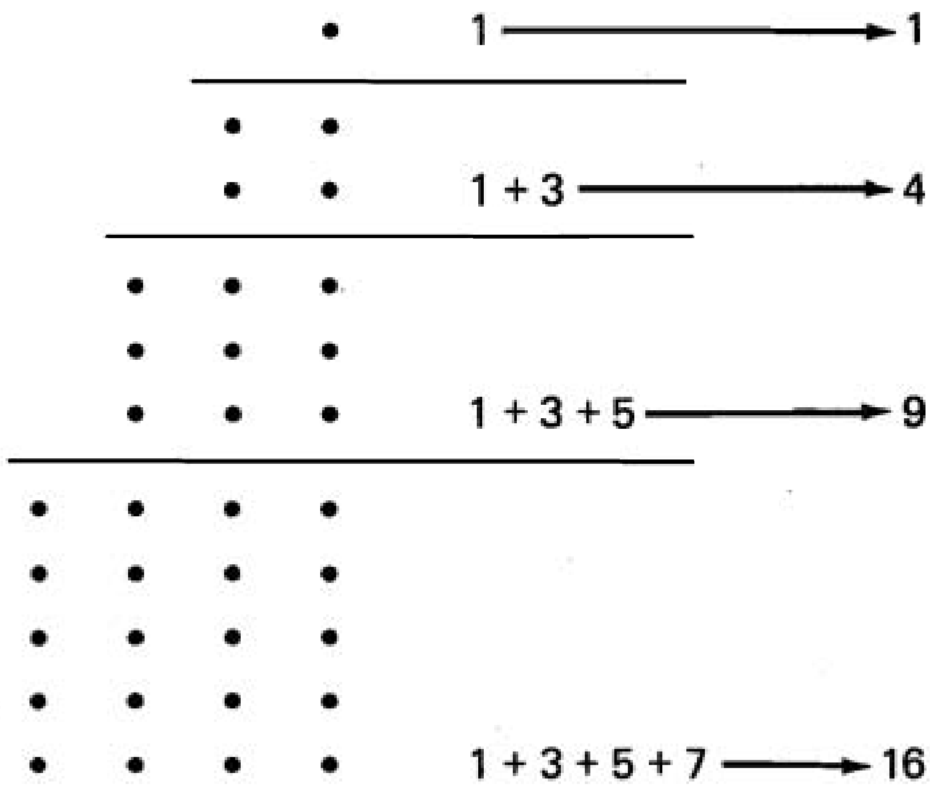
We get a sequence of numbers:

1, 4, 9, 16, 25, –, ...

These are square numbers, as mention earlier. They may be represented with counters.

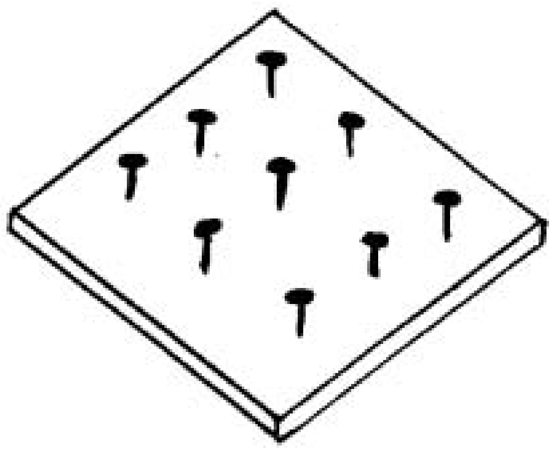


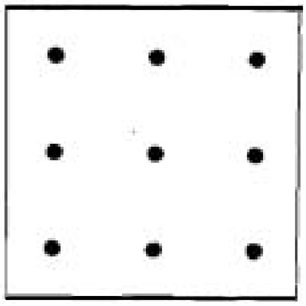
We see each square consists of:



PATTERNS ON GEOBOARDS

Geoboards are available from educational suppliers but can readily be made by using suitable plywood and knocking in panel pins arranged in suitable patterns hence the boards are also referred to as pin boards. Different arrangements of pins permit the students to make a wide variety of geometric patterns with elastic bands. Using different coloured elastic bands many comparisons and contrasts may be more easily recognised.

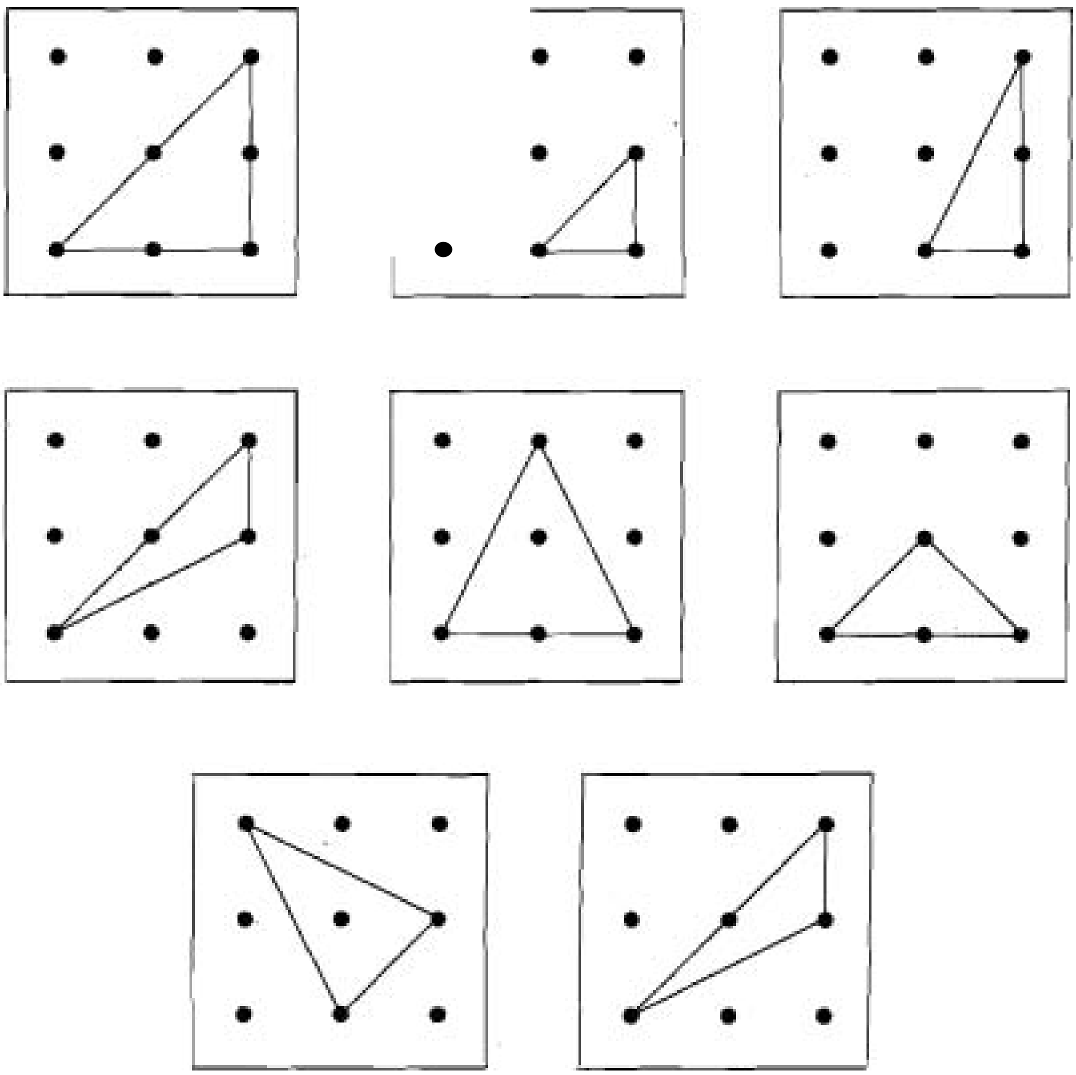




The number of different shapes which can be made on it is limited and this makes discussion about shapes which are the "same" easier to conduct.

An interesting investigation is to ask pupils to construct with elastic bands as many different triangles as possible on the 9 pin board. Discussion will arise as to difference and the "same as". This activity has proved highly motivating.

DIFFERENT TRIANGLES



THE POWER OF PATTERN

Recognition of pattern is an important and powerful way to give meaning and understanding to mathematical symbols. If the patterns are regular they permit a recognition of relationships. If the symbols and formulae are then used to summarise the meaning which may be derived from the pattern this leads to more powerful understanding of the underlying mathematical ideas.

Generalization is also more readily effected. It is the transfer of a property which is seen to be true in all familiar cases. Recognition of pattern facilitates abstraction and generalisation.

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