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Guest Editorial…

Mathematics for Teaching or Mathematics for Teachers?

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In Singapore, one topic of great interest in mathematics teacher education is the subject knowledge (SK) in mathematics of teachers. This topic has arisen in other places, often under the name "mathematical knowledge for teaching", or "MKT." Many mathematicians and mathematics educators believe that teachers should be given more content knowledge in mathematics. Such content knowledge should not be just higher mathematics but mathematics that is connected to mathematics teaching. In other words, it should be mathematics for teaching.

Many books have been published on this subject in the United States during the past few years. McCrory (2006) provides a review of twenty such books. In addition, the Fall 2005 issue of *American Educator* features the topic of mathematics for teaching (for example, see Ball, Hill, & Bass, 2005).

Issues

At Singapore's National Institute of Education (NIE)[1], at least 8 years ago, we introduced a course on SK in mathematics for elementary school preservice teachers in one program. It has presently been extended to all three training programs for elementary preservice teachers. For the past few years, we have engaged in a discussion on what to teach in SK, how to teach it, and who should be teaching it. It is a controversial subject, at least among some mathematicians and mathematics educators in Singapore. I shall list the five issues currently under discussion.

Issue 1: Is SK mathematics for teaching or mathematics for teachers?

It is generally agreed that we should give teachers more mathematics than what they need. The question is how far we should go. For example, should we teach the topic of base-five numbers? Some say yes, some say no, and others a qualified yes, meaning just the concept and not technical aspects such as the four basic operations. The argument is whether SK should be a course on mathematics for teaching or mathematics for teachers. If it is the former, we cover only the topics taught in elementary school. If it is the latter, we should go beyond what is being taught in the classroom. The idea behind mathematics for teachers is not only to learn mathematics but also to learn enough mathematics so that teachers become more confident when teaching.

Currently, the SK course being taught at NIE is meant to be a course on mathematics for teachers. One rationale is if we want our school pupils to attempt so-called challenging problems, then teachers themselves should have tried such problems. Furthermore, the problems should be challenging to teachers; that is, challenging at the level of teachers, not pupils. If teachers have never tackled problems challenging to them, how will they understand the difficulties that their pupils may encounter? Hence we must give them more mathematics beyond the elementary school.

Issue 2: Is SK a course in mathematics or a course in mathematics education?

By a course in mathematics, I mean the emphasis is on mathematics and rigor. By a course in mathematics education, I mean topics in the course may include problem solving heuristics. This does not mean that the two do not intersect. Of course, they do. The difference is in the approach. The mathematical approach may make use of heuristics implicitly, whereas the pedagogical approach may begin with heuristics and illustrate the heuristics using examples in mathematics. At NIE, it depends on who is teaching the course. Since the SK course at NIE is often taught by someone whose background is more mathematical than pedagogical, the approach naturally tends to be more mathematical.

Issue 3: Who should be teaching the SK course?

This issue is connected to Issues 1 and 2 above. Ideally, the SK course should be shared between a mathematician and a mathematics educator. In reality, it is often taught by one or the other and not jointly. The background training of the instructor seems to determine how the course will be taught. Though the

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course outline may be the same, once inside the classroom, the lessons look different. Sometimes we wonder whether it is necessary for it to be taught in a uniform manner. Perhaps not.

Issue 4: Is rigor a major learning objective for the SK course?

There are different possibilities for the degree of mathematical rigor used with preservice teachers. For example, should we use different notations for an angle and the measure of an angle? Should we distinguish between a line segment and the length of a line segment? Should we define an edge on a solid? Apparently, rigor means different things to different people. Some feel that we should be sufficiently rigorous when introducing concepts. At the same time, we want to keep the language dynamic and less rigid. For a mathematician, rigor comes before the Concrete, Pictorial, and Abstract teaching strategy (CPA).[2] I guess the issue is not rigor, but the degree of rigor.

Issue 5. Does every preservice elementary teacher need to take the SK course?

Another way of stating the issue is whether some preservice teachers can be exempted from taking the course. Perhaps we can have a placement test to determine who does not need to take the SK course. Perhaps we should practice differentiated teaching, meaning we may teach different versions of SK to preservice teachers under different training programs.

A course on subject knowledge

It was a long intellectual discussion at NIE on what to teach in SK, and how to teach it. The course has evolved over the years to what it is today. It is generally agreed that we should have a course on SK in mathematics for elementary school teachers under training and a certain amount of rigor should be maintained. It is also agreed that we should make it refreshing so that students find it useful and have a greater motivation to learn. Let me give two examples to explain what I mean by refreshing.

Example 1

The area of a circle can be given by the formula $A = \frac{\pi}{4} d^2$ where d is the diameter of the circle. If we take π to be 3 then the inscribed circle of a square has an area that is about 3/4 of the area of the square. This is more instructional than the formula $A = \pi r^2$. I saw a display of such a model in a department of mathematics at an old university in Rome. Furthermore, the ancient Chinese introduced the concept of diameter before radius. Indeed, there is a word for diameter in Chinese, but not for radius. Radius in Chinese is simply called half-diameter.

Example 2

The inequality $a^2 + b^2 \geq 2ab$ can be verified algebraically and geometrically. An alternative form is $(a+b)^2 \geq 4ab$. Consider a rectangle with area A and perimeter ℓ. If the length and the width of the rectangle are respectively a and b, then $A = ab$ and $\ell = 2(a + b)$. Hence the above inequality becomes $\left(\frac{\ell}{2}\right)^2 \geq 4A$. Fix ℓ, then A is maximized when the equality holds, that is, the rectangle is a square. Similarly, fix A and we can find the minimum ℓ in terms of A.

As we can see, the topics covered in the examples are school related. The presentation is accessible to teachers, and language used is familiar. The examples provide a glimpse of what I mean by mathematics for teachers.

Conclusion

I have written here what has been discussed at NIE on the subject of SK for mathematics teachers. While I may have personal views, I do not have answers for all the questions asked. In my view, the key factors in mathematics for teachers are "rigor" and "refreshing". Our direction should be to document what we have done, to build up a closer link between mathematics and pedagogy, and to nourish a better understanding between mathematicians and mathematics educators. Hopefully, in time, SK will become an integrated part of teacher training.

References

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McCrory, R. (2006). Mathematicians and mathematics textbooks for prospective elementary teachers. *Notices of the American Mathematical Society, 53*(1), 20–29.

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[1] The National Institute of Education is the sole teacher-training institute in Singapore. Aligned with initiatives set forth by Singapore's Ministry of Education, NIE administers preservice, in-service, and graduate programs for teachers.

[2] For more information on this strategy, see: Lee, P. Y. (Ed.). (2007). *Teaching primary school mathematics, A resource book*. Singapore: McGraw Hill.