Title: A state-of-the-art review on mathematics education in Singapore: An overview
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A STATE-OF-THE-ART REVIEW ON MATHEMATICS EDUCATION IN SINGAPORE - AN OVERVIEW

MS BERINDERJEET KAUR

Paper presented at the
International Seminar on
'State-of-the-Art Research in Science and Mathematics Education
Southeast Asia and the Pacific'
in Penang, Malaysia, from February 17-19, 1992
A State-of-the-Art Review on Mathematics Education in Singapore - An Overview

by

Berinderjeet Kaur (Presenter)
on behalf of the Singapore research team
Dr Chong Tian Hoo
Dr Khoo Phan Sai
Dr Foong Pui Yee
Mdm Kaur, Berinderjeet
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National Institute of Education
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Synopsis:

A state-of-the-art review in mathematics education in Singapore was carried out by a team of researchers in the National Institute of Education in late 1990. The report was published in April 1991.

As there has never been a concerted research agenda for research in mathematics education in Singapore, the studies reviewed originated from the interests of specific individuals or groups.

The review was part of a regional effort and commissioned by SEARRAG (Southeast Asian Research Review and Advisory Group). It was predetermined that each country classify the studies into five topical areas of interest (Teaching and Learning, Assessment and Examinations, Teacher Education, Development Projects, and Curriculum Development, Implementation and Evaluation). As bulk of the studies reviewed fell into the category Teaching and Learning, this category was further divided into seven sub-headings, namely affective variables and problem solving, types and levels of understanding, analysis of errors, low achievers and remediation, learning strategies, use of microcomputers, and other variables.

Forty-two selected studies were reviewed, an important criterion for selection being relevance and representativeness of the topical area of interest rather than rigour in methodology or thoroughness in conceptualization.

In addition to highlighting the major thrusts and strengths of research on mathematics education during the last decade or so, the review has pointed to gaps or areas requiring further attention.
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1. Introduction

1.1 BACKGROUND

This State-of-the-Art Review on Mathematics Education in Singapore was commissioned and funded by the Southeast Asian Research Review and Advisory Group (SEARRAG). It was one of a series of similar reviews being carried out in each of the ASEAN countries. In Singapore it was carried out by a team of researchers in the National Institute of Education in late 1990. The report was published in April 1991.

The Review had a dual purpose. The first purpose was the compilation of abstracts of selected research studies in mathematics education that had been conducted in Singapore in the last decade. A copy of the compilation of abstracts being kept in the National Institute of Education Library so as to be readily accessible to researchers in Singapore. Also, the compilation of abstracts together with those from the other ASEAN countries have been incorporated into the database SEABAS (Southeast Asian Bibliographic and Abstracting Service) operated by SEARRAG in cooperation with the Universiti Sains Malaysia in Penang, Malaysia. The second purpose was the writing of the report based on an analysis of the selected research studies. The selection of studies was fairly comprehensive as it was based on documented research reports, theses, dissertations, journal articles, and proceedings of conferences wherever the items were located.

1.2 OBJECTIVES OF THE REVIEW

The objectives of the Review were as follows:

(i) to survey the current status of mathematics education research as well as significant findings;

(ii) to suggest implications of the research work done; and,
(iii) to identify factors that should be considered in conducting future research aimed at the improvement of mathematics teaching and learning.

The objectives of the Review took into consideration the salient features of mathematics education in Singapore. For example, elementary mathematics has a crucial role in the school curriculum, being a compulsory subject in the primary school and an unavoidable subject in the secondary school. Thus the fact that mathematics is regarded as a fundamental core subject in the school stimulated questions like what are the pervading perceptions of the nature and usefulness of mathematics and how would these perceptions influence the teaching and learning of mathematics. Cognizance was also taken of the recent reformulation and promulgation of a new set of aims of mathematics education (Curriculum Planning Division, 1990):

"Mathematics education for the primary and secondary schools aims to enable pupils to:

- acquire the necessary mathematical knowledge and skills, develop thinking processes and apply them in mathematical situations that they will meet in life;
- use mathematics as a means of communication;
- develop positive attitudes and a sense of personal achievement in mathematics; and,
- appreciate the importance and power of mathematics in the world around them."

This set of aims of mathematics education has been incorporated into two revised syllabi (which will be effective from 1992), one for the primary schools, and the other for Secondary 1 and 2. The rationale for the revision, as claimed, was to reflect recent developments in mathematics education that had taken place in many other countries. In particular, the primary aim of the revised curriculum is to enable pupils to develop their ability in mathematical problem solving. The attainment of problem solving ability being dependent on five inter-related components: concepts, skills, processes, attitude and metacognition.

Hence identifying factors that should be considered in conducting future research ought to take into account not only the accomplishments of the past decade in research and curriculum development but also the challenges posed by such concepts as problem solving and metacognition.
1.3 SCOPE OF THE REVIEW AND SELECTION PROCESS

The five strands or topical areas of interest which delineate the scope of the Review were suggested by the SEARRAG review proposal. The five strands provided a convenient classification of the research studies.

They were as follows:

TL Teaching and Learning of Mathematics
AE Assessment and Examinations in Mathematics
TE Teacher Education in Mathematics
DP Development Projects in Mathematics

The category TE was originally designated as Pre-Service and In-service Training of Mathematics Teachers. It was however shortened to Teacher Education in Mathematics in this Review.

The process of selecting the studies began by acquiring a documented listing of research studies and reports from the Institute of Education Library through the use of the education information online database search. Requests were also made to relevant departments of the Ministry of Education (MOE) and the Curriculum Development Institute of Singapore (CDIS). Personal collections especially of project or research reports and conference proceedings were also consulted.

Forty-two selected studies were reviewed, an important criterion for selection being relevance and representativeness of the topical area of interest rather than rigour in methodology or thoroughness in conceptualization.

The following table highlights the distribution of the studies by year of publication.
The large increase in the number of studies from about 1983 onwards was attributed to the re-vamping of the M.Ed. programme in the IE (Institute of Education, Singapore) and the founding of the ERA (Educational Research Association of Singapore) in 1986. Of the 42 selected studies, only 16 or 38% are theses or dissertations presented for a M.Ed. or Ph.D. degree, a number which is somewhat less than what might be expected.

Table 2 below shows the distribution of the 42 selected studies arranged by each of the topical areas of interest as well as by the type of research method.

**TABLE 2 : Summary of Studies by Research Method**

<table>
<thead>
<tr>
<th>Study Type</th>
<th>TL</th>
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<th>TE</th>
<th>DP</th>
<th>CD</th>
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<td>ckm</td>
<td>kb2</td>
<td>gks</td>
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<td>ost1</td>
<td>csh</td>
<td>lsk</td>
<td>fhk2</td>
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<tr>
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<td>ccm1</td>
<td>tch</td>
<td>la</td>
<td>ost2</td>
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<td>wp1</td>
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<tr>
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<td>pkj</td>
<td>lmk</td>
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<td>tpk</td>
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<td>wp1</td>
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<tr>
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<td>School-based</td>
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<td>ysk</td>
<td></td>
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<td>Studies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Number</td>
<td>26</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>42</td>
</tr>
</tbody>
</table>

Each study is represented by the initials of the name of the researcher. TL, AE, TE, DP and CD stand for the topical areas of interest.
2. **Studies Reviewed**

The review provided a telescopic view of the nature and scope of the research work that had occupied the attention of researchers in mathematics education in Singapore in the last decade.

2.1 **Teaching and Learning of Mathematics**

26 out of the 42 studies i.e. 62% were classified under the heading of Teaching and Learning in Mathematics. The preponderance of studies in this area probably demonstrated the preoccupation of researchers on topics of research which had manifest implications for teaching and learning mathematics in the classroom.

The 26 studies were reviewed under 7 sub-headings as follows:

1. **Affective variables and problem solving**

2. **Types and levels of understanding**

3. **Analysis of errors**

4. **Low achievers and remediation**
   Fong (1987), Ee (1990) and Yap (1990)

5. **Learning strategies**
   Ng C.K.(1985), Wong K.Y. (1990), and Wong P. (1990)

6. **Use of microcomputers**

7. **Other Variables**
   Chai (1979), Ang (1984), and Kaur (1987)

2.1.1 **Affective variables and problem solving**

The study of Foong (1990) investigated successful and non-successful problem solving processes by observing a group of 57 pre-service (primary and secondary school) teachers solve non-routine mathematics problems. The verbal data was obtained through a think-aloud
methodology and the application of protocol analysis produced a taxonomy of five different aspects of problem solving activity such as problem orientation heuristics, problem solution heuristics, metacognition, domain-specific knowledge and affective behaviours.

The study of Wong P. (1989) was concerned with a survey of metacognitive processes as used by secondary students when solving mathematics problems.

The three other studies also classified under this subheading had, as a common feature, the focus on the same variable of anxiety towards mathematics. Foong (1985) however focussed her attention on only female Secondary 4 students and showed that there were various interrelationships among mathematical anxiety (as measured by the Fennema-Sherman Anxiety Scale) (Fennema-Sherman, 1976), test anxiety, mathematics achievement, and interpersonal variables such as perceptions of teachers' or parents' attitudes towards the students themselves.

Ng-Gan (1987) validated the Mathematics Attitude Inventory (MAI) developed by Sandman (1979) and explored the relationship between the various MAI sub-scales with mathematics achievement of Secondary 4 and 2 Express and Normal stream students.


2.1.2 Types and levels of understanding

The five studies reviewed under this sub-heading were considered in three groups

(a) an experimental investigation based on the distinction between instrumental and relational understanding (Purbrick, 1982);

(b) two studies concerned with the solving of addition and subtraction open sentences, and mastery of the subtraction algorithm (Lam, 1985 and Chu, 1987); and,

(c) levels of understanding of the concepts of area (Tan C.S., 1987) and of the concepts of reflection, rotation and enlargement (Tay C.H., 1986)

The trigonometric concepts of sine, cosine and tangent were used by Purbrick (1982) to determine whether an
approach based on a set of experimental materials and activities specially designed to promote relational understanding (following the notion of Skemp, 1976) would be more effective than the traditional approach of classroom teaching of these concepts, the latter being assumed to promote instrumental understanding.

In his study Lam (1985) confirmed much of the work of Weaver (1971), for example, pupils have the greatest difficulty in solving sentences of the following types: 

\[ a + b = c \quad a - b = c \quad a + b = c \quad a - b = c \]

The study of Chu (1987) was primarily concerned with determining the levels of understanding of the subtraction algorithm and succeeded in confirming much of the work of Cox (1975).

The concepts of area and the concepts of reflection, rotation and enlargement were researched by Tan C.S. (1987) and Tay C.H. (1986) respectively, using similar statistical and analytic techniques to determine which groups of test items should be grouped together to form a particular level of understanding.

2.1.3 Analysis of errors

The paper of Booth (1986) reported on an indirect method of getting pre-service teachers to be involved in the research process of identifying errors made by children in doing multiplication sums.

The three studies of Chai and Gan (1987), Ong and Lim (1987) and Kaur (1991) had the common feature that each relied on a test of algebra to obtain an insight into the errors or misconceptions of secondary students at different secondary school levels. Chai based her study on an adaptation of the CSMS Algebra Test and discussed her findings using the framework of Kuchemann (1978). Kaur examined the responses of 25 Secondary 3 Express students and classified their errors in terms of categories such as conjoining in algebraic addition, non-use or improper use of brackets and misconception of the meaning attached to letters.

The study of Ong and Lim covered 3 groups of students, ranging from Secondary 4 to pre-university students, and included an interview of 60 of the students in addition to the administration of an algebra test.
2.1.4 **Low achievers and remediation**

Fong (1987) carried out an experiment to determine the relative efficacy of a thinking strategies approach for learning the multiplication facts in conjunction with CAI (computer-assisted instruction) drills versus a traditional teaching approach plus the use of CAI drills.

The experiment of Ee (1990) compared the use of count-on and count-up strategies with count-all and take-away strategies for the learning of addition and subtraction facts by ESN (educationally sub-normal) children.

The paper of Yap (1990) illustrated the process of thinking, planning and conceptualization that are necessary before the implementation of a school-based action-research.

2.1.5 **Learning strategies**

The study of Wong K.Y. (1990) was a questionnaire survey aimed at setting base-line data concerning the types of learning strategies used in mathematics lessons by a sample of 1318 secondary students ranging in level from Secondary 1 to Junior College.

The research of Wong P. (1990) was however concerned with the effectiveness of an instructional programme which requires students to apply elaboration techniques to mathematics learning.

Peer tutoring was the focus of the study of Ng C.K. (1985) which was designed to compare the effects of a tutoring versus a non-tutoring programme on achievement as well as affective measures.

2.1.6 **Use of microcomputers**

The study of Tan P.K. (1987) and that of Woo-Tan (1989) were aimed at showing that an experimental CAI treatment was more advantageous than a traditional expository or teacher directed approach in the learning of (a) linear equations, and (b) transformation geometry concepts such as reflection, shearing and stretch.

The study of Ho C.C. (1990) was aimed at determining the efficacy of a CAI method for conducting remediation lessons for Secondary 3 Express stream students.
2.1.7 Other Variables

The study of Chai (1979) was carried out in the days when the so-called "modern mathematics" was in vogue. The study examined the preferences of Secondary 3 Science stream students for three modes of presentation, i.e. verbal, symbolic or diagrammatic.

The study of Ang (1984) covering a sample of 20 schools was aimed at comparing the mathematics attainment of Secondary 2 Express with Normal stream students on core as well as optional topics in the mathematical syllabus.

The study of Kaur (1987) was concerned with gender and mathematics attainment of Singapore pupils in the November/December 1986 G.C.E. "O" level examination.

2.2 Assessment and Examinations in Mathematics

Three of the five studies included in this section illustrated applications of item response theory (Drasgow, 1982). The study of Chang K. M. (1980) pioneered the use of the Rasch model by attempting to develop a scale of mathematics test items that would permit an objective assessment of pupils' progress from Primary 1 to Primary 6. The study of Ismail (1986) continued the work by using a national representative sample of Primary 1 to 3 pupils to determine performance levels in the basic computation skills. The paper of Koh and Cheung (1990) attempted to show how meaningful measurement can be constructed for a particular concept such as the part-whole concept of fractions.

The study of Plant (1985) concerned modes of presentation of mathematics problems and had implications for assessment.

The study of Chen-Yip (1990) examined learning difficulties in mathematics by using a set of test items constructed on the basis of the SOLO (Structure of Observed Learning Outcomes) taxonomy (Biggs and Collis, 1982; Collis et al., 1986) as well as analyzing the response data by using a particular error classification model (Movshovitz-Hadar, 1987).

2.3 Teacher Education in Mathematics

The five studies in this section reflected mostly the analytic and positivistic perspective. The study of Kaur (1990a) was prompted by a concern that pre-service primary school teachers should have a strong mathematical knowledge.
The study of Lim and Wong (1989) sought to identify the perceptions of the characteristics of an effective mathematics teacher.

The study of Wong K.Y. (1987) was aimed at determining whether the concerns about teaching were perceived differently by pre-service and practising teachers. The study of Ong (1987b) was aimed at investigating the validity and reliability of an Observer Rating Scale of Teaching Competence in Mathematics (ORSTCM).

The study of Lee-Leck (1985) was based on the argument that since teachers can influence their students' attitudes towards mathematics, then an instructional method (CAI in this case) for improving their own attitudes and at the same time enhancing their own mathematical learning, if shown to be effective, would have useful implications for teacher training.

2.4 Development Projects in Mathematics

The two reports of Ho W.K. (1983) and Chin et al. (1987) were classified under the heading Development Projects in Mathematics. Ho reported on a pilot project which was aimed at exploring the feasibility of adapting the mastery learning model (ML) of Bloom (1984) to meet the needs of Secondary 2 Normal stream students in a particular school.

The report of Chin et al. (1987) concerned the relative effectiveness of a cooperative versus an individualistic approach to teaching Mathematics and English.

2.5 Mathematics Curriculum Development, Implementation and Evaluation

In 1983, the team of Ko et al. solicited teachers' opinions regarding several aspects of pupils' texts, workbooks and teachers' guides.

Gan et al. (1984) carried out actual classroom observations to determine how teachers used the PMP (Primary Mathematics Project) materials in their lessons. In 1986, the team of Wong K.Y. et al. performed an intrinsic evaluation of the content and teaching method advocated by the PMP developers as contained in the written materials for pupils and teachers.

The study of Fong (1988) was undertaken to evaluate the mathematics curriculum (specifically, a tutorial programme) of a private organization, the Yayasan Mendaki, which is dedicated to the welfare of the Malay community.
2.6 Summary


Of the 26 studies classified under the topical area of Teaching and Learning of Mathematics, as many as 11 were concerned with some aspect of mathematical content knowledge, whether understanding of concepts, learning of skills, errors or misconceptions; for example, trigonometric concepts (Purbrick, 1982); addition, subtraction and multiplication (Lam, 1985; Chu, 1987; Booth, 1986); area and transformation geometry (Tan C.S., 1987; Tay C.H., 1986; Woo-Tan, 1989; Lee-Leck, 1985); algebra (Chai and Ang, 1987; Ong and Lim, 1987a; Kaur, 1991).

This preoccupation with research on mathematical content knowledge was contrasted with only four studies involving the use of the microcomputer (Ho C.C., 1990; Tan P.K., 1987; Woo-Tan, 1989; Lee-Leck, 1985).

A further contrast was provided by noting that there were as many as eight studies dealing with instructional intervention in one form or another (omitting studies dealing with CAI or remediation): cooperative or mastery learning, learning strategies or peer tutoring (Chin et al., 1987; Ho W.K., 1983; Wong P., 1990; Ng C.K. 1985); basic skills and thinking strategies (Ee L.C. 1990; Lam, 1985; Fong, 1987a); teaching trigonometry (Purbrick et al., 1982).

3. Synthesis and Conclusion

This section highlights the major thrusts and strengths of research on mathematics education reported in the review during the last decade as well as points to gaps or areas requiring further attention.

3.1 Research Methods and Levels of Subjects

Majority of the studies used a survey method involving the administration of a questionnaire, a test or an attitude scale, or a quasi-experimental method (most popular being the pretest - posttest control group design).
Many of the researchers of the studies reviewed were more comfortable or more experienced with analytic approaches although recent focus has been on the distinction between analytic and humanistic approaches (Brown et al., 1990).

Table 3 provides a summary of the studies classified by publication source (top half of Table 3) as well as by the levels of the subjects (bottom half of Table 3).

16 or 38% of the studies were theses or dissertations as reported earlier with the rest being research reports, journal articles or conference papers. It was evidently clear that there has been continued interest in research after the first presentation of a thesis or dissertation.

From Table 3, it is clear that there has been a preference for primary school and lower secondary levels and a lack of interest in Junior College or pre-university subjects for study.
### TABLE 3: Summary of Studies by Publication and Level of Subjects

<table>
<thead>
<tr>
<th></th>
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<td>15</td>
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<td>42</td>
</tr>
</tbody>
</table>

* Each study is represented by the initials of the name of the researcher. TL, AE, TE, DP, and CD stand for the topical areas of interest.
3.2 Synthesis of Research topics

Many of the studies reviewed were one-time and exploratory in nature. Thus, the findings even when significant can be considered tentative and perhaps not easily generalizable. Interestingly, there has not been a single case of a research finding in the studies reviewed which contradicted or opposed the trend of known results in the corpus of related literature. Hence much of the research can be said to be on the right track, except that each being carried out in isolation; lacked continuity, coordination or direction. The only exception being the studies of Wong P. (1989 and 1990) which were by-products of an ERU Project ITL1 Effectiveness of Learning Strategies (Chang S.C., 1988).

In an attempt to map the direction of future research the following domains of research interest have been placed in a wider context taking into account the studies reviewed.

3.2.1 Problem Solving

The new set of aims of mathematics education which has been initiated in 1992 has as its primary focus mathematical problem solving.

The research of Foong (1990) and Wong P. (1989) on a taxonomy of problem solving activity and the use of metacognitive skills by students are not only important contributions but also the beginnings of more research on problem solving.

Research on mathematics problem solving is growing apace (Charles and Silver, 1988) in other countries and with the introduction of problem solving into the mathematics curriculum, it is to be expected that interest in future research would be directed at topics such as (a) assessment of problem solving skills and knowledge; (b) teachers’ pedagogical knowledge or schemata which they employ to apprehend student understandings or misconceptions (Shulman, 1986); and, (c) teachers’ and students’ views of what constitutes a problem in mathematics and beliefs about what it means to do mathematics (Khoo, 1990).

3.2.2 Learning Strategies

Relevant to this section are three groups of studies reviewed: (i) surveys of learning strategies or metacognitive skills of secondary school students (Wong P., 1989; Wong K.Y., 1990) and experimentation with
elaboration techniques for learning mathematics by secondary school students (Wong P., 1990); (ii) teaching of primary school children by using thinking strategies for multiplication or count-on and count-up strategies (Fong, 1987; Ee, 1990); (iii) cooperative learning, mastery learning or peer tutoring intervention programmes for secondary school students (Chin et al. 1987; Ho W.K., 1983; Ng C.K., 1985).

While all the studies had the common aim of helping to improve student learning and/or promoting positive attitudes, an implicit assumption in the application of the intervention or treatment programmes seems to be the use of teacher control and direction of student learning in the context of what is known as direct instruction. There is ample evidence to suggest that (a) training on cognitive strategies can significantly enhance students' learning (Weinstein and Mayer, 1986), and (b) using small group cooperative learning as an adjunct to whole-class instruction can be effective (Slavin, 1991; Peterson et al., 1984).

Bearing in mind that in the new aims of mathematics education, ancillary to problem solving, there is also "emphasis on pupils' communication of mathematical ideas stressing both precise communication as well as unique personal expression ..." (House et al., 1988), a variety of factors affecting mathematical communication in the mathematics classroom merit attention. Future research might want to investigate (a) the context or setting which might influence the use or non-use of learning strategies and (b) the extent and nature of practices which encourage student autonomy, independence, self-direction and persistence in learning (Peterson, 1988).

3.2.3 Pupils' understanding

In section 2.6 it has been mentioned that 11 of the studies reviewed had mathematical content knowledge as focus.

An alternative view of these studies would be that they are attempts to determine levels of understanding or whether relational understanding would promote better learning, or identify various types of misunderstanding. All the studies however seemed to be based on an analytic perspective in the sense that subject matter content is seen as separate from the learner or in the sense that mathematics is a received body of knowledge.

It seems possible that insufficient attention may have been given to the idea that the errors or misunderstandings that students make are also a function
of the way that they are taught and not exclusively determined by the nature of the mathematical tasks (Thompson, 1989). It has been shown that students' mathematical errors do not occur at random but originate in a consistent conceptual framework based on earlier acquired knowledge (Nesher, 1987). This line of thinking, it can be argued, would lead eventually to the following position:

"Constructivist views and theories of the way pupils build representations of mathematical concepts strongly suggest that teachers cannot merely be broadcasters of facts and theorems. Teachers must provide classroom environments and activities that will facilitate pupils' active construction of mathematical knowledge" (House et al., 1988).

Various implications for research can be drawn from the above line of argument. It is sufficient here to consider possibilities such as: (a) What is the nature of children's informal knowledge in various mathematical topics and how can teachers build their teaching systematically on this knowledge? (b) What are the kinds of incorrect or "buggy" algorithms that involve the systematic misapplication of algorithmic procedures and how can "buggy" models be utilized for diagnosis and instruction?

3.2.4 Affective variables

Only three studies, namely, Foong (1985), Ng-Gan (1987) and Tan O.S. (1989) are specifically directed at the relation between affective variables and mathematics achievement.

With the introduction of problem solving into the mathematics curriculum, research interest is likely to be drawn to non-cognitive and affective variables. Already metacognition skills have received attention (Wong, P., 1989) but it is increasingly being recognized that a student’s success or failure in solving a mathematics problem is often due to such factors as self-confidence, motivation, perseverance and beliefs as much as the possession of appropriate mathematical knowledge. Thus future research might consider the role of factors such as affects (emotions, preferences and attitudes), beliefs, control (or metacognition) and contextual conditions in problem solving in mathematics (Lester et al., 1989).

Research might want to investigate conjectures such as (i) an individual's beliefs about self, mathematics, and problem solving play a dominant, often overpowering role in his/her problem solving behaviour; or (ii) metacognitive training is likely to be most effective
when it takes place in the context of learning specific mathematical concepts and techniques (Lester et al., 1989).

3.2.5 **Use of microcomputers**

Since there is no doubt that the microcomputer or the electronic calculator can support and enrich existing classroom instruction, it is rather surprising to find only four such studies reviewed and not more research reports or project reports or articles detailing the use of information technology in the classroom.

It is tempting to ask whether the kinds of misconceptions or errors reported in the three papers (Chai and Ang, 1987; Ong and Lim, 1987a; Kaur, 1991) would still occur in a modified environment of teaching algebra that relies heavily on the use of the microcomputer as a problem solving tool (Kieran, 1989).

3.2.6 **Mathematics curriculum**

There is a notable absence of research on matters pertaining to the curriculum in mathematics. Apart from the three evaluation studies on the Primary Mathematics Programme materials, there is the study of Kaur (1990a) into the basic mathematical knowledge of pre-service teachers. The study relates to the curriculum in the sense of the belief that a teacher should possess a sound knowledge of mathematics to a level beyond that at which he/she teaches.

With the emphasis on problem solving, doing mathematics investigations and communication in the classroom, perhaps some attention needs to be paid to questions such as the following:

(i) What is the role of the teacher as a change-agent in the mathematics curriculum? To what extent does the teacher drive the curriculum or does the curriculum drive the teacher?

(ii) Curriculum aimed at improving mathematics teaching and learning must be concerned with factors that contain the opportunity to learn, including matters of teacher quality and preparation. How can we identify these factors and manipulate them to our advantage.

(iii) It is not easy to achieve curriculum change. The outcomes are often very different from what is intended. For example, teachers may not play the roles in the classroom that would allow students
to acquire more independence in doing or using mathematics. How might we observe such changes (House et al., 1988)?

3.3 Conclusion

The following conclusion was reported not as inevitable but as a challenge to raise the level of attention in research on mathematics education towards improvement in mathematics education. In the following quotation, the variable xxxxxx can take on the value "mathematics" as well as the value "teacher".

"An obvious deficiency among studies purporting to be research in xxxxxx education is the lack of serious attention to subsequent improvements in xxxxxx education. Thus, at most a few of the studies reviewed reached Level 2, while the rest are at Level 1 in relation to improvement in xxxxxx education, as shown below:

Level 1: Little or no reference to possible improvements in xxxxxx education.
Level 2: Discussion of implication for possible improvements in xxxxxx education.
Level 3: Specific plans for improvements in xxxxxx education, based on the findings.
Level 4: Report of improvements from applications of the research.

(Source: Sim W.K. in Care Note 2, 3 April 1991, Institute of Education, Singapore.)

There is an assumption that research on mathematics education can make a difference to mathematics instruction. It is not obvious that this assumption is justified. Consider the following remarks of Wheeler (1986):

"The separation of researchers from practitioners, and of theory from practice, is common to most professional activity in many societies. The structure of academic institutions, which distinguish "pure" and "applied" science, and institutionalized differential career opportunities, reinforce the separation. In education the gap is particularly wide and distressing. This is not so much because teachers do not want technical help, nor because researchers would not be able to supply it, but because each group is embedded in a different situation with its own goals, responsibilities, and rewards, none of which relate to communication between the two groups. Researchers are not recognized by the extent to which they are informed about and use the latest research. The system of separation, meant to be rational and efficient, does not work."
REFERENCES


Curriculum Planning Division (1990). Mathematics syllabus. (Secondary 1 and 2; and, Primary Schools). Singapore: Ministry of Education.


**LIST OF THE STUDIES**

   A study of the relative attainment in mathematics of two matched groups of secondary two express and normal stream pupils. M.Ed. dissertation, National University of Singapore.

2. Booth, Lesley (1986) *(bl)*

3. Chai Chee Meng (1979) *(ccml)*
   A study of secondary school pupils’ performance in aspects of mathematics and their responses to a cognitive preference measure in some Singapore schools. MA in Ed. thesis, University of Reading.

4. Chai Chee Meng and Ang Gim Har (1987) *(ccm2)*

5. Chang Kwong Ming (1980) *(ckm)*
   Development of mathematical tests for Singapore primary schools. M.Ed. dissertation, National University of Singapore.
6. Chen-Yip Siew Har (1990) (csh)

The effectiveness of different teaching approaches in Mathematics and English: lessons from the fourth IE vacation learning camp. Monograph, Institute of Education, Singapore.


10. Fong Ho Kheong (1987a) (fhk1)

11. Fong Ho Kheong (1988) (fhk2)

12. Foong Pui Yee (1985) (fpv1)

13. Foong Pui Yee (1990) (fpv2)


15. Ho Chwee Choon (1990) (hcc)
16. Ho Wah Kam (1983) (hwk)  
Report on the mastery learning (pilot) project.  

17. Ismail, A. (1986) (ia)  
Applying prescriptive test theory to the  
assessment of the basic skills mathematics  
curriculum of Singapore.  

Sex differences in mathematics attainment of  
Singapore pupils.  
M.Ed. dissertation, University of Nottingham.

19. Kaur, Berinderjeet (1990a) (kb2)  
A study of basic numeracy skills: primary  
education student teachers at the Institute of  
Education, Singapore.  

Some common misconceptions in algebra.  
Teaching and Learning, 11(2), 30-36.

On meaningful measurement: primary school pupils'  
understanding of part-whole concept of fractions.  
Chap. 2 in: Cheung K.C. et al. (Eds.): Meaningful  
measurement in the classroom using the Rasch  
model: some exemplars.  
Monograph, Institute of Education, Singapore.

Summative evaluation of Primary Mathematics  
Programme: Sub-study report - Teacher Opinion  
Survey.  

23. Lam Tit Loong (1985) (ltl)  
An exploratory study of children's ability to  
solve arithmetic open sentences.  
M.Ed. dissertation, National University of Singapore.

The effects of computer-assisted instruction on  
attitudes and achievement in mathematics of pre-  
service primary school teachers.  
M.Ed. dissertation, National University of Singapore.

25. Lim Suat Khoh and Wong Khoon Yoong (1989) (lsk)  
Perceptions of an effective mathematics teacher.  
Singapore Journal of Education, Special Issue,  
101-105.
26. Ng Chey Keng, Priscilla (1985) (nck)
   The effects of peer tutoring in mathematics on secondary two pupils. M.Ed. dissertation, National University of Singapore.

27. Ng-Gan Lay Choo (1987) (nlc)
   Relationship between secondary school students' mathematics attitude and achievement. M.Ed. dissertation, National University of Singapore.

28. Ong Sit Tui and Lim Suat Khoh (1987a) (ost1)

29. Ong Sit Tui (1987b) (ost2)


32. Tan Chu Sing (1987) (tcs)
   Pupils' understanding of area. M.Ed. dissertation, National University of Singapore.

33. Tan Oon Seng (1989) (tos)

   An experimental investigation of a new approach to the teaching of algebra using microcomputers. M.Ed. dissertation, National University of Singapore.

35. Tay Choon Hong (1986) (tch)
   A study of concept levels in three aspects of transformation geometry. M.Ed. dissertation, National University of Singapore.
36. Wong Khoon Yoong (Ed.) (1986) (wky1)

37. Wong Khoon Yoong (1987) (wky2)

38. Wong Khoon Yoong (1990) (wky3)

39. Wong, Philip (1989) (wp1)
The effects of academic settings on students’ metacognition in mathematical problem solving. Paper presented at the Australian Association for Research in Education Annual Conference, Australia.

40. Wong, Philip (1990) (wp2)

41. Woo-Tan, Jeann Lay Beng (1989) (wlb)
Effects of computer-assisted instruction on the learning of transformation geometry. M.Ed. dissertation, National University of Singapore.

42. Yap Siew Kee (1990) (ysk)