LANGUAGE MATTERS IN MATHEMATICS:
SINGAPOREAN PRIMARY CLASSROOMS
BOOKLET 1

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THE TEAM

Dr. Sally Ann Jones  
Principal Investigator  
Senior Lecturer  
English Language and Literature Academic Group (ELL/AG), National Institute of Education

Dr. Yeo Kai Kow Joseph  
Co-Principal Investigator  
Senior Lecturer  
Mathematics and Mathematics Education Academic Group (MME/AG), National Institute of Education

Dr. Mark Fifer Seilhamer  
Co-Principal Investigator  
Lecturer  
English Language and Literature Academic Group (ELL/AG), National Institute of Education

Dr. Loh Mei Yoke  
Co-Principal Investigator  
Lead Curriculum Specialist, Curriculum Planning and Development Division (CPDD), Ministry of Education

Ms. Ho Hsien Lin  
Co-Principal Investigator  
English Language Teacher  
Ai Tong School  
Subject Literacy Officer (2015–2017), English Language Institute of Singapore (ELIS)

Miss Yeo Rei-Chi Lauren  
Research Assistant  
English Language and Literature Academic Group (ELL/AG), National Institute of Education

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Background of the Research

An interdisciplinary team studied pupil and teacher perceptions and experiences of teaching and learning mathematics and English in Singaporean primary school classrooms. We present the findings that are significant for teaching and learning in Singaporean primary classrooms in a series of three booklets about mathematics (booklet 1), English (booklet 2), and transfer of learning (booklet 3). The exploratory study sought to address the following questions:

What are pupils’ views about and experiences of learning mathematics and English at school and elsewhere at primary one (P1), three (P3), and five (P5)?

What are the pedagogic features and learning processes of mathematics and English at school and elsewhere which enable pupils’ learning of the language and concepts of mathematics?

What are English and mathematics teachers’ practices and perceptions of language teaching, and how in this regard does the subject English function in their English-medium schools?

The Participants

The participants of the study were pupils from P1, P3, and P5, as well as teachers of mathematics and English, that is, teachers of mathematics, teachers of English, and teachers of both subjects. Pupils’ and teachers’ responses and reflections were collected via interviews and surveys. Additionally, an equal number of mathematics and English lessons were observed. These data form the basis of the study.

206 pupils participated in focus group interviews
88 teachers were surveyed
17 teachers were interviewed
48 lessons were observed

Important Findings: The Significance of Words and Understanding

Some important findings of the study were as follows:

1. Language was viewed by a majority of pupils and teachers as important for mathematics, particularly in relation to interpreting and answering questions.

2. Understanding word problems was viewed by teachers as a challenge for pupils in mathematics. Additionally, the tendency for pupils to rely on the identification of key words without understanding the meaning of the question was seen as an obstacle to learning how to solve problems.

3. Understanding in mathematics was closely related to reading in mathematics, and both were viewed by pupils and teachers as highly important in the subject.

4. Although some degree of metacognitive awareness was evident among pupils in interviews, metalinguistic awareness was not as evident.

Two aspects of language stood out in pupils’ and teachers’ responses about mathematics: words and understanding. Each of these will be explored in subsequent sections of this booklet.
Language Profile of Pupils

The proportion of pupils who spoke English predominantly at home was found to be similar to the proportion of pupils who spoke their Mother Tongue (MT) predominantly. English is the main home language of 43%* of pupils, while for 40%, the main home language is their MT. According to pupils, 8% speak English and their MT equally, while 9% of pupils spoke mainly other languages or combinations of languages at home.

According to the Singapore General Household Survey 2015\(^1\), English is the most frequently spoken language at home for 51% of the Chinese, Malay, and Indian resident population aged 5-14. The remaining 49% speak in other languages predominantly.

Thus, pupils’ self-reported language use in our study highlights the multilingualism in Singaporean society and reminds us that almost half of the children said that they do not have a great deal of exposure to English at home.

Pupils’ Home Support

Pupils were asked about the quality and type of help they received with school work outside of school.

The most common form of home support for mathematics was help from parents (65%), followed by tuition (37%) and help from other relatives (34%). Support in the form of other resources such as the Internet, as well as assistance provided in student care centres, made up a small percentage of support - 3% and 2%, respectively.

Comparing the two most common forms of support, that is, help from parents and tuition, by level, P1 pupils were found to receive the most help from their parents, but the least amount of tuition. Parental help seemed to decrease as pupils got older; conversely, the percentage of pupils with tuition was higher at P3 and P5.

*Note: All numerical data are rounded to the nearest whole number.
The Purposes of Teaching and Learning Mathematics

Asked about the purposes of teaching mathematics, the vast majority (91%) of teachers identified real-world application as the primary purpose. Asked a similar question about the purposes of learning mathematics, pupils prioritised counting money (36%) and application of mathematics to their daily lives (32%). However, teachers’ second most frequent response, problem solving (46%), was rarely mentioned by pupils as a reason for learning mathematics.

Mathematical problem solving is the central focus of the Singapore Mathematics Syllabus (2012), which describes the key aims of mathematics teaching and learning as the acquisition and application of mathematical concepts and skills, the development of cognitive and metacognitive skills through a mathematical approach to problem solving, and the development of positive attitudes towards mathematics. This study finds that application is foregrounded by both teachers and pupils, while problem solving is central for almost half the teachers.

Pupils’ Reasons for Preferring Mathematics to English

Learning mathematics is meaningful to pupils in various ways. The main reasons identified by pupils who preferred learning mathematics to English were:

1. mathematics is easier than English
2. mathematics is enjoyable
3. mathematics is useful
4. love for numbers/problems

Two of the four main reasons given by pupils for preferring mathematics to English show a degree of practicality. Mathematics was perceived to be easier than English or useful by some pupils. However, interest was also a factor in pupils’ preference for mathematics. Pupils felt it was enjoyable, or they liked numbers and problems. The positive attitudes required by the mathematics syllabus are here detailed by pupils, from their own perspectives. They are to do with mastery (1), pleasure (2), and interest (4).
THE IMPORTANCE OF LANGUAGE IN MATHEMATICS

Teachers Recognised English as Important in Mathematics

All teachers surveyed felt that English is important for learning mathematics, with 67% of teachers identifying English as being very important for learning mathematics.

The teachers’ acknowledgement of the importance of language in mathematics shows that they are aware of the role of language, in this case English, in teaching and learning. Teachers referred to a certain ‘style’ of English in mathematics, or the ‘mood’ of mathematics, pointing to their recognition of the specific use of language in mathematics. We know that academic school language is different to everyday home language and that language functions differently in each school subject. As the teachers pointed out, pupils have to learn a specific mathematical language in order to gain access to the concepts of the subject. In the words of one teacher, each topic ‘comes with words that you need to know’.

What did Teachers Say about Language in Mathematics?

... there is a very big co-relation between being able to read and being able to do math.

... even with a good understanding of the math concepts, the moment the same math concept is translated into words, [students] are unable to comprehend.

... if they have difficulty recognising and reading words, then they cannot handle word problems.

... the topic of numbers would come with words that you need to know. For example ‘place value’ stands for … hundreds, tens …

... it’s all about being able to understand the question. So, without these terms they can’t solve the questions.

... for the sake of the maths they are taking, they need to be good at understanding a certain style of English …
**THE IMPORTANCE OF LANGUAGE IN MATHEMATICS**

**What is the Language of Mathematics?**

Mathematical language is different from the language we use in everyday interactions. It is more complex and specific in all aspects: vocabulary, grammar, and genre. Researchers\(^3\) make three points about language in mathematics: mathematics is a specific type of language in itself; language is used to teach and learn mathematics; mathematical language provides conceptual tools for the study of other school subjects. The language of mathematics also includes a variety of semiotic modes, such as visuals and symbols, in different combinations\(^4\). Thus, children in primary school face the challenges of learning a new technical language as well as new modes of representation when working mathematically\(^6\).\(^7\). O’Halloran (2015)\(^4\) describes mathematics as the learning of a particular variety of language involving the modes of print, visual, and iconic to ‘describe and predict patterns in space, number, quantity and arrangement’ (p. 73).

Apart from learning actual mathematical language, children must develop the ability to manipulate its terminology and connect the terms with the mathematical concepts they encode. The development of metalinguistic awareness is suggested to be just as important as metacognitive awareness for children to learn mathematics successfully\(^2\). Consequently, mathematics teachers themselves need to have a degree of Teacher Language Awareness (TLA)\(^8\) in order to help pupils notice how language is used in mathematics. Pupils’ metalinguistic awareness may be developed when they have opportunities to use mathematical language to articulate mathematical concepts and procedures to each other and the teacher. The effort to communicate mathematically helps children notice language, especially when given feedback and scaffolded by peers and teachers.

**How Long does Mathematical Language Take to Learn?**

Many children come to academic English at school via their dominant home languages and from there to mathematical English. Since the MT is not the medium of education but viewed as a cultural language in Singapore, MT teaching generally does not include the specialised vocabulary necessary for studying other subjects. Additionally, many children speak a colloquial variety of English outside lessons, which can also lengthen the journey to academic English. International research\(^9\)\(^,\)\(^10\) suggests that it takes a minimum of five years for children for whom English is a non-dominant language to develop competence in academic English. Thus, the interplay among different registers and varieties of English, as well as the registers and varieties of other languages, affect how children learn English for everyday purposes and for specific academic purposes across all the subjects on the curriculum.

**What does the Mathematics Syllabus Say about Language in Mathematics?**

The mathematics syllabus states that communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely, and logically. It helps pupils develop their understanding of mathematics and sharpen their mathematical thinking.

*Dr. Yeo Kai Kow Joseph*, National Institute of Education

Language is important in the mathematics syllabus. It is a vehicle for communication of mathematical ideas. Using precise mathematical language helps students to think about and articulate their mathematical ideas and understand those of their peers. Some examples are given in the syllabus and also in the Teaching and Learning Guide.

*Dr. Loh Mei Yoke*, Ministry of Education
The Importance of Language in Mathematics

Pupils Recognised English as Important in Mathematics

A majority of pupils (86%) felt that English is important for learning mathematics, demonstrating their awareness of language use in disciplines other than English. However, when asked to elaborate, most pupils gave rather general responses, showing that their metalinguistic awareness was limited or vague.

Despite this, a significant percentage of pupils (86%) felt that language is important for mathematics.

What did Pupils Say about Language in Mathematics?

- **PI Pupil**: "English is important to learn math because sometimes...some words are key words.
- **P3 Pupil**: "...if you never learn English, you cannot do math because math include English. You need to read the instructions.
- **P5 Pupil**: "...without the words you wouldn’t understand what is the question talking about.

Four ways identified by pupils in which English is important for mathematics were:

1. Reading/understanding questions
2. Answering questions
3. Expressing/communicating ideas
4. Identifying key words

Nevertheless, this awareness appears to develop with age and instruction, as seen from the increasing percentage of pupils from P1 to P5 who realised the significance of language.

Metacognition and Metalinguistic Awareness in Mathematics

The Mathematics Syllabus (2012) defines metacognition as ‘the awareness of, and ability to control one’s thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one’s own thinking, and self-regulation of learning’ (p. 17). Our interviews with pupils suggest that they are developing that awareness as they move up the levels of school, although in the observed lessons, there was not much time for pupils to actually articulate their thinking.

Another aspect of children’s thinking relevant for learning mathematics is metalinguistic awareness. This is the ability to identify language concepts, talk about them using a metalanguage, and more importantly to control and apply that conceptual knowledge. Pupils’ recognition of the importance of language in their mathematics learning, however vaguely expressed, is an important aspect of their learning. As theorists suggest, language and concept development must go hand in hand; language has to become more complex when it has to encode more abstract thinking. Furthermore, an awareness of language concepts can facilitate children’s cognitive development.
Features of Mathematical Language

Research has surfaced particular features of mathematical language that were also noted by the teachers interviewed. These have to do with the processes of mathematics (e.g., comparison, reasoning, explaining), thinking in the abstract (e.g., the word operation used metaphorically rather than to refer to a literal surgical operation), and the need to compress information (e.g., nominalisation).

- Everyday language and symbols that children are familiar with may be used in mathematical contexts with new technical functions and/or meanings – e.g., equal, similar, the colon, the dash
- Words in mathematics that refer to abstract concepts may be used to refer to material items in everyday language – e.g., table
- Words in mathematics can be used metaphorically, making them different from everyday use – e.g., operation
- Technical vocabulary in English is often from Greek or Latin roots and has prefixes or suffixes – e.g., sub-, -tion
- Many words are nominalised (made into nouns from other forms) in mathematics to describe processes – e.g., add (verb) to addition (noun)
- Noun phrases may be heavily loaded with information in mathematics – e.g., ‘the original price of a tube of toothpaste’ (p. 50), ‘the number of pupils in each level in a primary school’ (p. 60)
- Words in mathematics can have many synonyms and forms – e.g., subtract, subtraction, minus, take away
- Grammatical words in mathematics are used to show concepts – e.g., prepositions to show position

... some of the topics, the words won't be covered in English... let's say we are doing shapes. Or let's say maybe graphs... All the words that are associated with the topics, we rarely use in English.

‘Product’, we also have it in mathematics. But product means you multiply, you get this, this is the product. But in English, a product could mean something that's on the shelf, that's a product.

Teachers Identified Learning Words as Challenging, but Many Pupils did not

A majority of teachers interviewed (73%) suggested that learning mathematical words, or technical vocabulary, was challenging for pupils and affected their understanding of mathematics, particularly with regard to word problems. On the other hand, most of the pupils interviewed did not seem to notice the significance of technical vocabulary in mathematics, although some did exhibit an awareness, as shown below.

…when you spell the numbers you’re using English to spell numbers.

…when there’s no word problems like there’s no words, it feels like language is not even related to maths.

If you don’t understand … simple basic words like ‘more’ or ‘less’ or like ‘the same’, you won’t be able to solve the math question.
In addition to words being viewed by teachers as a barrier to understanding mathematics questions, an over-reliance by pupils on key words was also brought up as a challenge. Sometimes, at the lower primary, pupils learn to identify key words in word problems, drawing straightforward associations between these words and their corresponding mathematical operations (e.g., ‘more than’ or ‘altogether’ to mean addition, ‘less than’ to mean subtraction). The ensuing difficulty noted by teachers is that pupils search quickly for key words, and do not spend enough time understanding the meaning of the question, especially at higher levels. Thus, identifying key words provides short-term results, and may not be beneficial to pupils or sustainable in the long-term.

**What did Teachers Say about Identifying Key Words?**

… it helps but it can be also very dangerous
… Sometimes ‘more than’ means minus. You have to understand the question.

… sometimes when you go for the key words, you’re not getting the true picture of what the question wants.

They have no understanding of the words … they are just guessing, and they are only looking for key words.

**What did Specialists Say about Identifying Key Words?**

Plenty of research indicates that using key words is not always helpful, especially when pupils approach upper primary levels where problems become more complex both linguistically and mathematically. For instance, the key word ‘altogether’ can implicate either multiplication or addition. Therefore, simply relying on the specific word is not helpful for pupils to decide which operation they should select for the problems that include that particular word. In other words, pupils who just identify key words do not construct a qualitative representation of the situation described in the problem.

Identifying key words without understanding the context of the problem is ‘dangerous’. For example, ‘more than’ and ‘less than’ can both illustrate an addition or subtraction context. In another example of ‘1/2kg of the oranges’ versus ‘1/2 of the oranges’, ignoring the unit of measurement and not understanding the context of the word problem might lead to interpreting ‘of’ as ‘multiply’ in both cases.
Reading in Mathematics

Reading in mathematics is a particular kind of reading, specific to the subject and its genres. Research has found that reading in mathematics requires the memorisation of formulae during reading, the re-reading of specific phrases, and an extremely close focus and attention to function words. This is unlike the pleasure reading of narratives, which is quicker and chronological. Pupils have to learn how to read in different school subjects according to the purpose of the subject. In mathematics, this entails reading to make sense of the context of word problems and inferring the mathematical procedure(s) for solving them. In other words, pupils have to read like mathematicians.

However, both internationally and in Singapore, research has shown that children experience difficulties with reading word problems. In Singapore, reading and comprehending were found to be major areas of difficulty for P3 children when solving word problems.

Pupils may find reading in mathematics unexpectedly difficult due to the genre and grammar of word problems. Word problems may look like narratives, even though they function as procedures which signal mathematical operations; hence pupils, especially at P3, may apply their skills of reading narratives inappropriately to the problems. Additionally, the grammar of the word problem may differ from that of everyday English. This may surprise children, who do not yet know how to read like mathematicians, because it reflects a procedural sequence rather than a narrative one.

Pupils Recognised Reading and Understanding as Important in Mathematics

Pupils suggested that a primary purpose of language in mathematics is to be able to read and understand questions. The percentage of pupils who identified both reading and understanding as ways in which English was important in mathematics increased from P1 to P5, again suggesting a greater thoughtfulness or metacognitive awareness with age.

In terms of home support, the percentage of pupils who identified understanding as an area in which they received help in mathematics (19%) was significantly higher than the percentage of pupils who identified reading as an area in which they received help in mathematics (5%). These results suggest that understanding the concepts of mathematics is a greater concern to pupils than is reading the language of mathematics. In other words, these pupils see their difficulties as mathematical rather than linguistic.
UNDERSTANDING PROBLEMS & QUESTIONS

What did Pupils Say about Understanding in Mathematics?

... the most important thing in math is actually understanding. Because you must understand the question in order to get the right answer.

P3 Pupil

In some word problems if I don’t do it carefully or understand the question properly I find it difficult to get the answer correct.

P3 Pupil

When you are reading the question, you need to be aware of what words they put there, so you understand properly.

P5 Pupil

Teachers Recognised Understanding in Mathematics as Important and a Challenge

A majority of teachers (73%) interviewed felt that understanding concepts was important in mathematics. This was often, once again, expressed in relation to solving word problems. Many of the teachers (64%) explained that a lack of understanding would mean an inability to answer questions correctly.

However, when teachers and pupils used the phrase ‘understand the question’, it was not always clear whether they were referring to linguistic knowledge or conceptual knowledge. For some, ‘understanding the question’ seemed to imply being able to read the language, while for others, it seemed to imply knowing the mathematical concepts. The quotations from pupils also demonstrate this complexity.

What did Teachers Say about Understanding in Mathematics?

If they cannot understand [the question], then they will do it wrongly.

Mathematics Teacher

... when they do not understand ... if the answers are wrong ... they may not be able to make the judgement that their answers are wrong.

Mathematics Teacher

... ultimately math is all about practical experience. So, it’s all about being able to understand the question.

Mathematics Teacher
INTERVIEW:
DR. YEO KAI KOW JOSEPH

Dr. Yeo Kai Kow Joseph is a Senior Lecturer and mathematics educator from the National Institute of Education, Singapore. His research interests include mathematical problem solving at the primary and secondary levels, mathematics pedagogical content knowledge of teachers, and mathematics anxiety.

How important would you say language is in mathematics?

Language in mathematics is important as the language of mathematics is very specific, and pupils need to identify the correct meanings of words in order to communicate effectively and construct appropriate meanings. Moreover, pupils who have difficulty in reading and understanding the language of mathematics would have a challenging experience in grasping the mathematical concepts. It is important that primary mathematics teachers use simple language to establish linkages between the mathematical concept and task. It is also important for pupils to be able to identify the semantic structure of word problems and compare the problem representation and problem solutions.

What did teachers say was most challenging about teaching mathematics? Is there anything additional that you see as a difficulty?

Mathematical ideas are abstract mental constructs. Mathematics is often communicated through symbols, and mathematical symbols are quite abstract for many pupils. To help pupils grasp these ideas, they must be represented in a more concrete way using external representations. These external representations take the place of the abstract, mental concepts. Sometime students may have the concepts but not have sufficient representations of the concepts.

What are some effective strategies for teaching language in mathematics?

Identifying key words is not ideal because it makes minimal demands on working memory, and it does not depend on extensive knowledge of problem types. Moreover, this approach leads to incorrect answers when information implicit in the situation described by the problem is relevant to the solution because pupils who use identify key words fail to represent this situation. Researchers and mathematics educators found, for example, that rewording of word problems has systematic effects in the problem-solving performance of pupils when they are faced with ambiguous and abstract language. The rewording of word problems makes semantic structures more explicit and compensate for a less developed semantic schema. Moreover, researchers indicate that pupils show better understanding and solution performance when the problems are reworded to state more explicitly the relationship among known and unknown quantities. However, pupils using key words would focus more heavily on the numbers and relational terms in a problem rather than on the other words, because a pupil bases his or her solution plan entirely on this information.

Why are teachers and pupils so concerned with understanding in mathematics?

Understanding helps teachers and pupils reason and think analytically, as well as to note patterns, structure, or regularities in both real-world situations and symbolic objects in the mathematics world.

What do you see as the primary purpose of teaching mathematics?

The primary purpose of teaching mathematics should be to promote the process of mathematics, including problem solving, deductive and inductive reasoning, justification and proofs, and mathematical communication. Mathematical habits of mind such as curiosity and independent thinking should be inculcated.

What can teachers learn from this study?

This study provides greater insight into issues related to language in mathematics and understanding word problems.
I think teachers and pupils refer to ‘understanding’ in mathematics as understanding the mathematics in a word problem. Our students are generally good in applying procedures in solving computational problems and problems in simple contexts. However, it becomes challenging when the context in a word problem is complex.

INTERVIEW: DR. LOH MEI YOKE

Dr. Loh Mei Yoke is Lead Curriculum Specialist at the Curriculum Planning and Development Division (CPDD), Ministry of Education, Singapore. Her research interests include metacognition in mathematical problem solving, and she is involved in designing and reviewing the primary mathematics curriculum, as well as monitoring its implementation.

How important would you say language is in mathematics?

Language in mathematics provides the precise expression and description of mathematical ideas. It is also the vehicle for communicating mathematical ideas.

What did teachers say was most challenging about teaching mathematics?

Is there anything additional that you see as a difficulty?

Problem solving is not just about solving word problems. However, our teachers find teaching solving word problems the most challenging.

What are some effective strategies for teaching language in mathematics?

For some topics, it is important to give examples and non-examples so that students understand the precise description (or definition) of the mathematical vocabulary, notations, and symbols. For word problems, it is important to take time to understand the mathematical context of each word problem instead of taking the ‘short-cut’ by attaching specific mathematical operation to phrases or words in a word problem. If ‘Understanding the Problem’, the first step in Polya’s four phases of mathematical problem solving, is not done well, it would consequently lead to students not being able to make a plan that would lead to the solution for the problem.

Are there any ways in which you think teaching in English can support mathematics?

There are ways in which teaching in English can support mathematics. Having big books (or storybooks) that include context related to mathematical topics such as telling time and position that are related to everyday activities would help students to understand the language of English and mathematics as a whole. If both English and mathematics teach terms such as ‘morning’, ‘afternoon’, ‘evening’, ‘half past’, ‘first’ position, ‘second’ position, learning would be more meaningful and enhance students’ experience.

Why are teachers and pupils so concerned with understanding in mathematics?

I think teachers and pupils refer to ‘understanding’ in mathematics as understanding the mathematics in a word problem. Our students are generally good in applying procedures in solving computational problems and problems in simple contexts. However, it becomes challenging when the context in a word problem is complex.

What do you see as the primary purpose of teaching mathematics?

It’s the goal of the national mathematics curriculum to ensure that all students will achieve a level of mastery of mathematics that will serve them well in their lives. At primary level, this would mean that students acquire mathematical concepts and skills for everyday use and continuous learning in mathematics. They get to know their own environment better through mathematics.

What can teachers learn from this study?

Teachers can draw insights from this study on the importance of understanding the mathematical context of word problems, metacognitive awareness in mathematical problem solving and metalinguistic awareness.
Perceptions about the purposes of teaching and learning mathematics were rather pragmatic and, for pupils, grounded in the everyday and concrete.

**General implication.** Since some children did highlight the excitement and mental challenge they experienced in mathematics, an implication from the study is to capitalise on the potential interest and enjoyment of the subject by highlighting for pupils the beauty, patterning, and puzzle of mathematical thinking, in addition to its grounding in everyday existence.

### Classroom Implications of the Significance of Words and Understanding

1. **Language was viewed by the majority of the pupils and teachers as important for mathematics, particularly in relation to interpreting and answering questions.**

   This is a very positive finding as it agrees with research into the importance of language in teaching and learning the disciplines. The finding connects to theories of children’s development of thinking which occurs through language\(^{15,14}\). It also shows that there is a high degree of teacher language awareness (TLA) among Singaporean primary school teachers\(^6\). It is particularly significant given the fact the Singaporean pupils are multilingual and may speak a colloquial variety of English, which means that they take the whole of primary schooling to learn academic language and may benefit from the explicit teaching enabled by teachers’ TLA.

   **Implication 1.** Teachers and pupils were most concerned with words and understanding, which are aspects of language input or exposure to technical vocabulary and reading skills. While these are essential aspects of learning the language of mathematics, pupils also need opportunities for output via exploratory talk\(^{23,24,25}\). This will allow them to articulate their understandings using mathematical language. Through listening to pupils’ language output, teachers may assess their understandings and provide feedback, resulting in pupils’ greater metalinguistic and metacognitive awareness. This is part of assessment for learning (AFL)\(^{26}\).

2. **Understanding word problems was viewed by teachers as a challenge for pupils in mathematics. Additionally, the tendency for pupils to rely on the identification of key words without understanding the meaning of questions was seen as an obstacle to learning problem solving.**

   **Implication 2.** Although pupils who concentrate on identifying key words may be successful in the short term, it will not help conceptual development in the long term and will proceduralise learning. When learning becomes a procedure rather than an exercise in thinking, it can short circuit an individual’s metacognitive development and therefore the potential transfer of learning to novel situations. Techniques such as rewording may be used instead to help children understand the meaning of word problems thoroughly before translation to application.

3. **Understanding in mathematics was closely related to reading in mathematics, and both were viewed by pupils and teachers as highly important in the subject.**

   **Implication 3.** We can deepen our knowledge about the skills necessary to read mathematically. Since expert mathematical reading is specific to the discipline, it is unlikely that children will learn how to do it anywhere else in the curriculum apart from in their mathematics lessons. Strategies for teaching reading such as pointing, choral reading, annotation, among others are helpful.

4. **Although some degree of metacognitive awareness was evident among pupils in interviews, metalinguistic awareness was not as evident.**

   **Implication 4.** The finding that some pupils were aware of their own learning is a positive one. The vague expression of their understandings suggests the need for greater opportunities for them to express and articulate their understandings with teacher support. For example, this can be done through teacher-guided reporting\(^{27}\) of the process of problem solving whereby the teacher provides language to scaffold thinking where necessary, it is children’s metacognitive and metalinguistic awareness that will help them transfer their learning to fresh questions and problems with a thoughtful approach rather than a proceduralised one.

In sum, the findings of this study suggest that we draw on the potential for enjoyment inherent in mathematics. By developing our own awareness of language, we may highlight thinking as much as application in our mathematics classrooms. Providing opportunities for exploratory talk and teacher-guided reporting, we may scaffold and formatively assess children’s thinking in order to support their metalinguistic and metacognitive development in mathematics.
REFERENCES


