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Source	*Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 157-164)

Snapshots of Productive Noticing: Orchestrating Learning Experiences Using Typical Problems

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In this paper, we re-examine the commonly-held notion that typical problems, such as textbook exercises and examination questions, are not useful for orchestrating mathematically-rich learning experiences. Drawing from a larger design-based research project, we present a case study of Alice, a secondary school teacher, who orchestrated a productive discussion by using examination questions. We describe how she perceived and harnessed the affordances of such typical problems before and during her lesson. Findings suggest teacher noticing as a key mechanism to enable teachers to unlock the mathematical potential of such problems.

In Singapore, there has been a recent emphasis on orchestrating learning experiences, "the interaction between the learner and the external conditions in the environment to which he can react" (Tyler, 1949, p. 63), to develop mathematical thinking. We see students' learning experiences as their engagement with mathematical tasks selected, adapted, or designed by their teachers. Orchestrating learning experiences involves teachers providing "opportunities for students to discover mathematical results on their own" or "work together on a problem and present their ideas using appropriate mathematical language and methods" (Ministry of Education-Singapore, 2013, p. 20). Teachers are thus expected to select, adapt, or design tasks and orchestrate learning experiences by engaging students in mathematical activities through these tasks. However, this can be challenging for teachers, considering that it is not just the design of mathematics tasks, but also how these tasks are implemented in the classrooms that matters (Smith & Stein, 2011; Sullivan, Clarke, & Clarke, 2013; Tyler, 1949). We present preliminary results from an ongoing study on orchestrating learning experiences in Singapore. Specifically, we present snapshots of a teacher's productive noticing as she orchestrated classroom learning experiences using tasks developed from typical problems.

Typical Problems: An Untapped Resource

Mathematically-rich challenging tasks as an important vehicle for orchestrating learning experiences (Smith & Stein, 2011; Sullivan et al., 2014). For instance, Smith and Stein (2011) argue that tasks which are of a higher cognitive demand form the basis for engaging students in doing mathematics. Similarly, Sullivan et al. suggest that students' learning experiences are enhanced when they "devise their own methods of solution at least some of the time" (p. 124) to challenging tasks. Despite the affordances of challenging tasks in enhancing learning experiences, there are at least three obstacles that hinder the prevalent use of these tasks in the classrooms: (1) These tasks may be too difficult for many students, and so additional prompts or supports are needed (Sullivan et al., 2014), (2) It is time-consuming for teachers to select, adapt, or design challenging tasks to use, and (3) The inherent complexity of the tasks would involve mathematics from across the curriculum and is best implemented across several lessons, or after a few topics are taught. These demands on teachers' knowledge and time may

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 157-164). Melbourne: MERGA.

limit the incidence of such tasks. Moreover, teachers are mindful about the concurrent need to develop procedural fluency in students as part of their preparation for tests and examinations. Thus, it is common to use examination-type questions with a more teacher-centered teaching approach in Singapore classrooms (Foong, 2009; Ho & Hedberg, 2005). This preference for using typical problems (standard examination or textbook problems) may reflect teachers' belief that it is "important to prepare students to do well in tests than to implement problem-solving lessons" (Foong, 2009, p. 279), a classroom reality that cannot be ignored.

Here, we define typical problems as standard examination-type questions or textbook-type questions which focus largely on developing procedural fluency and at times, conceptual understanding (e.g., see Figure 2). These questions can be solved more quickly than challenging tasks, and are used frequently in mathematics lessons. Given the omnipresence of such questions in textbooks and other curriculum materials, we see typical problems as an untapped resource that can be used to orchestrate daily learning experiences. Using tasks developed from typical problems to orchestrate learning experiences would position mathematical learning experiences as an integral part of mathematics lessons, and not just reserved for occasional "enrichment" lessons. Preliminary findings suggest that experienced teachers not only see the development of procedural skills with such questions, but also adapt them for different student profiles to develop conceptual understanding. We present Alice as a case study of an experienced teacher to offer insights into how noticing the affordances of a typical problem is critical for orchestrating learning experiences.

Productive Mathematical Noticing

Mathematics teacher noticing is the emerging construct that lies at the heart of these components of teaching expertise. It refers to what teachers attend to and how they interpret their observations to make instructional decisions, or to have a different act in mind (Mason, 2002; Sherin, Jacobs, & Philipp, 2011). Several studies have demonstrated the importance of teacher noticing in enhancing teachers' reflection on teaching to improve their practices (Goldsmith & Seago, 2011; Star, Lynch, & Perova, 2011); while others have highlighted how teacher noticing can enable teachers to respond to students' thinking during lessons (Choy, 2014). More recently, Choy (2016) examines what and how teachers notice when they design tasks to enhance students' reasoning. Even though noticing appears to be part and parcel of teaching, what and how teachers notice may not always lead to mathematically productive instructional decisions. That is, not all noticing is productive.

The FOCUS Framework (Choy, 2015) characterises productive noticing in two ways: having an explicit focus for noticing and focusing noticing through pedagogical reasoning—how teachers justify their instructional decisions or claims about students' thinking using what they attend to. Choy suggests that an explicit focus is useful for supporting teachers to notice relevant instructional details. There are two key aspects of this explicit focus: First, the three components of the didactical triangle, namely the mathematics concept, students' confusion associated with the concept, and teachers' course of action to address students' confusion. Second, the alignment between these three components, that is, whether the teacher's course of action targets students' confusion when they are learning the concept. This alignment between teachers' instructional decisions and students' confusion is not intuitive. Instead, this alignment is mediated by teacher's pedagogical reasoning.

In this paper, we build on extant research on teacher noticing by examining what and how a teacher noticed when orchestrating learning experiences. We focus on what the teacher attended to in relation to the interactions between students, content, and the task. These interactions can be visualised as a socio-didactical tetrahedron (Rezat & Sträßer, 2012). In Figure 1 (left), we follow Rezat and Sträßer (2012) in seeing each face of the tetrahedron as an instantiation of the relationship between a task and mathematics education. For example, the task-students-teacher face represents the interactions that occur amongst teacher, students, and the task. Given our emphasis on teacher noticing, we put "Teacher" as the apex, as seen in Figure 1 (right) to reflect our focus on how the teacher managed these interactions. This paper is framed by the question: How does a teacher's productive noticing of these didactical interactions help in orchestrating learning experiences with typical problems?

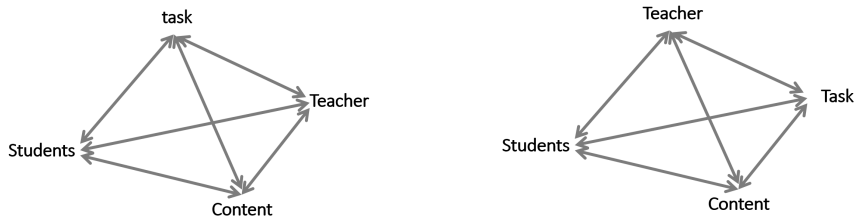

Figure 1. Socio-didactical tetrahedron for using task to orchestrate learning experiences.

Methodology

This ongoing project has adopted a design-based research paradigm (Design-Based Research Collective, 2003) to develop a toolkit to support teachers in noticing and a theory to describe their noticing when orchestrating learning experiences. We engaged in three iterative cycles of theory-driven design, classroom-based field testing, and data-driven revision of the Mathematical Learning Experience Toolkit (MATHLET) to provide a theoretical justification for the analytical frameworks on which the toolkit is based. By engaging with our teacher participants in designing, implementing, and reviewing learning experiences using the MATHLET, we aimed to develop a deeper theoretical understanding of how teachers orchestrate mathematically meaningful learning experiences. Four experienced mathematics teachers from three secondary schools, with different achievement bands and demographic factors, participated in this study. Each teacher designed and implemented a lesson during each design-cycle phase using the MATHLET, which resulted in 12 design cycles in total. Data were generated through voice recordings of planning discussions, pre-lesson discussions, post-lesson discussions, video recordings of lessons, and lesson artifacts. Findings were developed using a thematic approach (Bryman, 2012) together with the two characteristics of productive noticing as proposed by the FOCUS Framework (Choy, 2015).

Snapshots of Productive Noticing

Alice and the Context of Her Snapshots

Alice (pseudonym) is a Senior Teacher at a government-funded school with above-average performance in the national examinations. She has a strong mathematical background and has

been actively involved in mentoring novice teachers. We present an analysis of Alice's lesson on matrices for Secondary 3 (Grade 9) students. The syllabus document encouraged teachers to provide students opportunities to apply matrix multiplications to solve contextual problems, and for them to justify if two matrices can be multiplied by checking the order of the matrices. Prior to this lesson, her students had learnt how to multiply two matrices. For the lesson, Alice modified a typical problem and used it in an introductory task to orchestrate a mathematically productive discussion. Students then worked through a sequence of typical problems and presented their answers. Referring to Figure 1, we focus on Alice's instrumentalisation (Verillon & Rabardel, 1995) of the task using a typical problem in relation to the mathematics (teacher-task-content face), her use of the task with students (teacher-student-task face), and her attention to students' thinking about the concepts (teacher-student-content face). In managing these sets of interactions, Alice demonstrated productive noticing of the mathematics to be taught, her students' learning, and the affordances of the typical problem.

Alice's Instrumentalisation of a Typical Problem

For her introductory task, Alice selected a standard examination question (See Figure 2) which comprised of two parts. The first is a routine matrix multiplication involving pre-multiplying a 3×1 matrix by a 2×3 matrix to obtain a 2×1 matrix $\begin{pmatrix} 185 \\ 184 \end{pmatrix}$ as the solution; the second part asked for the meaning of the product in the context. Many teachers would have only focused on guiding students to solve the question as intended. However, Alice seemed to notice the mathematical affordances. During the post-lesson interview, she explained:

> Why I choose this question is because most of the exam style questions are based on solving problems involving matrices. And this question will extend their thinking and help them to transfer mathematical ideas into other representations. This is what I find challenging amongst some students…

Alice highlighted that the question could potentially extend students' thinking in terms of expressing the same information in using different representations involving matrices or otherwise. Students often do not have opportunities to express information using different matrices as the matrices are usually given in the question. As a result, they have limited opportunities to see the connections between arithmetic and matrix multiplication. To achieve her objectives, Alice modified the typical problem as follow:

> Teresa and Robert attend the same school. They keep a record of the awards they have earned and the points gained. Teresa obtained 29 Gold, 10 Silver, and 5 Bronze awards. Robert obtained 30 Gold, 6 Silver, and 8 Bronze awards. They gained 5 points from each Gold award, 3 points for each Silver award, and 2 points for each Bronze award. Find the total number of points that Teresa and Robert gained.

We note that Alice did not include any matrix in her modified question, and this expanded the solution space of the original question. For example, students could solve the problem using arithmetic without matrices. Alternatively, students who see the problem as a matrix multiplication problem would first need to formulate the matrices before deciding on the order of matrix multiplication. This provided opportunities for Alice to emphasise the connections between matrix multiplication and arithmetic which could potentially provide meaning to

matrix operations. In so doing, she attempted to develop both conceptual understanding and procedural fluency in the matrix operations.

Example 1

[Nov 2013] Teresa and Robert attend the same school. They keep a record of the awards they have earned and the points gained. The matrices show the numbers of awards and the points gained for each award.

$$\begin{array}{c} \\ \text{Teresa} \\ \text{Robert} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Gold} & \text{Silver} & \text{Bronze} \end{array} \\ \begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Gold} \\ \text{Silver} \\ \text{Bronze} \end{array} \begin{array}{c} \text{Points} \\ \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} \end{array}$$

(a) Find $\begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$.

(b) Explain what your answer to (a) represents.

Figure 2. The introductory problem (original form).

Moreover, Alice highlighted that she chose an "easier" question with a relatable context to start the lesson. Alice also revealed that she considered what students had found confusing from her analysis of students' errors committed in past year examinations. She weaved these questions as a sequence of tasks in her worksheet. Therefore, her noticing is classified as productive according to the FOCUS Framework: Alice attended to the key concepts in matrices, recognised students' possible confusion from their mistakes in past year examinations, and modified a typical problem and used it together with a carefully constructed sequence of questions to design a task that addresses students' confusion.

Alice's Orchestration of Discussion Using a Typical Problem

Alice demonstrated her recognition of the typical problem's affordances as she orchestrated discussions during the lesson:

1 Alice: (Walks around the class and comes to Student S1.) Can you write this for me on the board?

2 S1: Ok. (Walks to the whiteboard and writes the following:
$T = 5 \times 29 + 3 \times 10 + 2 \times 5 = 185$
$R = 5 \times 30 + 3 \times 6 + 2 \times 8 = 184$)

3 Alice: (Walks around while waiting for Student S1 to finish writing.) Ok. Most of you have written what [Student S1] has written. 5 points for 29 gold, 3 points for 10 silver and 2 points for 5 bronze. Most of you have written in this manner. The last few days, we have been talking about matrices, right? Would you like to convert this to a matrix problem?
(Looks at Student S2) Have you written it in matrix form? (Student S2 nods and Alice goes over to look at his answers.) Okay. Can you write your answer on the board?

4 S2: (Walks to the board and writes the following.)
$$T = \begin{pmatrix} 29 & 10 & 5 \end{pmatrix} \times \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = 185 \text{ and } R = \begin{pmatrix} 30 & 6 & 8 \end{pmatrix} \times \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = 184$$

5 Alice: Any other answers from [Student S2's] answer? (Walks around the class and selects Student S3's answer) Can you write this on the board?

6 S3: (Walks to the board and writes the following.)

$$\begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 29\times 5 + 10\times 3 + 5\times 2 \\ 30\times 5 + 6\times 3 + 8\times 2 \end{pmatrix} = \begin{pmatrix} 185 \\ 184 \end{pmatrix}$$

7 Alice: Thank you all three of you. [Student S1] has written using an arithmetic method. Most of you have written in this manner. This one comes very naturally to you, ok? [Student S2] has written Robert and Theresa's award separately. He has tried to use the matrix method, (points to Student S1's solution.) Something like this, ok? Let's check whether the order of matrix is correct or not.
(Alice goes through the method of matrix multiplication and gets the class to check the order of Student S2's matrices.)
… Ok. Student S3 has written Robert's and Theresa's together so that you only write this matrix once (points to the column matrix [5 3 2]). Don't need to write two times, correct or not? See. Over here. You have to write two times but here, [Student S3] only has to write it once. Let's check the order again…

8 Alice: (After a short time) I would like to bring this problem a little bit further. Notice that Student S3 presented the information this way. Is there another way to represent the same information?
(After some time, Student S4 highlights another possible way.)

Here, we see how Alice orchestrated a mathematically productive discussion (Smith & Stein, 2011). Alice carefully attended to students' answers before she asked for volunteers during the whole class discussion. However, it can be inferred that she was deliberate in her selection and sequencing of students' responses (See Lines 1 to 6). By beginning with an arithmetic solution, Alice connected Student S1's arithmetic operations to matrix multiplications through the sequencing of Student S2's and Student S3's matrix solutions. The reason for using a single matrix multiplication (Student S3's solution) was also made explicit when Alice moved from Student S2's solution to Student S3's using a matrix approach (Line 7) before she highlighted the different ways to express the given information as matrices (Line 8), which was an important idea for the lesson. As highlighted during her post-lesson interview, she knew "that certain students will give these answers" and hence, we see that Alice had anticipated students' solutions before the lesson. Therefore, Alice's noticing is classified as productive because she attended to the different solutions and orchestrated the discussion to highlight the key mathematical ideas.

Alice's Response to Students' Thinking When Using Typical Problems

Another snapshot of Alice's productive noticing could be seen from how she responded to Jason's (a pseudonym) ideas when working on a matrix formulation problem (See Figure 3).

31 Alice: Now we have got the cost to these two outlets already, what about your deliveries? How can you represent your deliveries in a matrix? [Jason], how can you represent the deliveries?
32 Jason: 27 by 25.
33 Alice: 27 and 25, so how to write? Row or column?
34 Jason: Column, 1 by 2.
35 Alice: (Writes column matrix as suggested by Jason) You can write like that right?
36 Jason: Yeah.
37 Alice: You can also write like that. (Points to a row matrix.) Which one do you choose?
38 Jason: The first one (Referring to the row matrix).
39 Alice: The first one? Why the first one?
40 Jason: Depending on the position.
41 Alice: Depending on the position, so [Jason] come and write out for us, what are the two matrices you have chosen such that the product will give you the total cost?

In this excerpt, Alice attended to Jason's ideas (Line 31) and asked questions (Lines 33, 37, and 39) to reveal what Jason was thinking. This reflected Alice's attempt to gain an awareness of Jason's thinking. She refrained from evaluating Jason's incorrect answer (Line 34) and instead, prompted Jason through a series of questions. By engaging Jason to think about his answers, Alice could direct Jason's attention to the need to check the order of matrices. Alice's noticing was productive because she "asked questions to reveal" Jason's thinking (Lines 39 and 41) and built on his understanding (Choy, 2015).

A bakery produces 3 different types of bread : white bread (W), wholemeal bread (M) and multi-grain bread (G). Delivery is made to 2 distribution outlets in the following way:

Outlet A receives	60 loaves of W,	50 loaves of M,	30 loaves of G.
Outlet B receives	40 loaves of W,	70 loaves of M,	20 loaves of G.

The costs of one loaf of W, M and G are \$2.10, \$2.70 and \$2.90 respectively.

It is given that $\mathbf{P} = \begin{pmatrix} 60 & 50 & 30 \\ 40 & 70 & 20 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2.1 \\ 2.7 \\ 2.9 \end{pmatrix}$.

(a) (i) Evaluate **PQ**.
(ii) Explain what the answer in (i) represents.

(b) In a particular month, Outlets A and B receive 27 and 25 such deliveries respectively. Form two matrices so that their product will give the total cost of the bread delivered to the 2 outlets. Find the product.

Figure 3. Problem discussed by Alice and Jason.

Concluding Remarks

The three snapshots presented above described how Alice orchestrated the interactions within the teacher-task-content face, teacher-task-student face, and teacher-student-content face of the didactical tetrahedron. First, Alice was able to tap the mathematical affordances of typical problems, and modified them appropriately to address possible confusions faced by her students when learning matrices. Her ability to see and use typical problems beyond their current forms provide an existence proof for the use of such problems to orchestrate discussions. Alice's use of typical problems highlights the critical role of productive noticing in enabling her to do this work (Choy, 2016). More importantly, it goes beyond Choy's (2016) work on productive noticing involving a single task, and extends the study of noticing into the realm of using a sequence of typical problems to bring about mathematically productive learning experiences for students. As seen from these snapshots, Alice not only attended to the mathematical possibilities of typical problems, but also exploited these problems fully during her orchestration of the lesson. The snapshots suggest that Alice had a bird's eye view of how a sequence of tasks is embedded in a lesson, which in turn is embedded in a sequence of lessons within a unit. This highlights her noticing of mathematical connections between tasks, lessons, and units within the curriculum. At the lesson level, she demonstrated the five practices of anticipating, monitoring, selecting, sequencing and connecting during the whole class discussion (Smith & Stein, 2011). Her orchestration of the classroom discussion is reflective of her productive noticing according to the FOCUS Framework (Choy, 2015). This work highlights the mathematical potential of using typical problems to orchestrate learning experiences. Alice's orchestration of this lesson opens an avenue for mathematics educators to support teachers in noticing and harnessing the untapped potential of typical problems.

Acknowledgments

This paper refers to data from the "Portraits of Teacher Noticing during Orchestration of Learning Experiences in the Mathematics Classrooms" project (OER 03/16 CBH), funded by the Office of Educational Research (OER), National Institute of Education (NIE), Nanyang Technological University, Singapore, as part of the NIE Education Research Funding Programme. The views are the authors' and do not necessarily represent NIE's views.

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