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MATHEMATICAL DISCUSSION IN THE PRIMARY SCHOOL

Khoo, Phon Sai

Paper presented in
Jagoh Primary School, Singapore, on October 5, 1991
Mathematical Discussion
In The Primary School

KHOO PHON SAI

Focus - What it is all about

'Who needs the most practice talking in school? Who gets the most?' Exactly. The children need it, the teacher gets it.

According to John Holt, it is the children who need to do most of the talking but it is the teacher who does the most talking in the classroom. This observation is probably true in most schools throughout the world. It is not talk for the sake of talking but exploratory or purposeful talk related to a learning task that is meant.

The promotion of pupil talk then is the focus of this presentation, and the main items for consideration are reflected in the following questions:

1 What is mathematical discussion?
2 Why is there a need for more mathematical discussion in the classroom?
3 What beliefs and practices work for or work against the provision of mathematical discussion?
4 How are teachers to promote mathematical discussion in the classroom?

Relevance - Why the topic is chosen

Various reasons may be given for the choice of the topic, among which are relevance to the mathematics curriculum in Singapore and significance of the pedagogical value of mathematical discussion.

Mathematical discussion is part and parcel of the idea of mathematical communication, and mathematical communication is due to become an important component in the newly revised Primary School Mathematics Syllabus which will be implemented in Singapore from 1992. Specifically, one of the aims of mathematics education in the revised syllabus is that pupils will become "aware that mathematics is a powerful means of communication". The idea of mathematical communication encompasses not only the ability to use mathematical language and to present ideas in
writing or through visual means, but also the ability to describe, illustrate, interpret, explain, present and discuss mathematical ideas and experiences in doing mathematics. In short, the emphasis on mathematical communication in conjunction with other key features such as problem solving, metacognition, investigations and mathematical thinking in the revised mathematics syllabus will necessitate more attention to be paid to the role of mathematical discussion in the classroom.

The inclusion of the key features of problem solving, metacognition, investigations and communication in the mathematics curriculum reflects a recent international trend in mathematics education in countries such as Australia, England and the USA. For example, the importance of mathematical discussion has been emphasized in the famous Cockcroft paragraph 243, which demands amongst other things: "discussion between teacher and pupils and between pupils themselves" as an essential feature of mathematics lessons at every level. Similarly, the NCTM Curriculum and Evaluation Standards for School Mathematics includes the Standard of Mathematics as Communication as one of four fundamental standards in mathematics education at all levels from grades K to 12. (By standard is meant a statement of what is valued.) In particular, Mathematics as Communication stresses among other things the need to provide numerous opportunities for pupils to "realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics".

The pedagogical value of mathematical discussion is based on the belief that language plays an essential part in the ability to formulate and express mathematical ideas. Although pupil talk is generally hesitant, imprecise and exploratory, it can be argued that pupil talk is indispensable to active learning because "such exploratory talk is the means by which the assimilation and accommodation of new knowledge to the old is carried out... By formulating knowledge for oneself one gains access to the principles on which it is based" (Murray 1985).

Limitations - Considerations to be taken into account

It is one thing to recognize the value of pupil talk in the learning of mathematics but it is another to carry out mathematical discussion in the classroom. It has been said that many classrooms are characterized by lack of discussion and discussion when it takes place in the classroom is dominated by teacher talk which is a one-sided affair concerned mainly with the teacher giving instruction, explanation or assessment.

However, it should be noted that mathematical discussion is merely the means to an end - that of improvement in the learning of mathematics. Improvement in the learning of mathematics in the
classroom entails many considerations, and one of them that needs to be explicitly stated is:

"Sustained change must begin with teachers themselves. Their beliefs about the nature of mathematics, how children learn and about their own role, are crucial factors in determining what actually happens in the classroom and therefore in determining the quality of children's mathematical development" (Better Mathematics 1987).

One implication of these remarks is that whether or not there will be more pupil talk in the mathematics classroom depends on the teacher, on his or her awareness and conviction of the role of discussion, and the support given by the school.

Current situation - Aspects of questioning as related to discussion

It has been said that the more things change, the more they remain the same. For example, in spite of the many changes that have already taken place in the mathematics curriculum and the many recommendations for better teaching and learning of mathematics, it has been noticed that it is still possible to find classrooms where children still "lack in confidence in the subject; spend a majority of their time reproducing their teachers' examples with different numbers; answer only other people's questions; ask 'What am I supposed to be doing then?'... dislike mathematics, seeing it as irrelevant and boring..." (Better Mathematics 1987).

More to the point is the pervasiveness of the Question-Answer pattern in many mathematics lessons. For example, imagine yourself as a pupil in a typical mathematics lesson. The teacher asks: "What are six sevens or six times seven?" You like many other pupils shoot up your hand because you think you know the answer. You have been taught well not to call out the answer but to put up your hand if you want to speak. Your neighbour by the name of How Lek also puts up his hand. He does it for a different reason: he doesn't know the answer but by putting up his hand he hopes he won't be called. The teacher not knowing the survival game of How Lek calls him and there follow embarrassing moments. Several hands drop down during these moments but you shake your hand even more vigorously. No luck, the teacher next calls the bright girl Sheila. "Correct! Well done!" says the teacher, "Now what are seven sevens or seven times seven?" You decide not to answer but by now several pupils begin to call out the answer and so you join in. "Forty-nine!" "Correct, but I told you before - put up your hand if you know the answer - don't call out."

This example shows how pupils learn to play their roles in a mathematics lesson and also shows the interaction pattern expected both by the teacher and pupils - the question and answer pattern. There is hardly anything in it that is related to discussion, and one can go as far as to claim that if teachers and pupils are to
break out of this pattern, then they will have to learn to play very different roles in the classroom.

There are quite a few people who find it hard to imagine how there can be any discussion in mathematics. They think "Surely there's only one right answer, so either you know it or you don't". These remarks refer to a common perception (or misperception) of mathematics, which is that in mathematics a question has a correct or incorrect answer. In the general case, however, if a teacher asks a question in mathematics, he or she usually has in mind a clear idea of a reasonable answer, that is a kind of "ideal response" that is expected of the pupil. Having the ideal response in mind, the teacher most likely cannot avoid matching the answer of the pupil to this ideal response, and so has the constant temptation to comment on everything the pupil says. The teacher's comments usually take the form of an evaluation which makes it clear whether the pupil's answer is accepted as "Right" or rejected as "Wrong".

If the above analysis is valid, then one can hypothesize that in many mathematics classrooms, pupils will have learnt that the traditional role of the teacher is to ask questions as well as to evaluate pupils' answers as right or wrong, and that pupils' own role is to answer the teacher's questions.

The question and answer pattern mentioned previously may now be enlarged to become what has been called a Three-Term Sequence (Brissenden 1988).

\[
\begin{array}{c}
\text{Teacher asks question} \\
\quad \text{(with an ideal response in mind)} \\
\downarrow \\
\text{Pupil responds} \\
\downarrow \\
\text{Teacher comments} \\
\quad \text{(matching pupil's response with ideal response)}
\end{array}
\]

Figure 1: Three-Term Sequence

A further feature of this interaction pattern is that if the teacher rejects a pupil response, another pupil may be selected or else the question may be modified in some way. Such an interaction pattern may be likened to a ball-bouncing game in which the teacher is playing against several pupils.
In this game, the pupils speak to the teacher, not to one another. Although the pupils listen to the teacher (usually), they seldom attend to the comments of other pupils.

In this pattern of interaction in the classroom, no doubt the teacher can find out how well the pupils can repeat the ideas, and some of the pupils may be active. But this is hardly discussion.

It will be instructive to look at examples of three-term sequences of question and answer in actual mathematics lessons. In the following extract of a lesson, the teacher is trying to explain the concept of the height of an obtuse-angled triangle in the awkward case where the base has to be extended and the line falls outside the triangle.

T: Where's the height of the triangle if BC is the base?
P1: Teacher, is it that line?
T: Which line? Can you name it?
P1: Is it line AB?
T: No, that's not the height, is it? It has to be perpendicular to the base. The base this time is BC.
P2: I don't think it's got a height, teacher.
T: Yes it has. You P3, where would you draw it from?
P3: I've got an idea, teacher
T: All right, come out and draw it then.
(Pupil 3 draws the line BN in Figure 3
T: Er, no, that's not the line I'm looking for. What's the base for your height?
P3: Er - er - it's AC
T: Yes, but I want the height when BC is the base.
(There is an awkward pause.
T: Look - suppose I extend BC like this - can you draw the height now?

(Brisenden 1988)

The lesson extract above shows clearly not only the presence of three-term sequences of question and answer but also that the attempt by the teacher to draw out pupils' ideas and encourage their thinking may have the effect of turning the lesson into a "Guess what I'm thinking" kind of game.
At least two effects have been observed as a result of situations such as those illustrated in the lesson above. The first effect, particularly in the hands of inexperienced or novice teachers, is that the teacher is most likely to shift from a question and answer pattern to straight outright exposition. The result of this switch in teaching strategy is that the pupils become inactive and the teacher takes over the mathematical thinking.

The second effect is that as the teacher is intent on comparing pupils' answers with his or her ideal response, the teacher fails to listen to what the children actually say. Indeed, there are cases where the teacher distorts a pupil's answer in order to match the ideal response. For example, in the above lesson extract, the idea of pupil P3 was ignored instead of being used to characterize how a line can be said to be perpendicular to another line from a given point.

In short, one conclusion of the above observations is that while teachers' questions remain vital in mathematics lessons, they should be used to provoke a problem-solving attitude and not just to search for ideal answers that the teacher already knows.

Exposition is a kind of safety position to which a teacher can retreat when things get too messy in a question and answer interaction (for example, when pupils' answers range from wild guesses, through those which are incorrect or reveal misunderstandings, to some which are correct in some sense but not what the teacher is looking for). In mathematics lessons, this position generally takes on the form of explaining "This is the way to do it" to the children. "This is the way to do it" shows the authority of the teacher and puts all the thinking and decision-making safely in one place – with the teacher! The next lesson extract shows how easy it is for a teacher to presume that only a preconceived rule is acceptable.

In this lesson, structural apparatus was in use and the children had four tens and two units in front of them.

T: Now from your forty-two it says to take away five – five ones. Can you do that?
P: No.
T: Why not? Why can't you do that?
P: Two ones.
T: You've only got two ones haven't you? You haven't got enough. Do you remember, when we went through these sums last week, what we said you had to do, if you haven't got enough?
P: I ... err ...
T: What did we say you had to do?
P: You take the ... erm ... that ...
T: You had to borrow. What did you have to borrow?
P: Three.
T: Did you have to borrow three? You had four tens and two ones. What did you have to borrow?
P: Four ... one ...

(Brisseenden 1988)

Notice that the teacher is here urging the pupil to remember the algorithm in which a 'ten is borrowed' and ignored the pupil's idea of three. Could it be that the pupil was trying to say that one had to take away three more, since two had already been taken from five? Since only the preconceived rule seemed acceptable to the teacher, who did not really listen to what the pupil said, so the idea of 'three' was lost and the pupil had to start to guess the answer or struggle to recall the algorithm which has already been taught.

To summarize, it is argued that in many mathematics lessons, there is a predominance of teacher talk consisting of exposition, question asking, praising or encouraging and controlling pupil talk in various ways. The interaction patterns of 'Question and Answer', 'Guess what I'm thinking' and 'This is the way to do it' are consequences of a pervasive Three-Term Sequence.

Discussion as we know it in the general sense is not anything like any of the interaction patterns mentioned above. For example, in a discussion there is no one whose role it is to comment or evaluate as right or wrong what everyone else says.

Beliefs and practices - how they affect mathematical discussion

Previously it has been mentioned that there is a perception that in mathematics there are right and wrong answers to everything as well as clear-cut methods to be taught and learnt. So according to this perception, how can there be discussion in mathematics when there is no place for opinion? While there might be open problems at the advanced levels of mathematics, the content of mathematics has all been sorted out and written down at the school level. In short, it is not common for mathematics to be viewed as a discursive subject.

It cannot be denied that the way a teacher teaches conveys to the pupils the teacher's view of what mathematics is about. One widely held view may be described succinctly as 'mathematics is a body of knowledge'. Teachers who hold this view see their role as one of imparting this knowledge to the pupils, chiefly by good explanation. For such teachers, questions are important to ensure that pupils have understood and can use their knowledge.

Related to this view of mathematics is the perception that good teachers are those who can transmit their knowledge through clear explanations so as to avoid confusing the pupils. Probable
consequences of this perception include the following:
(a) an answer oriented atmosphere in the classroom;
(b) an overriding dependence on the teacher;
(c) the inhibition of pupils' own ideas;
(d) a fear of 'getting it wrong'.

(Better Mathematics 1987)

In furtherance of the view of mathematics as a body of knowledge and the idea that a teacher’s task is to transmit knowledge and to test whether the pupils have received it, it can be argued that various common classroom practices such as the following may restrict pupils' mathematical development: the subject is broken down into easily digestible topics, there is an over-concern to simplify, difficulties are smoothed out for pupils, and often it is assumed that techniques must be learned and practised before problems are mentioned.

An important contrasting view sees mathematics as a natural human activity, so that the main task for the teacher is to get pupils to join in the activity to the fullest extent. This view may be termed succinctly as 'mathematics is a way of knowing'. This view leads to an outlook which believes that the pupil has an active role to play in learning, the pupil has to instigate rather than merely respond or listen, and in particular the pupil has to develop critical thinking. In contrast, the view of 'mathematics is a body of knowledge' leads to an outlook which encourages the pupils to believe that mathematics has to be learnt in an uncritical way.

The two contrasting views of mathematics cannot be said to completely describe the range of views held by mathematics teachers but they provide a useful and convenient way to distinguish between various personal tendencies that can be observed. For example, in observing a teacher's treatment of pupils' ideas, if it is often the case that pupils' responses are matched against the teacher's ideal response, are evaluated by the teacher and unacceptable responses are discarded, then one may conclude that most probably the body of knowledge outlook is at work. On the other hand, a way of knowing outlook is probably the case whenever the teacher frequently tries to use the ideas offered by the pupils and in addition the teacher gets them to decide as far as possible which idea will work and which fail.

When considered in relation to the use of language for learning, the contrast between the two views of mathematics may be summarized as follows:
"The Transmission teacher sees it as his task to transmit knowledge and to test whether the pupils have received it. To put it crudely, he sees language as a tube down which knowledge can be sent, if a pupil catches the knowledge he can send it back up the tube .... For the Interpretation teacher, however, the pupil's ability to reinterpret knowledge for
himself is crucial to learning, and he sees this as depending on a productive dialogue between the pupil and himself". (Murray 1985)

Benefits - pupil talk and discussion

It has been said that the problem in most mathematics classrooms is not the lack of pupil talk but too much of it. Clearly, pupil talk for the sake of talk itself is not necessarily a good thing. On the other hand, for reasons such as to reduce the noise level of pupil talk or to remain in control of classroom activities, the prevailing feature of most mathematics classrooms is one of teacher-talk. Teacher talk generally refers to activities such as exposition, question asking, praising or encouraging, controlling pupil talk in various ways and teacher-led discussion.

In teacher-led discussion, effects such as the following have been observed:
(a) individual pupils have little opportunity to talk;
(b) pupil answers tend to be short and undeveloped;
(c) pupils have difficulty concentrating because they are not involved;
(d) shy pupils will fear exposure;
(e) the teacher is wholly occupied. (Bain 1988).

Pupil talk refers to activities such as responding to the teacher's questions, responding to teacher's other interventions, discussion with other pupils, and discussion with the teacher. It should be noted that the term pupil talk is used here in a more encompassing sense than the term discussion. For the moment, it is useful to characterize mathematical discussion among pupils in the following manner:
Pupils, with or without the presence of the teacher, meet together to solve a common problem or achieve a common goal by sharing thoughts and modifying their opinions, ideas and understanding.

The following additional characteristics of mathematical discussion have been put forward by Brissenden:
(a) Pupils speak and listen to one another on an equal footing. Thus there cannot be someone present who comments and evaluates immediately on everything that is said.
(b) Pupils listen to what is actually said, that is, without some preconceived ideal response in mind.
(c) The various forms of a mathematical activity are potentially open to everyone taking part. Thus there is no clear division of roles as between the teacher and pupils.
(d) Mathematical discussion between a group of pupils is based on an effective mathematical situation. The crucial idea of an effective mathematical situation will be discussed later.
Based on the above characterizations of mathematical discussion, the following claims have been made as to the benefits of pupil talk.

(a) The very act of explaining what they do and do not understand will help pupils to consolidate their understanding.

(b) Pupils in talking to each other will explore more freely and hypothesize more willingly than pupils working individually or under the eye of the teacher.

(c) The teacher cannot be everywhere at once. Thus pupils talking to each other will free the teacher to give special help where it is needed.

(d) Sometimes pupils are able to explain difficulties to each other more effectively than the teacher could.

(e) Pupils talking together will exchange ideas, be more adventurous and more independent of the teacher. (Bain 1988)

Procedures - How to promote mathematical discussion

Even if the value of mathematical discussion has been accepted on the grounds cited above or simply because it will enable a teacher to discover a considerable amount of what pupils are thinking, there remains the considerable task of how to accomplish it. There are various practical problems to be considered, for example, how to manage and organize groupwork, as well as the problem of changing pupils' views and expectations of what should go on in a mathematics lesson. However, recent publications on the topic of mathematical discussion have indicated various practical suggestions that will help teachers develop their own discussion lessons.

One way to begin, according to Dawson and Trivett (1981), in exploring the role of discussion in mathematics, is to set aside one lesson in five to experiment with the following kinds of activities. Take any topic, for example, the 10 by 10 multiplication table or the addition in columns of three 3-digit numbers. The topic chosen is not material because it is the process of what happens during the lesson that the teacher should be intent on observing. In relation to the chosen topic, decide on the task to be tackled by the children; for example, for the multiplication table, have the children construct the table and ask them to identify any patterns that they can see. For the addition problem, ask the question: can we add from left to right instead of from right to left?

Following Dawson and Trivett, it is crucial that the teacher does not give any judgement on whether what the pupils say or write is correct or incorrect. The teacher's role is to encourage the children to talk to one another and to discuss what they are doing. The teacher is to accept what is done, that is, as far as possible to become neutral, in so far as the mathematical ideas are concerned. It is further advised that at least initially, the
teacher should remain silent in order to be better able to watch and listen. In this way the teacher is not preoccupied with his or her thoughts and thereby miss the chance of really seeing and hearing what the children are doing and saying.

In short, the procedures to be adopted in experimenting with conducting a discussion lesson have the following features:
(a) There is a meaningful situation to stimulate talk.
(b) The teacher has presented carefully thought out enabling questions in order to focus the learning activity of the children.
(c) All contributions to be listened to and accepted by all.

In a detailed analysis of the dynamics of mathematical discussion in the classroom, Brissenden points out two essential components in a discussion lesson:
(1) Well-planned materials which generate and support mathematical activity by groups of children.
(2) The teacher needs to change his or her role in working with the children, and to help them change their own response.

More specifically, the mathematical activity to be engaged in by the children will generate and support the tasks of doing, talking and recording. The framework of doing, talking and recording in turn supports the process of building and testing ideas by the children.

The teacher observes

\[
\text{TALKING \leftrightarrow \text{RECORDING}}
\]

framework supports a process of

\[
\text{Building and Testing of Ideas}
\]

by the children

**Figure 4: The Pupils' Role**
The process of building and testing of mathematical ideas can be described in more detail in the following way:

Children build ideas
- by thinking about a question (posed by them or the teacher)
- by manipulating materials
- by contributory discussion - sharing

Children test ideas
- by predicting
- by using materials
- by comparative discussion - comparing

Crucial to a discussion lesson is the role of the teacher. The teacher’s role involves the planning of effective mathematical situations which will generate and support the framework of Doing-Talking-Recording activities of the children. It is vital for the teacher to develop the skills of listening to and observing the activities of the children carefully. This careful attention enables the teacher to judge when and how to intervene. Such interventions will usually take the form of questioning or drawing out - all designed to keep the children doing mathematics rather than the teacher. Finally some form of groupwork is needed so that children can spend time working cooperatively in doing mathematics. Figure 5 below shows how the various component roles of the teacher are related one to another.

The teacher’s role

Plans effective math situations which generate and support

Is an acute observer and good listener (uses this information)

Intervenes in ways which keep children

Organizes group work systematically both with and without the teacher

Figure 5: The Teacher’s Role
The intervention role of the teacher is guided by two considerations. In the first place, the teacher needs to push the ideal responses to the back of his or her mind, avoid preconceived ideas of how to direct the activities of the children, and constantly avoid from evaluating pupils' responses. In this way, the third term in the Three-Term Sequence is broken. In the second place, intervention is guided by the principle of maximizing pupil mathematical work and minimizing teacher mathematical work. In other words, the teacher employs as much as possible a procedural type of intervention such as those shown below.

<table>
<thead>
<tr>
<th>Mathematical role</th>
<th>Procedural role</th>
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<tbody>
<tr>
<td>asks challenging questions</td>
<td>finds out about progress</td>
</tr>
<tr>
<td>asks helpful questions</td>
<td>draws out pupils' ideas</td>
</tr>
<tr>
<td>injects mathematical language</td>
<td>extends pupils' responses</td>
</tr>
<tr>
<td>directs attention to relevant ideas</td>
<td>ensures turn-taking</td>
</tr>
<tr>
<td>suggests testing ideas</td>
<td>encourages, supports</td>
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Finally, it is necessary to consider what makes an effective mathematical situation. Since the burden of carrying the pupils' mathematical discussion falls not upon the teacher but upon the situation, the situation should have certain desirable features. According to Brissenden, the following factors should be taken into account when planning an effective mathematical situation.

1. The situation must generate initial interest and continue to sustain it. The activity could extend over an extended period of time, for example, several lessons.
2. The activities should involve at least two of DOING, TALKING and RECORDING. The doing could be interpreted as the confident use of a mathematical idea, that is, doing does not necessarily have to involve manipulative materials.
3. The situation must enable the children to think and work at their own level as well as make their own judgements about their own ideas. In other words, the situation must enable the pupils to carry out evaluations with whatever skills they are likely to have, that is enable both building and testing of ideas to take place.
4. The situation should as far as possible enable the children to work on their own without the intervention of the teacher.

Conclusion - closing remarks

The above characterizations of mathematical discussion in the context of teaching mathematics provide useful guidelines for a teacher who wishes to explore an alternative framework in the conduct of mathematics lessons. It is hoped that the ideas presented will enable the teacher to reflect on his or her own beliefs and practices to see whether or not they work for or
against mathematical discussion. The presentation has provided sufficient material to answer the four major questions about mathematical discussion that were posed at the beginning. Finally, it can be said that the success of the presentation may be gauged by how much discussion actually ensues about what have been said.

REFERENCES


