THE EFFECTS OF INTEGRATING MATHEMATICS TO PHYSICS INSTRUCTION ON PHYSICS LEARNING AT JUNIOR COLLEGE LEVEL

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Abstract

This study was undertaken in view of the fact that many pre-university students had difficulties with mathematical skills as well as transfer of mathematical skills in solving physics problems. The study aimed: (a) to identify the main difficulties faced by the students and then devise teaching units to overcome the difficulties; (b) to investigate whether there was any improvement in achievement in physics if mathematical skills were being stressed explicitly in physics lessons. The topics chosen for this study were: Graphs, Errors and Vectors. Three different sets of pre- and post-tests were designed to accomplish the purposes of study. The aim of the pre-tests was to determine the weaknesses of the students in mathematical skills and transfer skills, from which three teaching units were specially designed for the experimental group. The fused curriculum was adopted to integrate the two subject areas and the combination of interrogatory, demonstration and prototype development instructional methods was implemented with emphasis on immediate feedback and corrective teaching. Students' achievements from both control and experimental groups were measured by the post tests. Analysis of treatment effects strongly supported the notion that with adequate emphasis on mathematical skills and fostering transfer skills in the physics tutorial lessons, students' performance in solving physics problems could be significantly improved.
1. INTRODUCTION

It has been reported that many students cannot estimate, cannot calculate with approximate numbers, cannot read graphs and do not know how and when to make approximations when being encountered with Physics problems. The difficulties surface more at the junior college level where such skills are integral part of the Physics course. Students should use concepts in Mathematics as tools to tackle situations met in Physics.

2. PURPOSE OF STUDY

This study aims to investigate the main difficulties with Mathematics knowledge in solving Physics problems and then devise teaching units to overcome these difficulties, if any. The areas of Physics chosen are Graphs, Errors, Composition of Vectors and Resolution of a Vector. The main task is to link Physics and Mathematics and use Mathematics as a tool to solve Physics problems.

The questions to be answered in this study are:

(a) What are the major difficulties, as far as Mathematics skills are concerned, that students face when solving Physics problems?

(b) Is there any difference in students' achievement in Physics when Mathematics skills are being stressed in Physics tutorial lessons?

3. DESIGN OF STUDY

3.1 Sample

The experimental group consisted of a total of 33 students. The subject combination of this group was Physics, Chemistry, Mathematics B and Economics.

The control group, comprised 37 students. The subject combination was Physics, Chemistry/Computer Science, Mathematics B and Further Mathematics.

3.2 Description of Instrument

There were a total of three sets of tests, the first being a pre-test and post-test on Graphs the second on Errors, and the third on Vector Resolution and Composition of Vectors. The aim of the pre-tests is to determine the weaknesses of the students in mathematics from which the lessons will be geared to combat these weaknesses, if any.
The pre-tests were administered to both groups. These tests were used to test if there is any significant difference in the two groups as well as to determine the weaknesses of the students in mathematical skills and transfer skills in solving physics problems.

The post-tests were designed as evaluative tests which were used to measure students' performances after the teaching units had been implemented to them.

### Conceptual Framework

The study attempted to make use of the remedy instruction, that emphasized on remedying deficient mathematical skills and fostering transfer of mathematical skills, to improve students' achievement in physics. The conceptual framework of this treatment process is shown in Figure 1.

The treatment consisted of two sections:

(a) **Treatment Phase**

- Identifying students' deficiency in mathematical skills and transfer of skills as a tool in solving physics problem.

- Remedying deficient mathematical skills in Physics tutorial lessons.

- Emphasizing interface between physics and mathematics by using various examples.

(b) **Treatment Phase 2**

- Highlighting the process of logical thinking and reasoning in analysing physics problems.

- Illustrating the application of physics concepts and the transfer of mathematical skills in solving physics problems.

- Encouraging students to be involved in solving physics problems.

- Implementing on-going and continual monitoring.
Figure 1. Conceptual Framework
3.4 Teaching Units

After each pre-test, the weaknesses of the students with regards to mathematics knowledge and transfer of skills in solving physics problems were carefully diagnosed. Based on the analysed results, three teaching units were specially designed for the students in the experimental group.

Graphs

Graphs construction and interpretation are important skills to both physics and mathematics education. However many students have not acquired these valuable skills especially the skills in graph interpretation. The important role of graphs in learning physics, coupled with deficiencies in graphing abilities suggests that remediation in instruction should be made to overcome the problems. We should ensure that the teaching unit we plan is able to assist students to achieve the following objectives.

(a) The content of the objectives in the area of mathematics includes:
- selecting appropriate axes
- locating points on a graph
- drawing lines of best fit
- interpolating, extrapolating
- describing relationships between variables
- interrelating the data
- transforming complicate equations into simple straight line equation

(b) The content of the objectives in the area of physics includes:
- selecting appropriate axes and assigning the corresponding physical quantities
- calculating of gradient with correct unit
- interpreting the dependent physical quantity correctly from the gradient of the graph
- presenting the result with appropriate significant figures and unit

The outlines of the content of the teaching unit as well as the procedure in presenting the unit are summarized in Figure 2.
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Errors

The common problems faced by students in physics practical are calculating errors and making estimation. Many students have difficulties in making approximations by discarding terms which do not affect the answer as well as in giving the answer in a reasonable number of significant figures. The objectives of this teaching unit include:

- differentiating mathematical expressions
- estimating the maximum error
- using ratios and proportions in calculating percentage errors
- expressing answers in correct significant figures

The skills to be illustrated and stressed are:

- manipulating skill
- transfer skill
- making approximation skill
- interpreting skill
- presenting skill
- skill in making estimation

Figure 3 shows the procedure of how the topic will be presented.
Mathematics functions

\[ z = x + y \]
\[ z = x - y \]

**Differentiation:**
\[ \Delta z = \Delta x + \Delta y \]
\[ \Delta z = \Delta x - \Delta y \]

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Mathematics function:

\[ z = xy \]
\[ z = x/y \]

**Differentiation:**
\[ \frac{\Delta z}{y} = x\Delta y + y\Delta x \]
\[ z = \frac{\Delta x}{y} - \frac{x\Delta y}{y^2} \]

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Mathematics Complicated functions

\[ z = x^3 \]

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Error (Physics)

Examples: Length and Volume

\[ l = l_1 + l_2 \rightarrow \Delta l = \Delta l_1 + \Delta l_2 \]
\[ V = V_1 - V_2 \rightarrow \Delta V = \Delta V_1 + \Delta V_2 \]

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Percentage Error (Physics)

Examples: Work done and Density

\[ W = F \cdot S \rightarrow f_W = f_F + f_S \]
\[ D = \frac{M}{V} \rightarrow f_D = f_M + f_V \]

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Physics

Example: Kater's pendulum

\[ g = 4\pi^2 \cdot \frac{h_1 - h_2}{t^2} \]

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Application of Mathematical concepts and skills in estimating error in Physics

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**Figure 3. Differentiation in Mathematics and Error Estimation in Physics**
Composition of Vectors and Resolution of a Vector

The concepts of vector, composition of vectors and resolution of vector are fundamental concepts in both Physics and Mathematics. The generic ideas of vectors is the foundation stone for many areas in physics and mathematics. A lack of understanding of the nature of vector quantities may restrict students from further understanding of many other physics topics. At the introductory level, pupils do find difficulty in assimilating the properties of vector in the discussion of force, acceleration, velocity and especially relative velocity. Analysis of the difficulties faced by the students indicate that they use the vector notation devoid of physical meaning.

To help students to grasp the notion of vector in mathematics as well as its application in physics, demonstration-observation-explanation teaching method is adopted. Simple but useful apparatus made of transparent perspex are used with an overhead projector to perform demonstrations such as:

- **vector addition**
  > simulation of a river crossing observed by an fixed observer at the river bank.

- **vector subtraction**
  > simulation of a bird flying directly over the river as seen by an observer floating down the river.

- **vector resolution**
  > the equilibrium of three forces.

There are many other demonstrations which can be carried out by using these simple apparatus. The concepts included in the teaching unit are listed in Table 1 and the general network for physics vector quantity is shown in Figure 4.
<table>
<thead>
<tr>
<th>Mathematics concepts</th>
<th>Application in Physics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Notion of Vector</strong></td>
<td><strong>Physics vector quantities</strong></td>
</tr>
<tr>
<td>- Direction</td>
<td>Examples: Force</td>
</tr>
<tr>
<td></td>
<td>Acceleration</td>
</tr>
<tr>
<td></td>
<td>Velocity</td>
</tr>
<tr>
<td></td>
<td>Momentum</td>
</tr>
<tr>
<td>- Magnitude</td>
<td></td>
</tr>
<tr>
<td><strong>Vector Addition</strong></td>
<td><strong>Resultant of forces acting on a body</strong></td>
</tr>
<tr>
<td>Methods:</td>
<td><strong>Resultant velocity-ferryman's problem</strong></td>
</tr>
<tr>
<td>- Parallelogram</td>
<td><strong>Three forces in equilibrium</strong></td>
</tr>
<tr>
<td>- Triangle</td>
<td></td>
</tr>
<tr>
<td>- Polygon</td>
<td></td>
</tr>
<tr>
<td><strong>Vector Subtraction</strong></td>
<td><strong>Relative velocity and relative acceleration with respect to a moving observer</strong></td>
</tr>
<tr>
<td>Methods:</td>
<td><strong>the direction of rain drop seen by an observer in a moving car</strong></td>
</tr>
<tr>
<td>- Parallelogram</td>
<td></td>
</tr>
<tr>
<td>- Triangle</td>
<td></td>
</tr>
<tr>
<td><strong>Resolution of a Vector</strong></td>
<td><strong>Resolution of Physics vector quantity (force, velocity, etc.) with its components along certain directions according to Physics situation and for the purpose of solving physics problems in an effective way.</strong></td>
</tr>
<tr>
<td>- Independent components (components perpendicular among each other)</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4 Network for vector concept of force, acceleration, velocity, .......
3.5 Instructional Methods

The instructional methods adopted in this study were combinations of the following methods:

<table>
<thead>
<tr>
<th>Method</th>
<th>Treatment components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype Development</td>
<td>Definition + Explanation of examples and non-examples</td>
</tr>
<tr>
<td>Interrogatory</td>
<td>Definition + Questions related to illustration</td>
</tr>
<tr>
<td>Demonstration, Observation</td>
<td>Prediction of the outcome of demonstration + demonstration and observation + discussion on the conflicts between predictions and observations or reinforcement on the correct prediction</td>
</tr>
</tbody>
</table>

The teaching method consisted of the components of immediate feedback and corrective teaching. A set of simple questions was designed to treat deficiencies in prerequisite knowledge and skills in mathematics. After the study of a unit, students were asked to solve a series of questions. Many of the questions were set to expose more common weaknesses in the application and transfer of mathematical skills in solving physics tutorial problems. Immediately after students having answered the questions, the teacher provided feedback, which gave the correct answers and information clarifying the incorrect responses.

4. IMPLEMENTATION

4.1 In implementing the study, certain constraints have been taken into consideration. The practical requirement is that the study should be carried out with no addition of manpower or special resources as well as without causing any administrative difficulty. With these in mind, a fused curriculum is adopted. In this study, the way of integrating the curricula is to have
the teacher agree in teaching two subject areas, physics and mathematics. It is also important to ensure that the topics selected for the study follow the scheme of work planned by the college.

In the study, the designed instruction constituted the treatment provided to the experimental group. For comparison, the control group was subjected to the conventional instruction, in which tutors discussed the tutorial questions prepared by the college during the tutorial sessions. The duration of the study was about five weeks. Both groups were given the same amount of time to cover the same topics. Same post-tests were conducted to both the experimental group and the control group.

RESULTS AND DISCUSSION

To determine the effects of integrating mathematics skills to physics instruction on students achievement in physics, pre-tests and post-tests were administered to the students in both the control group and experimental group. Analysis of the data collected was done by using t-test as well as pairwise comparison.

5.1 Analysis of Test-Scores Using T-Test

Pre-Tests Results

The means for both experimental and control groups, the standard error of the difference in means and the t-test result are given in Table 2.

<table>
<thead>
<tr>
<th>Experimental Group Mean</th>
<th>Control Group Mean</th>
<th>$S_{x_1} - x_2$</th>
<th>t-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.4</td>
<td>55.1</td>
<td>2.25</td>
<td>-2.53</td>
<td>Reject $H_0$ at $\alpha = .05$</td>
</tr>
</tbody>
</table>

From the pre-tests results, we can conclude that there is a difference in the two groups. The
results confirm that the control group is of better ability.

Post-Tests Results

The mean score for the experimental group is 73.4 and that for the control group is 67.1. The standard error of the difference in means is 3.53. The t-test result of the post-tests is given in Table 3.

Table 3 Post-Test Results

<table>
<thead>
<tr>
<th>Experimental Group Mean</th>
<th>Control Group Mean</th>
<th>$S_{\bar{X}_1 - \bar{X}_2}$</th>
<th>t-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>73.4</td>
<td>67.1</td>
<td>3.53</td>
<td>1.784</td>
<td>Accept $H_0$ at $\alpha = .05$</td>
</tr>
</tbody>
</table>

From the post-tests results, we see that the experimental group has managed to score higher than the control group. The t-test indicates that there is no difference in the two groups at $\alpha = 0.05$; nevertheless, the difference is significant at $\alpha = 0.10$. However, there is a significant difference in the two groups with the control group being better than the experimental group before the treatment. Thus it can be concluded that the teaching units have been successful i.e. by placing more emphasis in Mathematical skills as well as transfer of mathematical skills during physics lessons, students' achievement can be significantly improved.

5.2 Analysis of Test-Scores Using Paired Sample Theory

Since the pre-test and post-test scores of the same student are investigated, analysis can be done by using the theory of paired samples. The confidence limit for the difference between the means of post-and pre-test is given by:

$$\overline{D} \pm t (.975, n - 1) \times S\{\overline{D}\}$$

With a 95% confidence interval, the confidence limit for the control group as well as the experimental group are given in Table 4.
Table 4  Confidence Interval

<table>
<thead>
<tr>
<th>Group</th>
<th>$\bar{O}$</th>
<th>$S{\bar{O}}$</th>
<th>Confidence Limits (CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>25.6</td>
<td>3.60</td>
<td>18.3 to 32.9</td>
</tr>
<tr>
<td>Control</td>
<td>12.1</td>
<td>3.71</td>
<td>4.5 to 19.7</td>
</tr>
</tbody>
</table>

From the values of $C\text{L}_{\text{exp}}$ and $C\text{L}_{\text{cont}}$, one can conclude that better improvement in performance of the experimental group can be due to the instruction designed and the teaching approach adopted.

SUMMARY

The results obtained strongly support the notion that in the Physics classroom, with an emphasis on Mathematics skills and the transfer of Mathematics skills, students would perform better in solving Physics problem. Whenever possible, Mathematics concepts should be explained before introducing Physics concepts so that students are able to see the link between the two fields. Mathematical manipulation skill is one important area because when solving a physics problem, students would need to rearrange a given equation to suit certain conditions. The transfer of Mathematics skills to the Physics practice is also found to be important in fostering the growth of students’ problem-solving skills.
REFERENCES


