Students’ Metacognitive Problem Solving Strategies in Solving Open-ended Problems in Pairs

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This study examined the metacognitive behaviours of five pairs of Primary 5 of Singapore students as they solved mathematical problems. They were assigned a set of five questions, including one open-ended problem with missing data (Foong, 2002). This paper focused on students’ responses as they solved this problem. Each pair was video- and audio-recorded and their written solutions were collected to support the subsequent analysis. A think aloud protocol was used, and the protocols were then transcribed and analysed using a modified Artzt & Armour-Thomas’s (1992) framework. The analysis revealed that each pair has its own characteristics of regulation patterns ranged from low to high regulation, and two different cooperative levels. These results also suggest that students are not familiar with the nature of open-ended problems with missing data.

Since 1990, Mathematical Problem Solving (MPS) has been a central focus of the mathematical curriculum framework in Singapore (Ministry of Education, 2000). Figure 1 illustrates the framework.

Figure 1  Singapore Mathematics Curriculum Framework

In this framework, metacognition is deemed one of the main components of mathematical problem solving, with emphasis on students’ ability to monitor their own thinking. This concept is in line with Flavell’s (1981) definition of metacognition that refers to students’ awareness of their own cognitive processes and their regulation of these processes to achieve a specific goal. The strategies to generate such awareness and regulation are called metacognitive strategies (Foong & Ee, 2002; Teong, 2003) which include planning an overall approach to problems, selecting appropriate strategies, monitoring problem solving progression, assessing local and global results, and revising plans or strategies when necessary (Garofalo & Lester, 1985). Evidence from Schoenfeld (1985) suggests that good solvers’ metacognition differ significantly from novices’ in the effectiveness of their metacognitive
strategies. Thus, the explicit inclusion of metacognition in this framework underscores the importance of metacognition in the teaching and learning of MPS within the curriculum.

Emphasis in the syllabus aside, studies on students’ mathematical performance in Singapore suggest that students generally reflect very little explicit role of metacognition in their problem solving progression. In the school level, Yeap and Menon (1996) noted that metacognitive behaviours were exhibited while students solved non-routine mathematical problems. However, their metacognitive strategies often did not help them to solve the problems successfully. In the pre-service teacher level, Foong’s (1994) noted that unsuccessful solvers tended to focus their metacognitive behaviours on awareness of their own confusion and uncertainty. Both studies point to a need for mathematical instruction that supports students’ development of metacognitive strategies and systematic thinking.

In considering the design of such instruction, collaborative work may help enhance students’ metacognitive strategies. It provides opportunities for students’ spontaneous verbalisation, and for them to exchange and examine each other’s ideas (Artzt & Armour-Thomas, 1992), as well as develop the group’s monitoring and regulatory behaviours. However, not all collaborative work leads to successful problem solving behaviours. For example, Stacey (1992) found the performance of Grade 9 students diminished when they worked in groups. Stacey noted that students were able to propose and exchange a range of strategies, but they frequently overlooked the correct solution method due to a lack of checking and evaluation procedures. There are some complexities in group dynamics that mediate group’s effectiveness of metacognitive decisions (Goos & Galbraith, 2002; Goos, Galbraith, & Renshaw, 1996).

The task factors - the nature of tasks, difficulty levels, and familiarities to the solvers – also play an important and significant role in affecting group behaviours (Mevarech and Kramarski, 2003) as well as metacognitive behaviours generated by each group member (Garofalo and Lester, 1985). For example, basic procedural tasks would likely generate short conversation that focuses mainly on straightforward formula to find the unknown (Mevarech and Kramarski, 2003), and such tasks require minimal metacognitive decisions (Garofalo and Lester, 1985). On the other hand, more complex tasks could raise various mathematical conflicts that promote discussions, rich mathematical communications, exchange of strategies, and various metacognitive strategies. Phelps and Damon (1989) suggest that the effective tasks to be solved collaboratively are those require reasoning, both mathematically and in real-life context. The open-ended and investigative types of tasks in Foong’s (2002) mathematical problem classification match with Phelps and Damon’s criteria, especially those within real-life context (Cooper and Harries, 2002; Stillman and Galbraith, 1998).

To better understand students’ metacognitive problem solving strategies within collaborative group work, this paper examines the pairs of students working on open-ended problems.

**Methodology**

Five pairs of 11 to 12-year-olds from two Singapore primary schools were assigned a set of five questions, including one open-ended problem with missing data (Foong, 2002) as the last question in the problem set. While solving this problem set in pairs, they were video- and audio-recorded and their written solutions were collected to support the subsequent analysis. Since this study was aimed to examine students’ progression in solving open-ended problems, this analysis focussed on the last question. The problem consisted of two parts:

(a) With a full tank of petrol, we drove for 352 km up Malaysia and had to fill up at a petrol station. The tank took in 42 litres. How many km per litre did we get?

(b) On the way back, after driving 2/3 of the way, the tank was a quarter full. Do we have a problem if we don’t stop by a petrol station to fill up?

In part (a), one condition was omitted. No information was provided about whether or not the resulting tank was full after filling up at a petrol station. In part (b), no information was provided about whether the tank was full when they started the journey back home. Those insufficient conditions were expected to situate them in ambiguous situations and to probe their metacognitive strategies in determining the solution path. Moreover, part (b) also required logical reasoning in order to solve it, and the solution procedure was not so straightforward. In particular, part (b) could not be solved by the ordinary “model” method which is used prominently among primary school students. This problem was also situated within a real-world context designed to promote students’ discussion.
To analyse students’ problem solving progression and examine their metacognitive behaviours, the Artzt and Armour-Thomas’ (1992) framework was used and adapted, and the indicators to distinguish the cognitive and metacognitive behaviours were described as follows:

...metacognitive behaviours could be exhibited by statements made about the problem or about the problem solving process while cognitive behaviours could be exhibited by verbal or nonverbal actions that indicated actual processing of information (Artzt & Armour-Thomas, 1992, p. 141)

The original framework had eleven episodes with different level of cognitions to parse small-group protocols: reading (cognitive), understanding (metacognitive), analysing (metacognitive); exploring (cognitive); exploring (metacognitive); planning (metacognitive); implementing (cognitive); implementing (metacognitive); verifying (cognitive); verifying (metacognitive); and watching and listening (unassigned cognitive level). For the purpose of this study, two modifications were made in this framework. The first followed Teong’s (2003) modification to re-categorise the descriptors for understanding and analysing to analysing the word problem. This modification was based on Schoenfeld’s (1985) observation – “in analysing a problem an attempt is made to fully understand the problem.” The second modification was the addition of one more category others, a catch-all category with unassigned cognitive level.

**Analysis and Results**

Table 1 demonstrates the time and the percentage of behaviours coded as cognitive, metacognitive, or unassigned cognitive level for each student in the five pairs while solving the open-ended problem, as well as pairs; completeness and correctness of solution paths and answers.

<table>
<thead>
<tr>
<th></th>
<th>Pair 1</th>
<th>Pair 2</th>
<th>Pair 3</th>
<th>Pair 4</th>
<th>Pair 5</th>
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<tbody>
<tr>
<td></td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S4</td>
<td>S5</td>
</tr>
<tr>
<td>Metacognitive</td>
<td>05:29</td>
<td>04:07</td>
<td>06:33</td>
<td>04:15</td>
<td>04:45</td>
</tr>
<tr>
<td>(% of total)</td>
<td>(44%)</td>
<td>(33%)</td>
<td>(76%)</td>
<td>(49%)</td>
<td>(35%)</td>
</tr>
<tr>
<td>Cognitive</td>
<td>06:24</td>
<td>03:01</td>
<td>01:38</td>
<td>04:02</td>
<td>05:40</td>
</tr>
<tr>
<td>(% of total)</td>
<td>(51%)</td>
<td>(31%)</td>
<td>(19%)</td>
<td>(47%)</td>
<td>(41%)</td>
</tr>
<tr>
<td>Unassigned</td>
<td>00:44</td>
<td>04:39</td>
<td>00:22</td>
<td>03:22</td>
<td>06:18</td>
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<tr>
<td>(% of total)</td>
<td>(5%)</td>
<td>(36%)</td>
<td>(5%)</td>
<td>(24%)</td>
<td>(46%)</td>
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<td>(100%)</td>
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<td>Part (a)</td>
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<td>incomplete correct path</td>
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<td>complete correct path</td>
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<td>wrong answer</td>
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<tr>
<td>complete Could not settle the path</td>
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<tr>
<td>complete wrong path</td>
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<tr>
<td>wrong answer</td>
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<tr>
<td>incomplete in the midst of exploring the path</td>
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<tr>
<td>Part (b)</td>
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<tr>
<td>jumped into conclusion</td>
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<tr>
<td>complete wrong path</td>
<td></td>
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<tr>
<td>wrong answer</td>
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<tr>
<td>complete correct path</td>
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<tr>
<td>correct answer</td>
<td></td>
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<tr>
<td>jumped into conclusion</td>
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The findings in Table 1 appear to indicate that to some extent, metacognitive behaviours were indeed generated by pair interaction. The lowest percentage of time spent in metacognition was 29% by S10 in pair 5. The rest ranged from 33% to 49%, except for S3 in pair 2 who spent the highest percentage which was 76%. The total time spent to solve this problem ranged from 8.5 minutes to about 14 minutes.

None of the pairs obtained correct answer for both parts. Only pair 4 who did part (b) correctly. The detailed progression of each pair is discussed below and the sequences of behaviours generated are presented in the timeline representations (see Figure 2 to 6). In each diagram, odd-index student’s behaviours are indicated in blue/darker shade and the even one’s are in pink/lighter shade.

**Pair 1’s Progression of Problem Solving Activity (S1 & S2)**

In solving part (a), the protocol of S1 and S2 shows that they involved in a well-regulated progression of activity in the earlier part (first 5 minutes) but then broke down into irregularity. Before the irregularity occurred, their protocol could be summarised as reading → analysing/planning → implementing (C, M). The irregularity described by spending some time in exploring (C) followed by the random interplay between verifying (M), reading, and
In part (b), after reading and short analysing, they immediately concluded an answer using common sense logic.

In part (a), after reading the question, they discussed possible solution strategies. S2 initiated the discussion by straight away suggesting an incorrect plan, which is to minus away 352 by 42 to find the amount of petrol used up to drive for 352 km. S1 rejected this plan and came out with dividing 352 by 42. After reaching this point, S1 seemed to sense the ambiguity aroused by the missing data.

S1: Erm 42 litres, the tank took.. took in 42 litres, which is erm… full tank.
S2: But we don’t know the full tank.
S1: Yea, but… 42 and 352 are the only information given.
S2: Yea, lah.
S1: So we have no choice but just.. to take it as a full tank.

Based on this assumption, they proceeded to implementing their plan and finding out numerical values of 352 divided by 42.

The pair reached an impasse when the resulting division was not a whole number. They were confused with the existence of remainder. As a result, they were distracted by exploration to represent the division into a decimal. Despite their miscalculation, they still faced the nature of infinite decimal of dividing 352 by 42. As such, they rejected the answer. This rejection caused the subsequent irregular activity. S1 perceived that the answer must be “definite” and hence he went to series of reading and analysing back and forth to examine their solution strategy, while S2 mostly watched and listened to S1. Since the solution steps were already correct, these activities only verified the appropriateness of the strategies that was being used. While they were still figuring out on how to obtain the definite answer, the observer stopped the progression and asked them to skip to the next part to ensure similar time spent on the questions.

In solving part (b), very little discussion occurred within the pair. S2 felt that this part was related to part (a), so that to answer part (b) they needed the answer of part (a). However, S1 shot down S2’s task-assessment and immediately jumped into conclusion that 1/4 of a petrol-tank was too little to travel 2/3 of the journey. Judging by S1’s argument, it was inferred that S1 got the wrong interpretation of the statement “after driving two thirds of the way” and hence reached that conclusion.

**Pair 2’s Progression of Problem Solving Activity (S3 & S4)**

This pair spent the highest amount of time in metacognitive behaviours. From Table 1, we can see that S3 and S4 devoted 76% and 49% of their time to metacognition. In solving both part (a) and (b), the protocol of S3 and S4 revealed that their problem solving progressions were relatively well-regulated. In part (a), the protocol can be summarised as reading → analysing → exploring (C, M) → implementing (C, M) → verifying (M), whereas in part (b), the sequence of activity is described by reading → planning → implementing (C, M) → verifying (M). However, they were not successful in getting correct answers for both parts.
In part (a), after reading task-assessments were done. The first was an assessment by S4 about the authenticity of the problem situation which was immediately suspended by S3.

S4: The tank took in 42. 42? (pause) It’s not possible! My car…. You see, it’s not possible, you know! How come 42…
(This response raises the importance of real-world referents in assisting meaning).
S3: This is just a question!
S4: Okay.

The second assessment was done by S3 in response to the ambiguity of problem statement. She did not seem to realise the missing data as she straight away interpreted the result of filling up was a full tank.

S3: With a full tank… okay! If a full tank means 42 litres (pause for 3 seconds) drove for 352 km. So… 42 litres of…
litres can last 352 km.
S4: How about we try 352 divided by 44, no… 42.

S4’s suggestion led them to exploring possible division of 352 by 42. They could obtain the remainder “16” correctly, however they failed to represent it in decimal. From the conversation, it was inferred that they had same perceptions on such representation. After assessing the result of this exploration and its representation, they agreed with this solution path and hence wrote it down systematically, which was coded as implementing episodes. In this implementation, S3 had strong monitoring that determined their solution presentation direction. Short verification was done after the implementation, however S4 rejected the correct units (km per litre) stated by S3, and chose the wrong units (litre per km) without further clarified by S3.

Figure 3 A timeline representation of S3 and S4 solving open-ended word problem

In solving part (b), though the sequence of activity was relatively well-regulated, they encountered conceptual mistakes and difficulties in engaging with each other’s ideas. After reading, S3 suggested an incorrect plan to calculate two thirds of the 42 litres, which was accepted by S4 and being implemented by S3. Along the way, S4 was able to spot S3’s mistake, which was mixing up the “352” and the “42” for distance and tank-capacity. However, S4’s challenge was not strong enough to point this out to S3. S3 did not manage to catch S4’s ideas. Without examining the problem requirement as pointed out by S4, S3 tried to justify her correct procedural steps to convince S4. In the end, S4 was confused, so he abandoned his challenge and agreed with S3’s solution steps.

Pair 3’s Progression of Problem Solving Activity (S5 & S6)

There was no evidence of task-assessment done in this protocol. In the earlier part, S5 and S6 did not share the same activity. While S5 started reading the problem, S6 was still checking the earlier problem in this problem-set. It was followed by the discussion about the earlier question that drew them away from the task. They returned to the task after the 2.5th minute.

In solving part (a), S5 and S6 did not achieve solution strategy agreement. Each of them persisted with their own approaches. S5 was sure that the correct strategy was dividing 352 by 42, while S6’s strategy was the reciprocal division. Such disagreement affected their problem solving progression. Most time spent on arguing for own
strategy, abandoning peer’s strategy, and taking over implementation work. Therefore, their problem solving activity was predominantly spent on planning and implementing episodes. In terms of cooperative level of interaction, theirs was more to competitive mode of “whose strategy was correct” discourse.

In the 10th minute, they had not settled which formula to be used and the observer stopped them due to time contingency and asked them to proceed to part (b). The sequence of activity in solving part (b) was reading → analysing → planning → implementing (C, M) → verifying (M) completing within 4 minutes. They could settle with one solution path, which was by multiplying 1/4 by 2/3. Their plan was not correct and reflected their limited resources in dealing with this problem. It seemed that they just inferred directly a mathematical operation to combine two numerical data given in the problem statement. However, such inference may also be caused due to the time contingency. It was possible that they just tried to write something presentable in order to complete solving this problem.

**Pair 4’s Progression of Problem Solving Activity (S7 & S8)**

In solving part (a), S7 and S8 spent more time reading, before they proceeded to implementation. In between the reading activities, they engaged in intermediate activities such as analysing, planning, and exploring. They even jumped to read and analyse part (b) when they ran into an impasse, before coming back to solving part (a) (see Figure 5). Within these activities, they had rejected few plans. For example, one useful plan was brought up and self-rejected by S8. S8 switched into another plan and was not responded by S7.
S8: I think this one divided by 42 (referring to 352 km). Eh? Cannot!
S8: The tank took in 42 litres... (read the question again)
S8: This one minus 42 (referring to 352 km).
S7: We drove... for 352 km. And 42 litres. (did not respond to S8’s suggested plan)

Both students also assessed the authenticity of the problem noting the ambiguity caused by the missing data. It was first assessed by S8. However, this assessment was self-abandoned by S8. Later on, S8 also abandoned S7’s assessment.

S8: No, it depends! Because going up takes longer (laugh), but that’s nothing. It’s out of the question already. This one minus this one (re-suggested his original plan).
S7: Full tank, eh?
S8: Because.. 1km is 1000. This one is also 1000 (referring to 1 litre). So... there’s... so, you.. you take 352... (providing reason to support his plan: 352 minus 42)
S7: It depends on the car! Because some car.. say..
S8: It’s out of question!
S7: (laugh)

After that, they decided to use S8’s inappropriate plan; they implemented it and obtained a wrong answer. It was noted that their progression had similar pattern as Teong’s (2000) type Q of students’ cognitive-metacognitive word problem solving pattern.

In solving part (b), they also spent more time in analysing the problem statement before going to implementation. Not so much reading that may be a result of previous reading and analysing done while solving part (a). Their representation while analysing and the subsequent exploration were useful and “hit” the appropriate strategy and they arrived at a right answer for part (b). They did not seem to realise the missing information in part (b), and they straight away took the tank capacity as a full tank when starting the journey.

**Pair 5’s Progression of Problem Solving Activity (S9 & S10)**

In the protocol, there was no evidence of task-assessment done by this pair. After reading, S9 and S10 immediately jumped into exploring 352 divided by 42, and abandoned their exploration once they found a remainder. Then they returned to reading, analysing, and planning activities and gained insights that 352 divided by 42 was indeed the correct strategy. However, they were unsure because their exploration did not result in a whole number. Since they had erased their earlier exploration, they redid the exploration and again they were confused with the resulting remainder. Therefore, they decided to skip part (a) and proceeded to part (b). There was no implementation involved.

In part (b), after reading, S10 concluded that the answer for this question is “no”. On being probed by the observer, S9 questioned S10 for justification, but S10 could not justify and immediately changed the answer into “yes”. Only after that, they spent time to analyse the problem, and eventually S9 concluded that the answer is “yes” using common sense logic similar to pair 1.
Discussion

Analysis using modified Artzt & Armour-Thomas’s (1992) framework on these five pairs suggests that working collaboratively involves two possible cooperative levels and different characteristics of problem solving regulation. The *high* cooperative level occurred when there was high engagement of peer’s ideas. In this setting, the interaction that occurs within the pair was predominantly *interdependent* (Arzt & Armour-Thomas, 1992). Such *high* cooperative level was shown by pair 4 and pair 5. Pair 1 in solving part (a) did exhibit *high* cooperative level, however it did not stay for too long, after the impasse and irregularity occurred they engaged in another type of cooperative level.

Another level, the *low* cooperative level, was indicated by little occurrence of mutual exchange of ideas. There are two possible forms of *low* cooperative levels. The first one was the case when there is more dominant student, and the second one is the case when the engagement of each other’s ideas is very little. Such little engagement occurred when the differences of both students’ opinions are very strong and usually each tends to force the implementation of own opinion. In this setting, there is strong tendency to speak for their own arguments instead of to listen and challenge peer’s ideas. The first type was illustrated in pair 1’s protocol after they reached an impasse (see Figure 2). S2 appeared to be very passive and little contributed to the subsequent activity, though she was able to engage in S1’s ideas. The second type was illustrated in pair 3’s protocol (see Figure 4). Pair 2’s protocol when they solved part (b) also illustrated the second type (see Figure 3). Though they worked interdependently, S4 could not engage in S3’s ideas and S4’s challenge was not strong enough. S3 appeared to justify her own opinion without considering S4’s concerns.

Our analysis showed that pair’s regulation of behaviours ranges from *well-regulated* to *not-so regulated*, described in the five possible patterns as follows:

1. Regulated and sequence of behaviours described by orderly manner: *reading → analysing/planning → implementing (C/M) → verifying (C/M)*. This regulation had similar characteristic as type P model in Teong (2000). There was evidence of this type of regulation in pair 2’s protocol, especially when they solved part (b). In our data, this type of regulation corresponded to high percentage of metacognitive behaviours generated by one or both pair members (see Table 1).

2. The second one was described by spending more time in *reading, analysing/planning* before pairs proceeded to *implementing (C/M)*. Students occasionally engaged in *exploring (C/M)*, and their exploration “hit” correct target that determine subsequent activities. This pattern had similar characteristic as type Q model in Teong (2000). An example of this regulation is provided by pair 4’s protocol.

3. The third pattern was a transition from well-regulated behaviour to not-so-regulated one. Pairs engaged to a certain degree of regulation in the earlier part of problem solving protocol and changes to an irregular manner of *reading, analysing/planning, exploring/implementing (C/M), and verifying (C/M)*. In most cases, the source of the change was an impasse. Example of this regulation was provided by pair 1’s protocol when they solved part (a).
4. The fourth type was described by pair’s dominance in exploring episodes, mostly in cognitive level. When reaching an impasse, their tendency was to return to reading/analysing, followed by other exploration. The exploration might bring pair 5 into “wild goose chase” situation as described in Schoenfeld (1985).

5. The regulation was stuck due to pair’s inability to achieve agreement on which solution paths they needed to choose. This regulation was illustrated in pair 3’s protocol.

We noted that the source of impasse was students’ confusion about remainder. All pairs, except pair 4, were confused with the existence of remainder. To handle such case, their metacognitive strategies were not effective by their tendency to assessing the appropriateness of procedures that had been applied than making sense of the answers. Pair 4 chose non-fractional mathematical formula to solve this question, so that it could not be inferred whether they had the same confusion with the remainder.

In processing the problem statements, only pair 1 realised the insufficient conditions. As such, they created an assumption in order to proceed to implementing their plan through S1’s statement, “So we have no choice but just.. to take it as a full tank”. Pair 2 and pair 4 did not realise the insufficiency of the conditions, however their interpretations of the problem statement enabled them to proceed to a plan and obtained an answer. These two pairs were also the only pairs who assessed problems’ authenticity, though eventually those assessments were suspended. They seemed to believe that authenticity of the problem was not important and was not within the context of solving mathematical problems. Pair 3 and pair 5 were basically focussing on possible combination of numerical data provided in the question and did not really examine the actual conditions given in the problem. These behaviours are regarded by Foong & Koay (1997) as students’ stereotyped thinking of word problems. Furthermore, the analysis also revealed the possibility of students’ beliefs of the numerical answer’s uniqueness to every mathematical problem. This observation suggests that students are not familiar with the nature of open-ended problems with missing data, and students’ beliefs of word problems (Greer, Verschaffel, De Corte, 2002, p. 274) are prevalent among Singapore students.

Limitation of Study

During data collection

The problem set was administered during normal classroom hours so that time limit was imposed on all pairs. Since this open-ended question was put as the last question, observer could not help stop the progression of pair 2 and pair 3 due to contingencies of time. As such, for these particular pairs, the range of their metacognitive strategies captured may not be completely described. Besides, observer’s probing them to think aloud may have affected some of their metacognitive decisions.

Coding

The contents of pairs’ protocols were mainly based on students’ verbalisation during the problem solving process. Thus, the possibility of researcher bias towards some degree in making inferences is inevitable. This may be due to students’ incompleteness of verbalisations, technical aspects of the recordings, and researcher’s inconsistency in making such inferences.

It is noted that this modified Artzt & Armour-Thomas’s (1992) framework is able to keep track individual contributions to pair problem solving progression. However, as being pointed out by Goos et al. (2002), it overlooks the reciprocal nature of metacognitive activity generated by pair interaction. For example, the instances when students could not settle a solution path and spoke for each own arguments in pair 3’s protocol were coded as both students engaged in the same episodes – planning and implementing (C, M). In fact, they hindered pair’s metacognitive decisions that were qualitatively different from the case when both students engaged in the same episodes and their exchanges were mutual. Thus, further refinement is needed to better describe interactivity structure that mediates pairs’ metacognitive strategies.

Conclusion

This study provides a glimpse into problem solving progression of five pairs of Primary 5 students when they solved open-ended problem with missing data. Though the framework used does not provide details in pairs’ reciprocal nature of metacognitive activity, the findings suggest a range of pairs’ problem solving regulation that was determined by their metacognitive strategies as well as the two different cooperative levels involved. Students’
responses on this open-ended problem also provide salient points of their beliefs in solving word problems. It gives a picture that would hopefully contribute to better understanding of pairs’ metacognitive characteristics in solving such open-ended problems as well as pairs’ problem solving mechanisms. With these insights, a basis to develop students’ metacognitive strategies through pair interaction is established, which hopefully could facilitate the design of such pair work instruction in the classroom.

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