Helping Children Develop Algebraic Thinking Through Word Problems
(Grades 3-8)

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Abstract
This hands-on session will engage participants in a problem-solving heuristic that is widely practiced in Singapore elementary schools. The model approach will be used to solve word problems common in Singapore classrooms. Participants will also learn how this method can be incorporated into lesson plans to encourage algebraic thinking.

Introduction
The Singapore mathematics curriculum is a problem-solving one. The primary aim of the curriculum is to develop students' ability in mathematical problem solving.

One of the emphasis of the curriculum is to help students use an array of problem-solving heuristics to solve different types of problems.

This session focuses on one heuristic that is very prevalent in Singapore primary classrooms in the solving of word problems. The examples used in this session are the kinds pupils in primary school are expected to be able to solve.

The session comprises:
• demonstration of the model approach
• experience in using the model approach
• demonstration on how the model approach is used to solve a range of arithmetic problems including those that include fractions, ratio and percent
• discussion on how the model approach help students develop algebraic thinking

The Model Approach
This approach basically involves the use of simple (rectangular) diagrams to represent quantities.

The purpose is to provide opportunities for younger students to engage in algebraic thinking without the use of abstract symbols. This allows students to solve difficult word problems prior to formal algebra.
The Model Approach & Principles and Standards

Algebra Standard (NCTM 2000)

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

<table>
<thead>
<tr>
<th>Grades</th>
<th>Expectations</th>
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<tbody>
<tr>
<td>K-2</td>
<td>Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.</td>
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<tr>
<td>3-5</td>
<td>Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.</td>
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<tr>
<td>6-8</td>
<td>Model and solve contextualized problems using various representations such as graphs, tables, and equations.</td>
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The Model Approach & Algebraic Method

Mr Wong is four times as old as his son. Ten years ago, the sum of their ages was 60. Find their present ages.

Using the Model Method

Mr Wong

Son

5 units = 60 + 10 + 10 = 80
1 unit = 16

The son is 16 years old and Mr Wong is 64 years old.

Using the Algebraic Method

Mr Wong

Son

x + x + x + x = 10

x + 10 + x = 60

x = 16

Therefore, the son is 16 years old and Mr Wong is 64 years old.
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Typically, students may write

\[ (4x - 10) + (x - 10) = 60 \]

\[ 5x - 20 = 60 \]

\[ 5x = 80 \]

\[ x = 16 \]

Mr. Wong is 4x (= 64) years old and his son is x (=16) years old.

The *Model Approach and Its Benefits*
- Students are able to solve semantically difficult problems without the use of formal algebra.
- Students are able to engage in algebraic thinking years before they are ready for formal algebra. It can subsequently help students make sense of formal algebra.
- The method is a simplifying tool. Many constraints can be handled simultaneously. Fraction problems can be solved without cumbersome computations involving fractions.

The *Model Approach and Its Limitations*
- Like all heuristics, it is more suitable for some problems than others. It cannot be used for all problems. In others, solutions using other methods may be more elegant.
- Initial use of the method is problematic. A suggestion has been made to incorporate the use of concrete materials such as the fraction strips to overcome this initial problem.

**Further Reading & Resources**
The following materials are included for your further reading and use:

- Fong Ho Kheong (January, 1994). Bridging the Gap between Primary and Secondary Mathematics. Teaching and Learning, 14(2), 73-84.
- http://intranet.moe.edu.sg/math/
The Part-Whole Model

- Given a and b, find c.
- Given a or b and c, find b or a.

The Compare Model (Multiplicative)

- Given a.
- Given b.

The Compare Model (Additive)

- Given b.

The Change Model

Before

After