Using Open-Ended Mathematics Problems
A Classroom Experience (Primary)
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Abstract: With the call to promote thinking in schools, it presents opportunities to enhance the teaching and learning of mathematics through means that move away from the traditional approach. The use of contextual open-ended mathematics problem tasks is discussed in this paper. What makes open-ended problems an attractive approach towards teaching and learning is their open and challenging nature that can engage the students’ minds. Besides touching on the proponents for open-ended problem-solving, the teacher’s classroom experience in engaging Primary 6 students to work on the problem tasks is presented along with samples of the students’ solutions whereby the mathematics is discussed. The problem tasks involved have characteristics of simplified real-life situations and they require the exercise of some form of cost-benefit analysis. The intended purpose then was for students to use mathematics in real world situations via an open approach towards finding plausible solutions. The students worked in small groups of 4 or 5, made assumptions to the related problem tasks, and developed various options to help them make an informed choice towards their goals. The students’ solutions show the use of listing, categorizing, and graphing as their strategies. The teacher through this experience shares the positive implications and difficulties encountered.

Introduction

With the advent of “Thinking Schools, Learning Nation” (TSLN) as the vision for the education sector since 1997, schools have been seeking ways to make this vision materialize. That year was also significant in that the IT Masterplan was rolled out, and the Desired Outcomes of Education cascaded. It was a welcome opportunity for schools to place more emphasis on improving structures and conditions to facilitate the development of thinking amongst students. Not that the education system was not producing “thinking” students, but the culture of teaching and learning, the common curriculum and the assessment-driven mode were perceived to have shortcomings which could affect the future quality of workforce if these were not reviewed and addressed.

Before the turn of the century, mathematics education in Singapore was perceived to be “traditional” in that it was mainly teacher-centred, with lessons focusing on routine exercises and written tests (Chang, Kaur, Koay, & Lee, 2001). TSLN was introduced to change that. It has been the principle of mathematics education that pupils’ mathematics abilities be developed conceptually than procedurally. Textbook approaches that emphasize computation, rules, and procedures at the expense of depth of understanding are disadvantageous to students as they encourage learning that is inflexible, school-bound, and are of limited use (Boaler, 1998).

Now with the call to “teach less, learn more”, the approach towards developing student’s relational understanding will gain greater momentum. Mathematics educators and teachers are stepping up the use of meaningful mathematics assessment tasks such as project work, journal writing, self-assessment, and portfolios. As problem-solving is central to the mathematics education here, creative approaches like problem-posing and the use of open-ended problem tasks are making themselves manifest in the classroom.

In this paper, the roles of open-ended problem tasks are presented, along with a sharing of a classroom experience whereby Primary 6 students were engaged in the solving of contextual open-ended problems. This is followed by a brief insight of the mathematics involved and the implications thereof.
The Open Approach

In the light of educational reforms, the 1990s increasingly saw growing support for open, or processed-based forms of learning of mathematics where students were engaged in open-ended, practical and investigative tasks.

What makes open-ended problems an attractive approach towards teaching and learning is their open nature that poses the challenge to engage the students’ thinking. Open-ended problem tasks are often thought of as tasks for which more than a single correct solution is possible, and that they offer students multiple approaches to the problems by placing little constraints on the students’ methods of solution. (Hancock, 1995). They can range from simply asking a student to show the work done on a problem to involving complex situations requiring formulating hypotheses, explaining mathematical situations, writing directions, creating new related problems, or making generalizations (Kulm, 1994). Foong (2002) describes open-ended problems as “ill-structured” because they comprise missing data or assumptions with no fixed procedures that guarantee a correct solution. Students in the face of such cognitive conflicts therefore will need to extend their existing knowledge by drawing from other means and sources to engage the problem situations.

Effective uses of open-ended problem tasks are believed to foster higher-order thinking and promote, produce, and provide fodder for pondering (Dyer & Moynihan, 2000). When a student learns mathematics through such a problem-based approach, struggling with the difficulties facing him instead of relying on memorization or any pre-determined rules to search for solutions, it promotes “deep understanding” of the mathematics that is valued (Hiebert, Carpenter, Fennema et al, 1996).

Van den Heuvel-Panhuizen’s (1996) study of using open-ended questions saw the benefits of students solving realistic problems when incomplete information was given, where they were required to make their assumptions on the missing information. Other benefits of using the open-ended approach include increased enjoyment (Perez, 1986) and providing the teacher with meaningful information on how the students managed the problem-solving process (Van den Heuvel-Panhuizen, ibid).

One key feature in solving open-ended problem tasks is the opportunity for students to work collaboratively. By tapping on each other’s experiences and talking about the problem task, they are engaged in reflecting, conjecturing, and justifying. Hiebert, Carpenter and Fennema (ibid) put it that students who reflect on what they do and communicate with others are in the best position to build useful connections in mathematics.

The Open-Ended Problem Tasks

Students do come across terms like “insurance”, “subscriptions” and “premiums”, just to name a few, in their daily lives. Particularly evident today are the widespread advertising in the sales of mobile phones in the newspapers, and conversations at school and home pertaining to the types of insurance policies purchased by students (for accidents) and adult family members for the students respectively (life or medical policy). However, many a times, the opportunities to talk about these and the mathematics involved are missed except for the common “GST” concept that students are now familiar when discussing how mathematics is involved in real world transactions. These are opportunities for teachers to engage students to know more about real-life situations facilitated through solving open-ended problem tasks.

In this classroom experience, two contextual open-ended problem tasks were crafted by the teacher with the belief that they were able to promote thinking, metacognition and application of problem-solving strategies as mentioned in the earlier segment. What was important to the teacher then was that the students would also be able to make a decision with respect to their solution choice after deliberating on the solution options they had worked out. The two problems are presented in Appendix 1.
The Classroom Experience

The initiation
A Primary 6 EM2 (mixed-ability) class comprising 42 students took part in this experience of solving two open-ended tasks.

A six-period time frame was dedicated to the class to work in groups of 4 (with two groups of five pupils) to discuss how to problem-solve. These six periods were for each group to solve only one of the two problems they had chosen. As the mathematics subject was allotted two periods per day (1 hour), the problem-solving activity was spread over three days, coupled with an additional two periods for selected groups to present their cases. Instructions were provided by the teacher with respect to what was expected from the problem tasks and their roles in the discussion. As these problems dealt with some form of cost-benefit analysis, the teacher spent time explaining the two problem situations and difficult words such as “manager”, “premiums”, “productivity”, and “insurance” prior to the problem solving process. It was emphasized that the students should work out the optimum plan, in that they should go for products that offer the best value at a minimum cost if possible. There would be no right or wrong answers and that they would have to make assumptions and justify how they arrived at their decision.

The teacher had to practise management by walking about, and would sit with individual groups at different times to listen to their discussion and elicit their thoughts through questioning their assumptions, bearing in mind that without some form of support and encouragement, the problem-solving process could deteriorate into a frustrating experience for them. The teacher used “what if…” questions to prompt them to think further and deeper. The teacher encouraged the students to do likewise, to question students who provided inputs, so as to challenge them to rationalize their viewpoints.

Observation of the Problem-Solving Activities
Based on observation, the responses of the groups were mixed. Groups that had students who were vocal were more engaged in the discussion. The more vocal students took lead roles in sharing their thoughts, which in a positive way became the catalyst for the others in the group to contribute and feel involved. Groups that had quieter students remained rather passive, and needed the teacher’s attention more as they proceeded from one thought to another. The boys appeared to enjoy the problem-solving activity more than the girls. They were more “daring” as could be seen from their jibes at their teammates’ responses, and offering their “superior” viewpoints.

The students were often heard trying to seek the “right” answer. They were uncertain if they were working towards the “right” answer. The teacher, had to, from time to time remind them that there was no right answer but to think about working out several options, comparing costs, values, and deciding which option to accept. The more vocal groups which showed greater promise were, as expected, able to accomplish the problem tasks.

Evaluation of Sample Students’ Solutions
Samples of students’ solutions were evaluated under four categories, namely, “Approach”, “Processing”, “Explanation” and “Overall Quality”.

Figure 1 – Students’ solution of The Manager’s Dilemma
From Figure 1, the group’s solution was evaluated as follows:

**Approach** – This group used a diagrammatic and graphical approach which is very organized and neat in their presentation. Using bars to represent the number of workers, they stacked the bars for each day to obtain maximum productivity for the day.

**Processing** – The group worked out two solution options while trying to optimize. Using different combinations of number of workers by the companies to add towards a maximum of 14 per day, they found that Method 1 gave a larger productivity value of 6600 units, while Method 2 generated 6100 productivity units for the 5 days. Method 1 and Method 2 would cost the manager $6500 and $5600 respectively for the 5 days.

**Explanation** – The group tried to explain the outcomes of their findings with respect to productivity and cost. They acknowledged that Method 1 “produces a lot of work” and “costs a lot”, while Method 2 in comparison “does not produce much work” but “it is cheap”. A decision on which solution to choose however is not captured in writing.

**Quality** – High for overall solution clarity and clear understanding of problem.

![Figure 2](image)

Figure 2 – Another group’s solution of The Manager’s Dilemma

Figure 2 shows another group’s solution. It was evaluated as shown below:

**Approach** – This group used a listing approach where each day’s selection of the companies is stated. By listing the 5 days and the different companies attached respectively, the total cost and productivity is computed.

**Processing** – By pairing the companies, the group intended to work out the best possible combinations for productivity for the 5 days and thus the total costs as well. The group came up with 3 solutions options. The first yielding a productivity of 6300 units and costing $5800, the second with an output of 6600 units costing $6500, and the third 6000 units costing $5500.

**Explanation** – No explanation was given in their written copy with respect to how they decided on their chosen option though it had been verbally presented. From their solutions, they tried to ask themselves “The best is which method? 1, 2 or 3?” as can be seen in Figure 2.

**Quality** – High for overall solution clarity and clear understanding of problem.
For the solution to the of the Best Insurance Plan in Figure 3, the evaluation is described below:

**Approach** – This group’s approach is direct, weighing the three options A, B and C in terms of their costs.

**Processing** – The group worked out the payment of the premiums for each plan at the end of 10 years (as seen in Plan B, 30K means $30000 computed by taking $250 x 12 months x 10 years, and likewise, similar computation done for Plan C to arrive at “$12K”). The group seemed to have taken into consideration given payouts and health conditions of an individual to arrive at a conclusion.

**Explanation** – The group made assumptions and tried to associate income earners to the type of plans. They recommended Plan C which they believed had a bigger payout with lower premiums.

**Quality** – Moderately high for overall understanding of problem and presentation of solution
Another group attempting the same problem (Figure 4) was evaluated as shown below:

**Approach** – This group tried to make comparisons of the three plans by juxtaposing them.

**Processing** – No evidence showing costs being computed. Evidence of comparisons made mainly by specifying reasons based on consequences of getting diseases and profile of people (earning power) who should be purchasing these respective plans. Students appeared to relate earning power to cost of the premiums.

**Explanation** – Although no “mathematical” information was provided, the explanation revealed associations with other topic strands like taking health status of a person into consideration. The group made a decision to go for plan B. They argued that it had the best returns if one contracted a disease in future or no disease in 10 years.

**Quality** – Moderate for overall clarity and conveyance of thought.

**Positive Implications**

Open-ended questions of a higher-order nature used in this activity pose a challenge to the students. They are non-routine in that they draw students away from standard answers towards seeking plausible solutions.

The creative aspect is that students could come up with different ways to present their solutions and justify their case, as in real-life situations where there is no one solution to an issue. Because they depict “real-world” scenarios, yet simplified to a level adequate for their discussion, students are challenged to deal with these “issues” that they see happening around them.

The “real-world” depiction of these problem tasks provides opportunities for personal values and beliefs to be raised through the discussion, for example, when students think about whether it is worthwhile to spend more on a particular premium if they take good care of their health.
The experience in this context saw students applying mathematical thinking and reasoning in the likes of comparing costs, attempting to attach realistic monetary values to items, and communicating ideas to plan towards attaining a solution that is value for money. The task also enabled them to make decisions with respect to choosing the best option.

The group discussion fosters group dynamics. It brings out the leadership potential amongst some students as they lead discussions or play the devil’s advocate. Groups that are more relaxed seem more able to scrutinize the problem solving process better as they comprise members that are open to each other’s inputs and showing better sense of humour.

The provision of scaffolding helps maintain a high level of active discussion for groups that are task oriented. This is consistent with studies that indicated that scaffolding and encouraging students towards giving meaningful explanations enabled them to be actively engaged at high levels (Anderson, 1989; Doyle, 1988).

**Difficulties**

The lack of vocal students in some groups meant that these groups had quieter students. Their quiet disposition made it difficult to work as a team, and therefore the teacher’s presence was sought more often. Their problem of not being able to proceed further in the discussion was escalated, most probably due to a dip in group’s self-esteem when they sensed the other neighbouring groups with more vocal students having more fun and laughter in the problem solving process.

The need for scaffolding was eminent as these open-ended tasks were new to them. The teacher’s presence was often sought to clarify thoughts to determine if they were on the right track and whether they were getting the “right answers”. The teacher himself had to be patient and avoid getting tempted to “lead” them. The teacher had to be conscious to help them construct knowledge through questioning assumptions.

Such open-ended tasks require much time to plan and complete. It does not allow one to witness or be certain that with one or few of such problem-solving experiences, students’ performance in mathematics will improve significantly. The time frame to complete the subject syllabus played on both the teacher’s and students’ minds when the latter over the days asked about being focused in preparing for the Primary School Leaving Examination (PSLE) rather than engage in such exercises, especially since these sessions took up eight “precious” periods in their critical year. This speaks of the difficulty of bridging the rationale for such open-ended activities to instruction. Of what values do the outcomes generate towards mathematics instruction? While the outcomes reveal something about the students’ knowledge on working with costs, comparisons, conversions and other mathematical strands, the open process left questions unanswered in relating back to aligning with the mathematics instructions required to cover the syllabus. The teacher had to assure the students the benefits of having such problem-solving exercises, in that there had to be a start towards rethinking how mathematics could be learned. It should also prompt the teacher towards thinking about how outcomes could be better aligned to instruction.

While there are studies that have advocated the use of open-ended problems to engage students, there are also studies which cautioned that the use of such problems should build on students’ prior knowledge (Bennett & Desforges, 1988) rather than on issues that are beyond their required understanding at this age. The unfamiliar contexts could have caused some to have difficulty in proceeding with the problem solving process (Rogoff & Lave, 1984). This led the teacher to reflect on the appropriateness of the problems constructed, in particular, to introduce open-ended problems with a more familiar context to the quieter groups in such problem solving sessions in the future.

Reflecting on the whole exercise, the teacher’s concern was also whether the problem-solving activity had served its intended purpose of students using mathematics in real world situations via an open approach. As Wu (1994) pointed out, would students go away with the impression without knowing any solutions to the problem or that mathematics is like a jumble of disjointed formulas, partly because the teacher was unable to handle the students’ assumptions and guesses? To manage this, the teacher felt that incorporating the use
of open-ended problems into a clear assessment framework could provide students with greater assurance of their use and intent.

Open-Ended Problem Tasks and Assessment

Not all open-ended problem-solving tasks need to be assessed formally. How students can be assessed in these tasks depends on the type of information the teacher needs. The above classroom experience is an example of the teacher making anecdotal observations to determine the readiness level of the class in solving open-ended problems as that was their first time working with open-ended problems. Apart from making observation, interviews can be used to ascertain the level of response students will give to open-ended problems. As in the classroom experience, the teacher sat with the groups to listen to their discussion, and as a form of scaffolding, challenged their assumptions by asking them questions.

Students’ solutions can be analysed through constructing simple rubrics with pre-determined criteria based on the type of tasks given. The evaluation of the four group solution samples shown above have been gauged to range from moderate to high under approach, data analysis and explanation.

Concluding Remarks

Contextual open-ended problems can be challenging to students when they deal with issues that they have heard of but did not have the opportunity to talk about since the notion appears that such issues pertain more to the adults. The exposure to such issues by simplifying them allows the students to appreciate the complexity that adults face in the real world and yet through their problem solving process, connects the mathematics learning to dealing with such issues.

The open nature of the problem solving task requires much communication amongst members to generate thinking, and making sense of the mathematical ideas in relation to the problem. Such communication promotes metacognition and learning. The teacher is an important determinant in enabling the meaningfulness of the experience through scaffolding and encouragement.

Challenging as these open-ended problems can be, one cannot dismiss the make-up of the members in a group. Students that are vocal and have a sense of humour liven up the problem solving process within the group, while groups with quieter students could find the problem solving process painful. It would be more assuring if the outcomes of such problem tasks could be related back to instruction.

References


Appendix 1

Problem 1 – The Manager’s Dilemma
A construction firm wants to hire workers for a project. This project must be completed within 5 days and only a maximum of 14 workers are allowed to be hired per day.

Imagine you are the manager of the construction firm and you are to hire the workers from various companies (A to J), how would you do it so that your cost is minimized? Assume that each worker produces the same amount of work per day.

Below is a table of the cost of hiring the number of workers from the companies. You need not choose all the companies but if you choose any company, all the workers in that company will be hired for the day. You can only choose a company once.

<table>
<thead>
<tr>
<th>Company</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of workers</td>
<td>8</td>
<td>12</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>600</td>
<td>1400</td>
<td>300</td>
<td>400</td>
<td>500</td>
<td>600</td>
<td>700</td>
<td>800</td>
<td>1200</td>
<td>500</td>
</tr>
</tbody>
</table>

Problem 2 – The Best Insurance Plan
In Singapore, the most common types of diseases which have a high death rate are diseases A, B, D and H. Diseases A, B, D and H incur high medical costs of up to $100,000. Numerous insurance plans have been set up for the people to consider.

<table>
<thead>
<tr>
<th></th>
<th>Plan A</th>
<th>Plan B</th>
<th>Plan C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premiums/month</td>
<td>200</td>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>One-time payout</td>
<td>$100,000 for any of the above</td>
<td>$150,000 for any of the above</td>
<td>$80,000 for any of the above</td>
</tr>
</tbody>
</table>

Mr Tan earns a moderate income (about $3500 per month). He has a wife who is a homemaker and two school going children. He also smokes, does not exercise often, and works late because of a stressful job. His wife is concerned for him and is afraid that he may get diseases A, D and F which are related to smoking. If she wants to get an insurance plan for him, which should she choose? State your assumptions. (number of * denotes increasing severity of diseases)