COGNITION IN MATHEMATICS ACHIEVEMENT PERCEPTION AND PROCESSING

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INTRODUCTION

Cognition, which is a psychological domain of learning, generallly refers to the act or faculty of apprehending, knowing or perceiving. It can be synonymously termed cognitive styles, cognitive controls, cognitive dimensions, or information processing. The complete study aims to investigate into the individuals' application of cognition specifically to mathematics learning and the effects of different ways of information processing on mathematics achievement. Do individuals perceive and process mathematics content and concepts differently than others? Do these differences in cognitive styles affect their mathematics achievement? Do successful mathematics achievers use predominantly one side of their brain to process mathematical concepts whereas poor achievers the other side? We believe the answers to these questions will have profound implications to mathematics teaching and learning in schools.

In a sense, an individual's cognitive styles have been confined to his/her typical, habitual, consistent, and preferred manner or strategy of acquiring, storing, recalling, and processing information. At a higher level, these are trait-like modes or preferred learning sets of perceiving, encoding, reacting to, retrieving and drawing inferences of information. With this repertoire of information which is subject to these various rudimentary modes of operations, an individual is then capable of organising and re-organising, transforming, classifying, integrating, constructing new connections and interconnections, and restructuring relationships and inter-relationships (Gordon, 1986; Gregorc, 1979; Smith & Kolb, 1986; Kuchinskas, 1979; Nassp, 1986; Savacho, 1983).

Individuals have numerous cognitive controls that are descriptive of how they learn, namely, field dependence and independence, scanning, breadth of categorisation, cognitive complexity and simplicity, reflectiveness verses impulsivity, levelling verses sharpening, and tolerance verses intolerance. This study looks into the following three aspects of cognition in relation to mathematics achievement:

1 How an individual recognises or gains insight of ideas and concepts, that is his/her perception. Some perceive best from real experiences (concrete), while others are imaginative (abstract).

2 What an individual does with the acquired information, that is his/her processing of knowledge. Some listen and observe carefully prior to making a decision (reflective observation), while others learn through doing and practical approaches (active experimentation).

3 The tendency of an individual to use one side of the brain to perceive and function more than the other, that is his/her cerebral dominance and hemisphericity.

According to Kolb's (1986) experiential learning model, there are four dimensions to cognitive controls, namely, concrete experience (CE), abstract conceptualisation (AC), reflective observation (RO) and active experimentation (AE) (Figure 1).

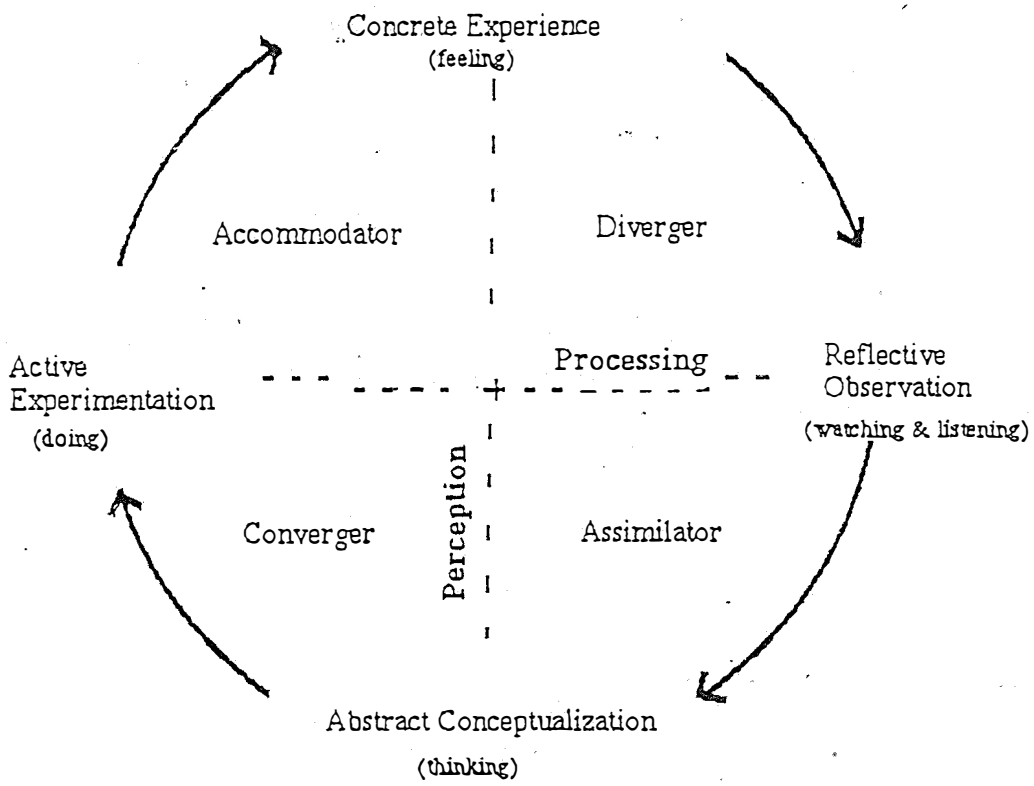

Figure 1: Modified version of the Experiential Learning Model (Kolb, 1976; Smith & Kolb, 1986)

The first two dimensions relate to how individuals perceive information and the latter two to the methods of processing information. Although they may co-exist with one another in an individual, one may predominate over the others. An individual using CE learns through specific experiences or being involved in feeling and intuition, whereas one using AC prefers thinking, logic and systematic analysis. In processing information, a learner with a cognitive control of reflective observation listens, watches, and observes, and one with a cognitive control of active experimentation learns by doing.

Can the explanations as to why and how individuals are different in mathematics achievement also be attributed to another cognitive dimension, that of hemisphericity? Hemisphericity is the tendency of a person to use one side of the brain to perceive and function more than the other. Research on the human brain indicates that there is a differentiation of functions between its right and left hemisphere. The left hemisphere is primarily concerned with language, verbal and logical processes while visual, spatial, gestalt perception and imaginal functions are primarily in the right hemisphere. The analytic-holistic distinction has been the most influential in moving thinking about hemispheric differences away from the original verbal-nonverbal dichotomy (Springer & Deutsch, 1985, p. 49). Studies have also shown that there is a distinct relationship between hemisphericity and academic achievement (Cody, 1983; Gwany, 1985; Koh, 1982; Yeap, 1989). In her study of Singapore students, Yeap (1989) found hemisphericity as a dimension that is predominantly different among three achievement groups. Will this finding still persist across ability groups in different content areas?

In acquiring mathematical knowledge and concepts and performing skills and procedures that require the competent use of mathematics, a successful learner possesses those aspects of cognition that enable him/her to master the discipline without much difficulty. On the other hand, inability to cope in understanding and processing the many complexities and intricacies inherent in the realm of mathematical knowledge may be attributed to the lack of the capacity to utilise high level cognitive skills, such as deductive and inductive reasoning and applying heuristics for problem solving, in processing mathematical information.

An individual's maturation, experiences and intellect presumably promote the growth of mathematical abilities. General principles which are the focus of most psychological research may not explain adequately behaviours in specific situations of mathematical learning, for example, what is involved in the process of acquiring mathematical knowledge and skills, of developing thinking processes, and of communicating in mathematics. A natural context with real life tasks is needed for the application of concepts in mathematical situations. This study focuses on behavioural patterns and mind qualities that constitute mathematical performance. In addition, it includes less observable behaviours such as perception, processing, and problem solving in cognitive psychology pertaining to questions such as: What do mathematics learners actually do when they are engaged in tasks? What goes on in their minds when an intellectual activity in mathematics is being carried out? What mental activities occur between posing the problem and the student's offering of an answer? What constitutes mathematical ability?

That individuals differ in mathematics achievement is a reality. The reasons why some individuals have mathematics ability, some suffer from mathematics anxiety or irrational fear

of mathematics or mathophobia are less obvious.

IMPORTANCE OF STUDY

The formulation of educational policy in Singapore has always taken into account the importance of the school curriculum and manpower training in the economic growth and development of the country. Within the framework of the educational policy, a curriculum is provided so as to develop the potential of every child to be an informed, responsible, thinking and creative person. A firm foundation in Mathematics has been recognised as the key to mastering technological skills. This again has been stressed in the Review Committee's report on the primary school education (1991). A child's mastery of mathematics will enhance his/her overall performance in school and, later, contribution as a skillful and productive worker in a sector of the country's economy. The findings of this study will be a valuable input towards realising the goals of mathematics education in Singapore.

The implementation of the 1992 new primary and lower secondary Mathematics syllabuses in Singapore schools brings with it a new philosophy and approach in mathematics teaching and learning. The emphasis on problem solving as the main focus is a shift away from the more traditional content curriculum of the past. The common notion that rote memorisation of tables, rules and algorithms makes a mathematician out of a person is no more regarded as absolutely relevant. Instead, a modern mathematics curriculum encompasses the development of mathematical concepts and skills as well as the underlying mathematical processes. This new perspective does not necessarily bring about a more enlightened learner without first having an enlightened teacher who is able to re-orientate himself/herself to the spirit of the new approach. This study is timely and is an attempt to understand the working of the intellectual capacity of learners which has bearing on how an individual engages in mathematical activities.

The assessment procedures in the Singapore's education system rely heavily on achievement scores but these do not necessarily cause an improvement in the method of educational intervention. Because of little local research as to how students process information and in our context, mathematical information, curricular changes have not advanced as fast as they should. The findings from this study will have serious implications on the type and mode of educational intervention for improving learning of students especially those needing remediation.

An individual's fear of mathematics or mathophobia or severe helplessness in mathematics may be learnt because of consistent failures in mathematics or exposure to uncontrollable, and usually, punishing outcomes related to mathematics learning. Such emotional state of intellectual block can vary among people in terms of level and intensity. To identify specific blocking or obstructing points and understand the cognitive processes

involved in the learning of mathematics may throw some light on the plausible causes of mathophobia for an individual thus making it possible to alleviate, if not eradicate, this fear and anxiety by providing cure or remediation.

Successful remediation requires correct diagnosis and prompt action leading to selection and carrying out of appropriate remediation programmes. In diagnosing students' learning weaknesses, teachers need to identify students' learning problems and their causes. Knowledge of the cognitive processes is a prerequisite for a teacher to better able to perform his/her function as an 'expert' in effectively overcoming students' learning difficulties.

OBJECTIVES

The study aims to find answers to the following research questions:

1 How are mathematical concepts perceived and processed among the three categories of mathematics achievers, namely, the high, average, and low ?

2 What are the cognitive profiles of the three categories of mathematics achievers?

3 How different are mathematical concepts perceived and processed among the three categories of mathematics achievers at two different levels of mathematics attainment?

4 What are the hemispheric profiles of the three categories of mathematics achievers at two different levels of mathematics attainment?

5 What is the inter-relationship among hemisphericity, perception, and processing of mathematical concepts by the three categories of mathematics achievers?

In this paper, the researchers will look into the first two questions.

SAMPLE

The sample consisted of 263 nineteen year old student teachers admitted into the former Institute of Education in July 1990 for the Diploma in Education programme which prepared them for teaching in the primary schools. All of them had graduated with 'A' level qualifications. They were divided into three categories of mathematics achievers based on their GCE 'O' level Elementary Mathematics results. The three categories were the High, the Average, and the Low achievers.

Table 1: Distribution Of Students By Sex

	Frequency	Percentage
Male	27	10.2
Female	235	89.4
No Response	1	0.4
Total	263	100.0

The distribution of the sample by sex is shown in Table 1. The small percentage of males in the sample was not unusual as it was representative of the whole population of student teachers enrolled in the various programmes in the Institute. It is possible that sex might make a difference in the way students perceive and process mathematical information. However, the part of this study dealing with sex difference would not be presented in this paper.

Table 2: Distribution Of Students By Highest Level Of Mathematics Taken

	Frequency	Percentage
GCE 'O' Elem Maths	58	22.1
GCE 'O' Add maths	16	6.1
GCE 'AO' Maths	93	35.4
GCE 'A' Maths C	90	34.2
GCE 'A' Further maths	5	1.9
No Response	1	0.3
Total	263	100.0

In Table 2 above, although all of the applicants had to have at least a pass in Elementary Mathematics in the GCE 'O' level examination for admission into teacher training, only 22.1% of them did not have a higher level of Mathematics beyond this minimum requirement. In fact, slightly more than one-third, i.e. 36.1%, were holders of at least a Mathematics qualification at the GCE 'A' level.

Table 3: Classification & Distribution Of Students According To Grades Obtained In GCE 'O' Elementary Mathematics

	Grades	Frequency	Percentage
High Achievers	1	46	17.5
Average Achievers	2 & 3	168	63.9
Low Achievers	4, 5 & 6	40	15.2
No Response		9	3.4
Total		263	100.0

The classification of student teachers into 3 categories of mathematics achievers by using the grades as shown in Table 3 is considered appropriate as the distribution approximates the proportions which divide a normal distribution into the lower 18%, the middle 64% and the higher 15%.

INSTRUMENTATION

The following instruments were used for this study:

1 Demographic Data Inventory (Yeap & Chong, 1990)

The inventory of 10 items was designed to obtain the background information of the student teachers in terms of gender, mathematics ability and level. The information would identify the commonalities and differences of the student teachers in the study. The commonalities would enable the results to be generalisable through logical inference to a larger population having similar characteristics.

2 Hemispheric Mode Indicator (McCarthy, 1986)

The Hemispheric Mode Indicator is a self-descriptive and self-scoring preference inventory of 32 items that distinguishes two modes of information processing emphasising on the psychological domain of learning, that of hemisphericity which is defined as the tendency to use one side of the brain hemisphere more than the other.

3 Learning Style Inventory (Kolb, 1986)

The Learning Style Inventory is a self-descriptive and self-scoring preference inventory of 48 items designed to measure a person's preferred way of perceiving and processing information. The scoring process sums the rankings of the options into 4 scores to determine the individual's learning modes.

Preference inventories tend to depend on the individual's interpretation of self-perceptions of preference. Their choices could be influenced by the individual's past experiences related to the context, culture, socioenvironmental considerations, sex role and self image. Accordingly, these various contextual situations provide a wide variety of background experiences that induce large variability (Knolle, Gordon & Gwany, 1987).

Concern exists in the field of psychological measures that many psychological theories are based not on what people do but on what people say they do in self-descriptive inentories. People can be fairly accurate self perceivers and self descriptors and that can be one of the most powerful perspectives on behaviour (Yeap & Wong, 1991).

PROCEDURE

The three inventories were all administered to the student teachers in different classes, 30 subjects at a time and over a period of a week in November 1990. It took a total of about one and a half hours for each class.

The responses were recorded in the optical machine readable forms and keyed into the computer for processing. The Statistical Analysis System package was used to analyse the data collected.

DATA ANALYSIS AND DISCUSSION

Research Question 1: How are mathematical concepts perceived and processed among the three categories of mathematics achievers, namely, the high, average, and low achievers?

Research has shown distinct individual differences in cognitive controls or dimensions and information processing habits across achievement groups and gender (Brennan, 1985; Canning, 1983; Clark, 1984; Cody, 1983; Griggs & Dunn, 1984; Gwany, 1985; Koh, 1982; Marcus, 1979; Yeap, 1987; Yeap, 1989). Implicitly, the research question subsumes the following sub-questions: Would there be equally distinct differences in the ways individuals learn content? Will different mathematics achievers perceive and process mathematical concepts differently? Which cognitive dimensions are a general characteristic of mathematics achievers? Will there be cognitive dimensions that predominate among all types of mathematics learners? Which dimension or dimensions can distinguish the three mathematics ability groups in their common search for the same intellectual goals?

Figure 2 graphically illustrates the phenomenon of 'more' or 'less' of each cognitive dimension in relation to mathematics achievers' methods of perceiving and processing concepts.

Finding No. 1

Irrespective of whether they are high, average, or low mathematics achievers, the three groups use a combination of cognitive processes in the perception and processing dimension to acquire mathematical knowledge and concepts (Table 4 and Figure 2).

Table 4: Means and standard deviations of the cognitive dimensions/controls of different mathematics achievers

Cognitive Dimensions / Controls	Mathematics Achievers: High n = 46		Average n = 168		Low n = 40	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
Concrete Experience (CE)	30.9	8.3	33.7	8.7	34.8	6.7
Abstract Conceptualization (AC)	36.5	6.6	34.1	8.1	34.2	8.2
Reflective Observation (RO)	37.2	5.8	36.0	6.8	34.7	8.1
Active Experimentation (AE)	36.4	7.8	36.3	7.9	34.9	9.4

CE, AC } perception RO, AE } processing

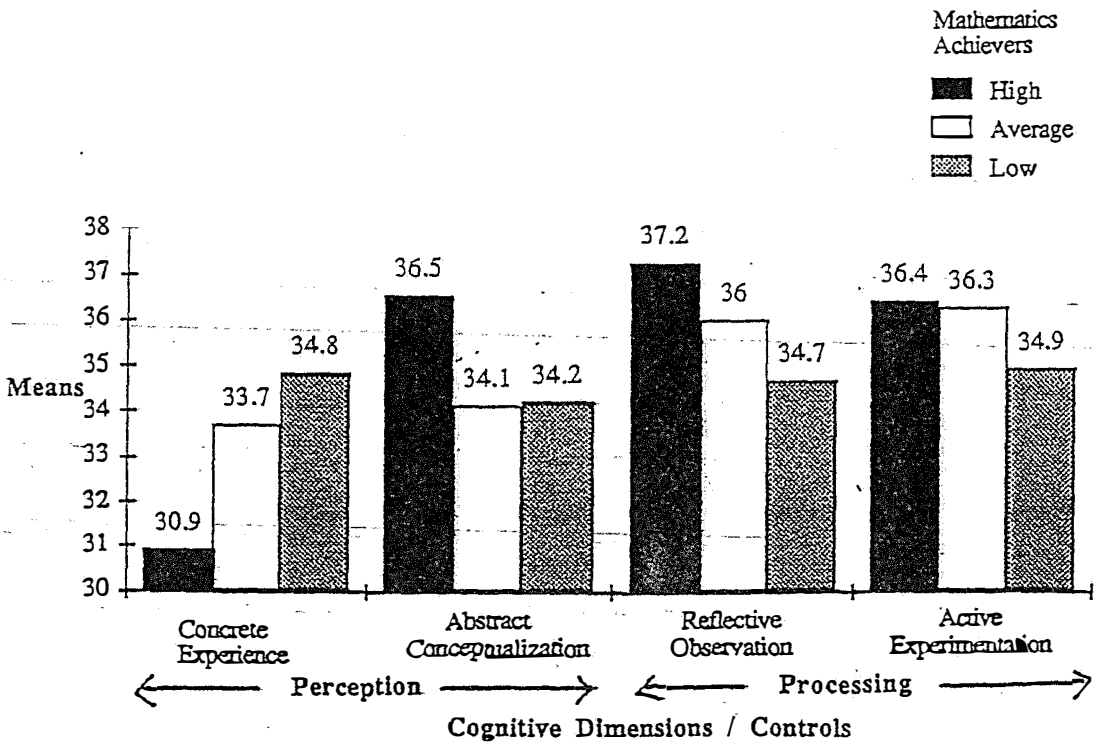

Fig 2: Means of the cognitive dimensions/controls of different mathematics achievers

All the groups perceive mathematical concepts using polar opposite cognitive controls of concrete experience (CE) and abstract conceptualisation (AC). Having acquired the knowledge, they process it by also using polar opposite cognitive controls of reflective observation (RO) and active experimentation (AE).

In understanding mathematical concepts, individuals need to use different degrees of the perception and processing dimensions in the learning cycle, namely, in a continuum of learning through specific experiences or being involved in feeling and intuition (CE), to logical and systematic analysis (AC). Having recognised or gained insight into the concepts, the individuals would process the acquired knowledge through a learning continuum of listening and observing (RO) to doing (AE) prior to coming to a decision.

There was no one mental quality that predominantly persists across the three mathematics ability groups. Rather, the mind qualities vary with the type of ability groups. It is the degree of the use of the different cognitive controls in the learning cycle that distinguishes the mathematics achievers.

Research Question 2 : What are the cognitive profiles of the three categories of mathematics achievers?

Figure 3 shows that there is a distinct cognitive profile for each category of mathematics achievers. In what ways are the cognitive profiles similar or different? Are they more similar in perception or more different in processing?

Finding No. 2

The cognitive profiles seem to suggest that the high mathematics achievers perceive predominantly less of concrete experience (CE) than abstract conceptualisation (AC) whereas the low achievers do not seem to have any predominantly cognitive controls.

High achievers do not have to acquire mathematical concepts by getting too involved in specific experiences, or through feeling and intuition (CE, 30.9). Instead, the continuum of perception is more towards that of learning through thinking, logic, and systematic analysis (AC, 36.5). Prior to making a final decision, the high mathematics achievers observe, watch and listen further (RO, 37.2) with the processing continuum decreasing towards learning by doing (AE, 36.4).

Compared to the high achievers, the average learners' perception becomes more of learning through involvement (CE, 33.7) but less of that of thinking, logic and systematic analysis (AC, 34.1); they also spend less time observing, watching and listening when they process the acquired information (RO,36.0); but they are just as involved in learning by doing (AE, 36.3).

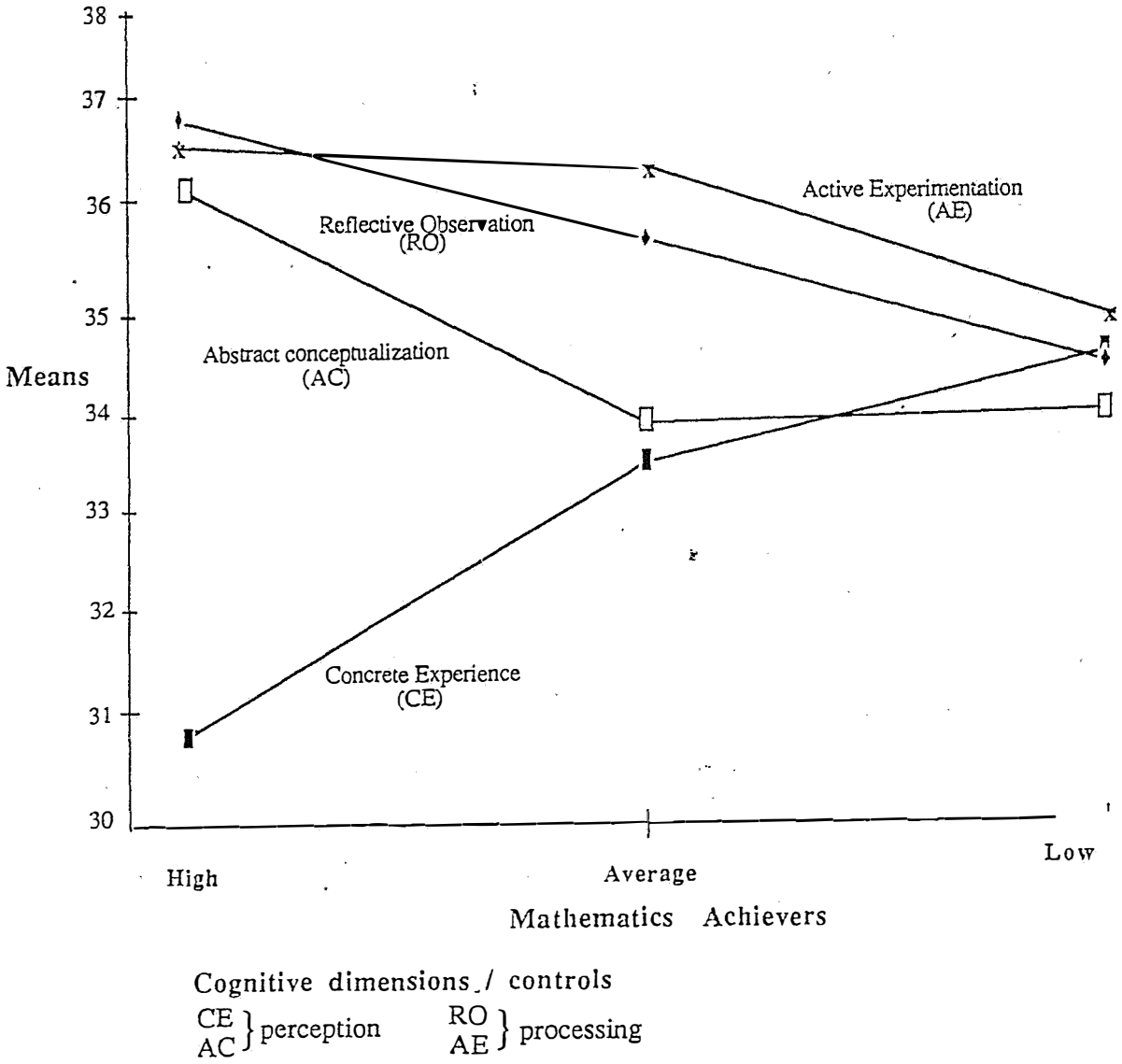

Figure 3: Means of the cognitive dimensions / controls of different mathematics achievers

The characteristics of the cognitive controls of the low achievers are distinctly different. In comparison to the high mathematics achievers, they learn very much through involvement in specific experiences, feeling, and intuition (CE, 34.8) but less through logic, thinking, and systematic analysis (AC, 34.2). This phenomenon of 'less of' is similarly shared in the low achievers' methods of processing the acquired knowledge. They use less of reflection, watching, and listening (RO, 34.7) to process the concepts and equally less by doing (AE, 34.9)

From the graph, one can draw the conclusion that as mathematics achievement increases, there is a decreasing dependency of learning through the concrete way of having to be involved in specific experiences (CE). Instead the continuum of perception is now more dependent on the abstract conceptualisation where thinking, logic and systematic analysis persist as the mind qualities. Similarly when mathematics achievement increases, there is an increasing use of reflective observation (RO) and active experimentation (AE) as cognitive controls to process the acquired knowledge.

Though all the three groups of mathematics achievers use a combination of different cognitive controls, certain cognitive controls predominate in certain mathematics achievers. High achievers think and use logic and systematic analysis a lot to recognise and have insight into ideas (AC). They watch, listen (RO) and practise (AE) more when it comes to deciphering the ideas.

The other extreme group, i.e. the low mathematics achievers, also uses a combination of cognitive controls, but the means of the different cognitive controls cluster together at a comparative low mean of 34. A totally balanced profile where there is a tendency to emphasise all the four modes usually is not necessarily best. This could be interpreted as the non-selective and non-effective use of distinct cognitive controls appropriate for a particular task. In the words of Kolb (1986), "The key to effective learning and adaptive coping is the ability to be flexibly competent in each mode when it is called for and not to use all modes in every situation" (p. 6). Because of their dependency of learning through involvement in specific experiences, feeling, and intuition (CE), and the lower means on thinking, logic, and systematic analysis (AC) and listening, observing, or practicing (RO), the absence of reliable concrete experience control would leave the low achievers helpless.

While there is also a clustering at the mean of about 36 for three of the four cognitive controls for the high achievers, they have a dominance at a very crucial control, that of abstract conceptualisation (AC) where the mind qualities like thinking, logic, and systematic analysis exist. The high achievers are even more dominant in the way they process information as they do not learn blindly without understanding or from feelings and intuition. They think and pay attention (RO). They also do and practice a lot (AE). For this group of high mathematics achievers, their dominant cognitive controls tend to be those of RO and AE. The average and low mathematics achievers appear to fall in Kolb's learning style classification of accommodator. The low mathematics achievers are divergers.

CONCLUSION AND IMPLICATIONS

Learner analysis is an extremely important component in the instructional design process as it identifies the characteristics of the learners that in turn influence the selection of learning resources and determine the type of appropriate instructional strategies. There are several factors about the learners that are crucial for pedagogical decisions in relation to media and methods selection. These factors include the learners' general characteristics, their specific entry competencies, and cognitive processes.

The findings of this study have identified yet another less observable dimension of individual differences to explain why certain students achieve highly in mathematics and others have

cognitive obstacles to the learning of mathematics. This would create dilemma for teachers in trying to meet the needs of individual differences, while at the same time facing the reality of a large number of students in each class. Yet the success of education depends on adapting teaching to individual differences among the learners. There may be many implications but in this paper, discussion is limited to one implication that follows.

Instruction should be around the students' actual cognitive learning processes appropriate to mathematical concepts. This could include consciously teaching students and teachers skills in observing, direct learning, listening, and thinking.

Attributing the high attrition rates to the inadequacies of the education system not catering to individual differences has led to frequent changes in educational policies which include revising the curricula, matching pace of learning with learners through streaming, adjusting the number of years required to complete certain levels of education, and introducing modern technology to convey course materials.

Instruction around the mental processes of the mathematics learners may have to include the development of a curriculum that incorporates the teaching of process awareness and the use of the process strategies in the learning of content areas. It can only be effective if all these become an explicit part of the classroom activities.

While research (Jarsonbeck, 1984; Brennan, 1984; Stice, 1987) has shown increased learning and achievement through matching teaching and learning styles, yet it may not be sufficient only to use persistently their preferred cognitive processes. They may have to diversify by adopting process flexibility which can be essential and useful in a complex and demanding society that warrants learning through multidimensional modalities. Modern society places increasing value on the students' abilities to read and write well, to reason in numeracy, to manipulate the computer keyboard, to think critically and to solve problems.

One such possibility is to teach students certain process skills to make them aware of the modes and strategies which are essential but which they have been avoiding or rarely put into use without realizing their effectiveness. With process awareness, students begin to assess their own cognitive controls. They are in possession of lifelong tools not only to learn mathematics but to learn other subjects and yet to be applicable to situations beyond the school.

(a) Observation

Compared to the high mathematical achievers, the low mathematical achievers were found to be very concrete experience learners (feelings), but less of the abstract conceptualisation

(thinking, logic, and analysis) to gain insight into the learning of concepts, and process the acquired information in no distinctly dominant way. They use less of observation to process, and less of thinking to perceive (Figure 3). They are classified divergers who can view concrete situations from many perspectives. Being imaginative, they can generate ideas well which is characteristic of humanities and the arts major.

Each category of learning types has its strengths and weaknesses. However, according to Stice (1987), "people tend to learn more effectively as they develop learning skills in their areas of weakness" (p. 291).

Low mathematical achievers can be taught observation skills (reflective observation) which are dominant in the high mathematical achievers. For example, Devine (1987) explored the possibility of using drawing to train observation skills. Drawing, graphing, contructing, and recognising patterns are good strategies as it requires careful observation of details, overall spatial relationships, complete wholes, visual representation, the different perspectives of two or three dimensional objects.

(b) Direct Learning

High achievers were found to be very much less of a concrete learner. They think and analyse a lot (abstract conceptualisation). Because their interest in the logical soundness of ideas, theories, and abstract ideas, "they may probably be less interested in people... making them unable to apply what they know in practical situations" (Stice, 1987, p. 292). There may be a need to develop the high mathematical achievers in their learning modalities through the concrete experience to enable them as teachers to relate with this group of pupils.

Major learning modalities of direct learning are multisensory through auditory, visual, kinesthetic, and tactile learning. Auditory and visual modalities are already widely practised modes of learning and teaching in the schools but schools often overlook the kinesthetic and tactile components of learning as they are usually outside of conscious awareness though they are the third major channel for taking in information and remembering it. Tactile and kinesthetic learners take in information most easily through their hands and through movements. The concreteness of kinesthetic experience may help them if they have difficulty over abstract concepts.

In mathematics, materials that rely on manipulation of objects provide concrete experience as the basis for understanding concepts. Patricia Davidson (cited in Williams 1983) distinguishes between materials that use a "discrete or set approach and those that use a continuous or length approach..... ...a discrete (set) approach uses counters and grouping of objects, for example, a single popsicle stick equals one, ten sticks in a cup equals ten, ten cups of ten sticks each on a

plate equals one hundred. A continuous (length) approach relies on measurement and spatial sense - for example, (cuisinaire rods) ten unit blocks placed in a row are the same length as the single ten rod" (p.153)

Such manipulative objects using easily available materials allow for the teaching of mathematical topics such as simple logic, number concepts, pattern recognition, mathematics operations, mensuration, geometrical shapes, solids, spatial relationship through the kinesthetic approach to learning.

(c) Listening

Is it not a common complaint among teachers that their students do not listen? Yet Wilt (cited in Devine, 1985) found 60% of the pupils' time was spent listening which is the primary means by which all incoming information is taken in before one speaks, writes, or organises the oral presentation. It is even more serious as one's understanding of instruction depends on how well the student listens correctly. This becomes incredibly important because teachers still use a lot of the verbal mode to explain, clarify, and instruct, though not necessarily effective, inspite of the many available and highly advocated educational approaches to effectively reduce talking during teaching through the practice of silent reading, directed reading, independent study, individualised systems, group interaction, cooperative learning, and mediated instruction. But it is this very cognitive process that is not a strength of the low and average mathematical achievers unlike the high mathematical achievers where listening (reflective observation) is dominant in them.

To listen effectively means being able to listen with understanding, take relevant notes including the main points, follow the flow of the presentation, paraphrase, summmarise, relate, evaluate and use the ideas. Listening is more than being attentive as the process involves the spoken language being translated to meaningful interpretations that assist the individuals in their academic performance. Wanting to listen also depends on the students' motivation, the state of the mind, and the physical being of the listener.

Are the students listening when specialised vocabulary like cone, angle, mean, plane, degree, parallel are explained? Could the written form help in their registering the concepts? Are they following when symbol systems like decimal point, equal and inequality signs, and a range of algebraic, geometrical and calculus symbols are taught?

Studies cited in Devine (1985) showed that listening skills can be taught but are neglected because of the myth that listening is inherent, the time factor to cover the teaching objects within a limited time, and the little training given to teachers.

(d) Thinking

A very distinct cognitive process associated with the high mathematical achievers is abstract conceptualisation, where characteristics like thinking, logic and systematic analysis are prevalent. However these characteristics become less dominant when the mathematics achievement in the learners decreases.

Thinking is a difficult term to define as it deals with too many abstractions. But Devine (1985) defined it as "a course of ideas, symbolic in characteristics, initiated by a problem or task, and leading to a conclusion" (p. 103). Thinking involves high order cognition dealing with mental conflicts involving defining purpose, drawing inferences and relationships, making anticipations, and comparing and contrasting.

The key thinking skills in mathematics include analysing, reasoning, and inference which are central to mental processes in mathematics. In manipulating number concepts, solving algebraic equations and word problems, using geometrical ideas, and interpreting statistical data, students need to use their thinking skills to analyse, reason, and infer. They can practise on drawing inferences by having teachers explain and discuss inferences, distinguish inferences from facts and opinions, give practice in inference making, have students make inferences from printed texts, explain how inferences work as explanation (Devine, 1985, p. 114).

In conclusion, to remove cognitive obstacles in mathematics learning and achievement, individuals need to have or to cultivate in them cognitive dimensions that minimize their learning helplessness. From the findings, 'good' cognitive controls that are dominant in the high mathematics achievers appear to be those of abstract conceptualisation for the perception dimension continuum and the reflective observation for the processing dimension continuum. There should be less reliance on concrete experience where acquisition of knowledge is without understanding, thinking, or analysis.

The study identifies cognitive processes among mathematics achievers that can positively or negatively affect the students' mathematics achievement. Cognitive testing gives a qualitative picture of cognitive profile of a person's strengths and weaknesses. Instead of the traditional vertical dimension of who is better or who is worse on a performance test, a horizontal dimension of the relative performance between information perception and process tasks is available for comparison. Neither can one conclude that concrete experience or abstract conceptualisation can be claimed to be better. They only differ in function as they are differentially aroused by the types of incoming information. As people do not process tasks in the same way, there is a need to recognise and accept the fact that there are equally valid methods of acting upon, processing, perceiving, and storing information.

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