LISTENING TO CHILDREN SOLVING MATHEMATICS QUESTIONS

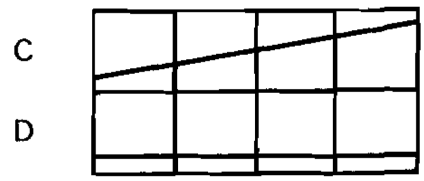
LESLEY R BOOTH

Perhaps one of the most important things we have learned from the work of Piaget, is that children's minds often work in ways which are quite different from those of the adult. In order to teach children, therefore, we need to know something about how children think, so that we can communicate more meaningfully with them and so that we can plan more effective learning experiences for them.

One way of finding out how children think is to give them a particular problem and ask them to explain how they would solve it. This is, of course, essentially the method used by Piaget. Indeed, the importance of Piaget's work owes much to his use of this procedure and to the kinds of problems which he gave children to solve.

In the case of mathematics, we should have no difficulty in finding problems to ask children. Are some kinds of problems, however, more useful than others in helping us discover how children think? Many of Piaget's most interesting observations came from studying children's *wrong* answers and the reasons which gave rise to them. Studying children's wrong answers in mathematics, and the explanations which children give for these may similarly provide us with some very useful information concerning how children think in mathematics, and indeed this is the approach which many recent research projects in mathematics education have taken.

For example, the results from a British survey of secondary-school children's understanding of various topics in mathematics (described in Hart, 1981) showed that nearly 50% of children aged 12, 13 or 14 years chose answer (3) for the following question (Figure 1).



Tick (√) the answer you think is true:

(1) Line C is longer . . .

(2) Line D is longer . . .

(3) C and D are the same length . . .

(4) You cannot tell . . .

Figure 1

When some 12 year-old children who made this kind of error were asked to explain their answers, the reasons they gave provided some very useful insights into their ways of thinking about length:

Adam: They're the same.

Interviewer: Why do you think they're the same?

A: Because they both start on the same line, and end on the same line (indicates the grid lines).

I: And does that make them the same length?

A: Yes, they both go across 4 squares. It's just that one's sticking up a bit.

I: And if I got a ruler, and measured along there (indicates along C), and then measured along there (indicates along line D) . . .

A: Yes, you'd just get the same answer both times.

Araba: They're both the same length.

Interviewer: Why do you think that?

A: Well, that one's (C) just the same, it's just going up. If you pulled it down . . . if you could pull it down straight, like that one (D), it would be the same as D.

I: So if I pulled it down straight, where would the end of the line be?

A: On the same line there (indicates the end grid line). You'd see it was the same as D then.

Sometimes even the correct answer can reveal some interesting information:

Lily: C's longer.

Interviewer: And why do you say that?

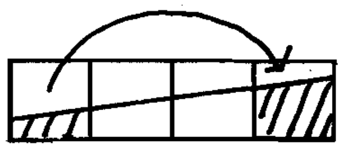
L: Because C's 2 long, and D's only one.

I: How do you know C's 2 long?

L: Well, that one goes with that one (indicates first and last segments of C) to make one, and then those two (the middle two segments) make another one.

I: Sorry, I haven't quite got that.

L: If you take this bit (first segment), and fit it on top of that bit (last segment), you'll get a whole one

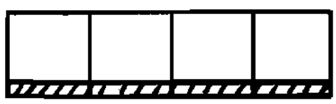
(indicates: C ).

I: Oh, I see! Do you mean the shapes under the line will fit together to make a whole square?

L: Yes. And the same for the middle two bits.

I: And what about line D?

L: Well, that's just one, because you've only got 4 quarter bits

(D ).

I: So if I were to measure along those lines with a ruler . . .

L: C would be longer, because it's got more space underneath.

Adam and Araba both considered that the important thing in comparing lengths was the position of the end-points of the lines. They both subsequently went on to say that any line drawn across a one-centimetre square on a grid, including the diagonal, was also one centimetre long (Figure 2).

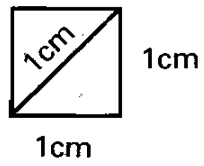


Figure 2

Lily had an interesting conception of longer lines having more space under them, but later got into difficulties with finding the perimeter of rectangles, because she confused the length of the perimeter with the number of squares inside (i.e. with the area of the rectangle). Clearly all three children had ideas about length which were not quite what the teacher might have expected.

Work on fractions with younger children (aged 10 years) has given rise to some equally interesting insights. For example:

Question: What fraction of the shape is shaded?

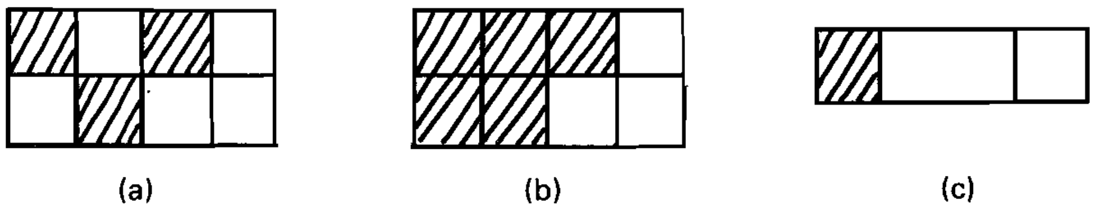


Figure 3

(Each shape is drawn on squared paper so that it can be seen that all the parts in (a) and (b) are equal while those in (c) are not. In fact the part shaded in (c) is one quarter.)

Louise (Fig. (a)):	One third.
Interviewer:	Why do you say one third?
L:	Because there are 3 squares shaded.

I: What about this one (Figure (b))?

L: One fifth.

I: How did you get that?

L: Because 5 are shaded this time.

Matthew (Fig. (a)): One third, because there are 3 shaded.

Interviewer (Fig. (b)): And this one?

M (pause): One fifth . . . Oh . . . wait . . . (long pause).

I: What's the matter?

M (pause): Well, . . . I was going to say one fifth, but I don't think that's right.

I: Why don't you?

M: Well, because there are 5 shaded . . . I thought that was one fifth. But 5 shaded is more than a half, but a 'fifth' is *less* than a half, so I don't think that can be right.

I: Any idea what fraction it *is*, then?

M (pause): No . . . not really. (Pause) It's got to be a quarter more than a half, but I don't know what you'd call that!

Justine (Fig. (a)): Three eighths.

I: How did you get that?

J: Well, it's divided into 8 parts altogether, and 3 of them are shaded, so that's 3 of the 8 parts . . . three eighths.

I (Fig. (b)): And this one?

J: Five eighths because this time there are 5 of the 8 parts shaded.

I (Fig. (c)): And this one?

J: One third.

I: How did you get that?

J: Because it's divided into 3 parts and only one of them is shaded.

We can see from these answers that Louise has her own view of how we name fractions and of what we mean by 'one third' or 'one fifth'. Matthew also seems to share her ideas, but has an independent concept of the size of fractions in that he knows that 'one fifth' is smaller than 'one half', and furthermore is able to check whether the answer he gets makes sense (something children often don't do!). He also knows that the fraction he needs is one portion more than one-half, even though he mistakenly thinks that the portion shaded is 'one quarter' more than one-half (you can doubtless see how he arrived at the idea that it should be one quarter more). Joanna's answers are interesting, in that her first two responses seem to indicate that she thoroughly understands what fractions are, and how we name them. However, we can see from her answer to shape (c) that she has not fully grasped that 'thirds', 'fifths' and 'eighths' etc. refer to *equal* parts of a whole. This is perhaps not so surprising when we remember that the diagrams in many textbooks only show shapes which *have* been divided into equal parts, and do not include shapes which have been unequally divided. Consequently the child's attention is directed only to the *number* of parts, and the child is given little opportunity to discriminate between the cases of equality and inequality of part-size. (Clearly we need to include both kinds of examples in our teaching.)

Listening to children's answers in this way can therefore tell us quite a lot about their understanding of the ideas involved in a problem, and of how they think in trying to solve it. This information can in turn help us to plan better ways of teaching, not only for the particular children we listen to, but for all the other children who may share the same way of thinking. Using this approach can be quite easy, but does require the teacher (or researcher) to accept two principles. Firstly, the idea is to find out what the *child* thinks. This means that the teacher must resist the temptation to tell the child what he or she 'should' be thinking.

Secondly, we need to take a different view of 'wrong' answers. Like Piaget, we need to accept that a wrong answer may *not* be wrong from the child's point of view, but indeed may be very sensible in terms of the child's concepts and ways of thinking. Consequently, the wrong answers which a child gives should not be ignored or merely corrected. Instead they can be used to help us firstly discover more about how children think in mathematics, and secondly to plan better teaching approaches based on this information.

Reference

Hart, K. (Ed.) (1981) *Children's Understanding of Mathematics: 11–16.* London: John Murray.

***(Note:** Interested teachers might like to try these or similar questions with some of their own children. The author would be very pleased to hear from any who do.)*