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Refining Teaching Expertise through Analysing Students’ Work: A Case of Elementary Mathematics Teacher Professional Learning during Lesson Study in Singapore

Heng Jiang, Ban Heng Choy, and Christine Lee Kim Eng

Abstract

This article provides a concrete illustration of how teachers in a primary school in Singapore discuss students’ learning in a lesson study cycle and grew professionally as a community. Specifically, we examined how collaboratively analysing students’ work serves as a useful practice for teachers to learn to work with diverse learners. The findings suggested that open discussions around students’ work helped teachers to reflect upon their unwarranted perceptions of their students and their teaching. The study provided insights into how teachers’ understandings of their students’ diverse backgrounds, as well as teachers’ understanding of subject content and pedagogy, developed as they participated in lesson study activities that were focused on analysing students’ work. Our findings found that lesson study provided the following affordances to foster such changes: (1) eliciting hypotheses in dialogue; (2) creating space for alternative perspectives; (3) collaboratively scrutinizing student learning evidence for follow-up teaching; and (4) identifying problems for further discussion. While the illustration of this case is uniquely Singaporean, implications include concerns about teacher professional learning and teaching for equity common to many other educational contexts.

Key Words: Teacher Learning; Teaching Expertise; Analyse students’ work; Lesson Study
Introduction

In the past decade, there has been a “practice turn” in conceptualizing teacher learning, which suggests that the core teaching practices (such as providing feedback and classroom management) can create opportunities for teachers to develop a repertoire of these practices and provide the basis for developing and understanding specialized teacher knowledge and competencies (Ball and Forzani 2011; Grossman 2008; Janssen, Westbroek, and Doyle 2014). But practice-based teacher learning has been focusing mostly on pre-service teacher education and novice teacher learning. What was largely ignored is how such core practices can be deliberatively used to help experienced in-service teachers develop and refine their adaptive expertise. Only a few studies tapped into specific discussions that teachers engage in a community to adjust and develop their acquired expertise in particular instructional practice (Brodie and Borko 2016; Vescio, Ross, and Adams 2008).

Further, despite the emerging consensus about the core practices teachers should learn and how they can learn these practices, researchers identified a major tension between decomposing specific core practices in building routines and recomposing grain-sized practices into the adaptive expertise (Darling-Hammond and Bransford 2005; Janssen, Grossman and Westbroek 2015). Herein lies the gap between the routinized and the adaptive dimensions of teaching.

Routines, defined as tried-and-tested experience or knowledge representations, fit well into familiar and stable circumstances. They do not automatically require teachers to apply their usual understanding or teaching practices in novel ways (Soslau 2012). Although seemingly
contradictory to creative and adaptive teaching, Bohle Carbonell and colleagues (2014) have pointed out that routines are the basis for adaptive approaches. Teachers often need to modify their routines by slowing down and making conscious efforts to deal with the ever-changing complex classroom situations. Furthermore, routines allow teachers to free up their attentional resources to consider day-to-day teaching situations from new perspectives, possibly leading to more adaptive ways of knowing and doing (Schwartz et al. 2005). For example, when routines do not work out the way teachers have envisioned, teachers are pressed to reconsider their prior ideas and behaviors in a new light (Schön 1987). However, retrieving essential information is challenging if teachers lack an awareness of how they can exploit their routines for adaptive perspectives, or if they do not have an enriched repertoire of alternative understandings and coping strategies.

This article intends to address this issue and to provide a concrete illustration of how a focus on analysing students’ work can support teachers to hone their expertise—namely, discussing each other’s teaching practice, monitoring students’ performances, becoming more aware of their own conceptions of teaching and learning, and adapting their instructional approaches with the support from the lesson study community. It is based on a discussion episode selected from data collected during a two-year research project involving teachers participating in lesson study as a professional development platform. We examined the experiences of a group of mathematics teachers engaged in lesson study (Lewis and Hurd 2011; Stigler and Hiebert 2016) in one Singapore elementary school, inquiring how professional discussions on students’ work played a role to facilitate teacher learning. The findings from this study suggest that lesson study provides affordances for teacher learning via open discussions about the evidence of student learning. This can help teachers to reflect upon their assumptions.
about their students, adapt their teaching expertise, and learn from colleagues and knowledgeable others to find out more about how to teach their students. In this way, teachers’ intuitive judgments about students’ performance and learning progression can be deliberatively examined and tuned to contribute to adaptive teaching expertise.

Literature review

Recent research calls for more practice-based teacher learning (Ball and Forzani 2011; Janssen, Westbroek, and Doyle 2014). Diverse aspects of this general model of practice-based teacher learning have been elaborated by Grossman and McDonald (2008), Ball and Forzani (2011), and other researchers in the past decade. One of the major tensions in this practice-turn of teacher learning is the specification and the building of routines with the decomposed teaching practices and the need to help teachers develop fluid expertise that can adapt to the complexities in classroom teaching.

For our purposes, we focused on three recent extensions of practice-based teacher learning that seemed especially useful for understanding teacher learning to develop adaptive expertise: (a) modularity of instructional activities encompassing related core practices for teacher learning (Janssen et al. 2015); (b) adaptive and intuitive expertise fostered by working on student learning data (Vanlommel et al. 2018); and (c) collaborative learning communities, such as lesson study, that contribute to the creativity and knowledge building for expertise (Dudley 2013; Cheng 2019).

Core practices and modularity of instructional activities for teacher learning
Representatives of practice-based approaches propose that teachers learn through a core set of high-leverage instructional practices, such as eliciting student ideas, leading a discussion, explaining and scaffolding, creating a classroom culture, and analysing students work or assessment of student learning (Forzani 2014). Generally, researchers agree that core practices are those that (Grossman and McDonald 2008; Janssen, Grossman and Westbroek 2015):

- frequently occur in teaching;
- novices can enact in classrooms across different curricula or instructional approaches;
- novices can begin to master;
- allow novices to learn more about students and about teaching;
- preserve the integrity and complexity of teaching; and
- are research-based and have the potential to improve student learning.

To help teachers learn these core practices—managing a classroom, providing feedback to students, and so on—researchers have suggested means to provide the necessary learning opportunities as follows: (1) represent the target practice by modeling or by analysing video or written cases of teachers that enact the practice. (2) decompose the elements of the practices into their constituent parts; and (3) provide opportunities for the teachers to rehearse the approximation of the practice in a setting that is removed from the full complexity of the authentic practice (Grossman et al. 2009; McDonald et al. 2013).

Although such high-leverage practices or core practices are mostly discussed in the area of pre-service teacher education, they can also be considered important tasks for in-service teachers as part of their professional improvement efforts. For novice teachers, the goal of practice-based teacher learning is to help them unpack and later achieve certain fluency and automaticity in enacting the core practices. For in-service teachers with some experience in
teaching, practice-based teacher learning tends to gravitate towards the shifting of current practices and teachers’ mental models. For instance, researchers in the Netherlands brought into focus the practical tasks teachers regularly face within the classrooms, and used “bridging method” for teachers to re-examine their regular teaching practices and their goal systems in comparison with the target practices, and gradually acquire the innovative practices advocated in the educational changes (Janssen et al. 2013; Westbroek, Janssen and Doyle 2016). American researchers have also tried to support and measure changes in teaching throughout the professional development in the foundational core practices with the use of observational protocols (Cohen et al. 2016). These studies, which made use of core practices to promote in-service teacher learning suggest that teachers take heterogeneous approaches in their professional learning and are guided by their existing goals and knowledge when negotiating the target innovative teaching practices. That is, teachers make sense of their own teaching using their expertise built via daily routines and years of experience which is resistant to change.

Hence, advocates of the practice-based teacher learning recognize the need for teachers to move from decomposition to the re-composition of teaching practices. This will involve the infusion of the skills in their practice and the will to refine it in the development of adaptive expertise (Janssen et al., 2014; Wetzel et al., 2015). Recent developments in this line of research suggest locating core practices within larger instructional activities (Lampert and Graziani 2009), or a modular system or sequence of lesson elements (Janssen et al., 2015) that provide a frame for teachers to understand the meaning and function of the practices as a structured whole. In this study, we do not attend to a series of lesson elements but focus on one set of instructional activities revolving around analysing student work, which is a major aspect of teacher’s daily work in Singapore. After marking the student worksheets done in the research lesson, the
teachers discussed as a group and together analysed the pattern of mistakes students made to
gauge the effect of the teaching. They not only identified “typical errors” that signaled the
common misconceptions of the students, but also compared and challenged different hypotheses
raised to understand students’ thinking. These students’ misconceptions were highlighted and
addressed in the lesson study cycle that modified their understanding of the students, the lesson
content, and their follow-up lessons. This paper will focus on this series of instructional activities
to explore the opportunities embedded for teacher learning.

Adaptive expertise with intuition and rationality in analysing students’ work

Teaching practices can provide opportunities for teachers to improve their teaching
through developing adaptive expertise for teaching. Teachers might need some procedures to get
started, but they have to move beyond routines and rules to achieve adaptive expertise. When
teachers become proficient in practice, they see situations instead of calculating procedures. As
Tsui (2009) noted, adaptive expertise is developed through ‘the processes of reflection and
conscious deliberation in which practical knowledge is theorized, and theoretical knowledge is
interpreted in practice’ (p.437). However, such expertise was found to rely heavily on tacit and
intuitive dimensions as well as rational decision making (Klein 2015). It is challenging for
teachers to be aware of the often implicit processes underlying the established teaching routines
(Mannikko and Husu 2019).

Experienced teachers often rely on their immediate intuitive expertise without needing to
search for it (Harteis and Billett 2013), which is also described by the researchers as personal
practical knowledge or tacit knowledge (Clandinin 1985; Elbaz 1991; Mannikko and Husu
2019). That is how they can orchestrate productive discussions with more than 20 learners with
different learning abilities and needs and work with the students around their misconceptions while maintaining an orderly classroom. However, the disadvantage of intuitive expertise is that it may lead to confirmation bias when teachers focus their attention on what they expect to see, consequently missing the information that may challenge their assumptions. Thus, their intuitive judgment of student learning progress may be far from reliable (Allal 2013; Kaiser et al. 2013; Vanlommel et al. 2018). In addition, without assessing students’ learning using the evidence from classroom practices, their unreliable intuitive judgment may be crystalized as biases.

Berliner (2001) cited Bereiter and Scardamalia (1993) using concepts from the psychology of intelligence to distinguish between crystallized and fluid expertise: “Crystallized expertise consists of intact procedures that have been thoroughly learned through experience, brought forth and used in relatively familiar tasks. Fluid expertise consists of abilities that come into play when novel or challenging tasks are confronted by an expert. Adaptive or fluid experts appear to learn throughout their careers, bringing the expertise they possess to bear on new problems and finding ways to tie the new situations they encounter to the knowledge bases they have.” (p.203). He also suggested that adaptive expertise relies on a set of constructs and relationships that proves crucial to frame the practitioner’s understanding.

These findings have led to an increased expectation that teachers will collect solid evidence of student learning to enhance the quality of their rational decision making (Berliner 2001; Carlson, Borman and Robinson 2011; Schildkamp and Lai 2012). Many studies have therefore investigated factors that might promote or hinder teachers developing their adaptive expertise using data about student learning (Author 2018; Coburn and Turner 2011; Levin and Datnow 2012). For example, Furtak, Bakeman, and Buell (2018) developed an intervention in which teachers learned about student ideas in a three-year professional development program.
(subject: biology; topic: natural selection). They studied the influence of this professional learning experience on teachers' professional knowledge-in-action as measured by the quality of questions the teachers asked in classrooms to elicit student thinking, as well as the extent to which teachers selectively responded to particular types of student ideas in the course of classroom conversations. They demonstrated that teachers' “data collection” in the form of questions that elicited student ideas, and their actions (responding to student ideas) shifted as the teachers acquired new knowledge on how to think about and represent ideas about natural selection.

While most of these studies largely relied on the rational model of developing teacher expertise, recent research has started to incorporate the intuitive dimension of teacher expertise and recognize that teachers actively use their beliefs and tacit knowledge such as conceptions of good teaching, and their usual teaching heuristics to evaluate and adjust new approaches to the practice of teaching (Buchanan, 2015; Dam, Janssen, and van Driel, 2018). We adopt both intuitive and rational approaches to understand teaching expertise and position the instructional activity revolving around one type of “using student data”--analysing student work--in relation to teacher learning. This is especially important when teachers have to work with students from diverse backgrounds since they have to exercise their professional discretion on how to respond to the students’ different needs when interpreting the students’ work..

Lesson study and improving teaching expertise

Studies on the rational and intuitive use of student learning data are mostly on the individual level and often used to explain the relationship between teacher using evidence of student learning and student academic performance (Mandinach and Gummer 2013; Williams
and Coles, 2007). More recent studies examine how to intervene in teacher learning about how to effectively work with student data in teacher groups. For instance, Alonzo and Kim (2018) explored how discussions of classroom videos focusing on student thinking in the context of professional development workshops can help teachers improve their judgment of student thinking as required in-the-moment in their classrooms. The authors found, for example, that the quality of teachers' judgments was related to the quality of their discussions. Their findings also suggested that elaborated focus questions and interactions with colleagues could support teachers in their teaching.

In-depth insight is needed into how teachers’ spontaneous responses to interpreting student learning evidence shape and are being shaped by the discussions in the professional learning community. Following Little’s (2012) call to examine data use from a micro-process view, we focused our lens at the grain-size of teachers’ discussions to examine the relationships between the practice of analysing students’ work and opportunities for teachers to learn to teach mathematics to diverse learners. Further, we situate this study in a professional community for teachers to acquire their expertise about and beyond analysing student work. In Singapore, teachers’ professional discussions often happen in various professional learning communities as strongly recommended by the Ministry of Education and are commonly implemented at the school level.

Lesson study, which was introduced to Singapore in 2004, has been one of the major structures in schools to support teachers’ professional learning to promote student learning (Lim-Ratnam et al. 2011; Saito and Atencio 2013). According to Lim-Ratnam et al. (2014), 190 schools out of a total of 354 schools (53.7%) in Singapore had implemented lesson study by 2014, and teachers commented positively on the effectiveness of lesson study. Teachers have
reported that their pedagogical content knowledge has improved and their understanding of student learning has increased as a result of the opportunity to learn from colleagues in discussing student learning and pedagogical strategies, and through visiting each other’s classrooms (Lim-Ratnam, et al. 2014). Specifically, lesson study is described as a process consisting of the following steps: (1) collaboratively planning the study lesson; (2) implementing the study lesson; (3) discussing the study lesson; (4) revising the lesson plan (optional); (5) teaching the revised version of the lesson (optional); and (6) sharing thoughts on the revised version of the lesson (Fernandez and Yoshida 2004; Lewis et. al. 2009; Shuilleabhain and Seery 2018; Stigler and Hiebert 2016; Verhoef, et al. 2015). By engaging teachers to work together to develop a unit of work including a detailed research lesson plan, teach and observe the research lesson, and discuss its enactment for instructional improvement with a focus on student learning, lesson study provides a major platform for researchers to delve into teacher conversations about student learning.

Although lesson study has a straightforward structure including four common procedures (i.e., studying materials, planning lessons, observing research lessons, post-lesson discussion) which are said to target student learning (Lewis 2006; Dudley 2013), researchers have identified several features embedded in the cycle of lesson study that contribute to teaching and teacher learning (Cheung and Wong 2014). These features can be categorized into four characteristics: (1) Meaningful sharing of teaching and learning resources (Fernandez 2005; Lewis, Perry and Hurd 2004; Ono 2010; Watanabe, Takahashi and Yoshida 2008); (2) Explicating, anticipating and hypothesizing student learning to bridge teacher thinking and student thinking (Bocala 2015; Perry and Lewis 2010; Tolle 2010; White and Lim 2008); (3) interpretive discussions that are cognitive and content specific (Gutierez 2015; Masduki 2016); and (4) emotional support for
collegial collaboration and reflections based on feedback from both within the team and external knowledgeable others (Saito and Atencio 2015; Simmons 2016; Takahashi and Yoshida 2004; Lewis 2015; Simmons 2016). Given these features, lesson study is found to be effective to help teachers learn to understand their students’ thinking (Kriewaldt 2012; McDuffie and Eve 2009), deepen their content knowledge (Leavy 2014; Meng and Sam 2013; White 2007), collaborate with colleagues (Pektas 2014; Saito and Atencio, 2015), improve instruction (Lewis 2015; Utami, Mashuri, and Nafi’ah 2016), and highlight their professional identity and efficacy (Mintzes et al. 2013; Sims and Wash, 2009).

More recently, researchers have begun to attend to the small-scale improvement procedures at the level of discursive analysis in lesson study (Dudley 2013; Vrikki et al. 2017) to delve into the thinking process of teachers to create pedagogical knowledge and practices. For instance, Cheng (2019) used Nonaka’s knowledge creation model to advocate studying the “kaizen kata” (continuous small improvements) embedded a spiral knowledge-creating frame (SECI—socialization, externalization, combination, and internalization) of tacit and explicit knowledge interaction and conversions. We followed this line of research and examined how the teacher discussions before and after the research lesson were enriched by the analysis of the students’ work and how this particular engagement influenced teachers’ interpretations of diverse student learning and their classroom teaching.

This research considered the question: How does analysing students’ work in lesson study influence teachers’ learning to teach mathematics to diverse students in Singapore? We employed qualitative research methods to investigate this question as will be elaborated later.

**Context of the study**
Singapore is a small city-state with a population of 5.6 million composed of 76% ethnic Chinese, 15% ethnic Malays, 7% ethnic Indians and 1% other ethnic groups (Singapore Department of Statistics, 2018). Singapore is a particularly fascinating context for exploring how educators learn to work together to polish their expertise in teaching diverse learners. The tracking system, although now less explicit at the primary school level, remains a robust feature of local schools in Singapore (Anderson 2015; Kam and Gopinathan 1999). Singapore is also different from other high-achieving Asian countries, with its higher income inequality and lower social mobility (Corak 2013; Ng 2014), larger gaps in achievement between students from different socioeconomic backgrounds and sustained low academic achievement of students from the indigenous Malay ethnic group (Talib and Fitzgerald 2015). Teachers have to work with diverse student populations in a highly competitive education system that bands students into different ability groups. The ethnic- and class-based disparities and home language differences are manifested as the educational gaps for students from diverse backgrounds. To be sure, this educational gap is already present at lower levels of schooling; compared with the Express course (track) attended by roughly 65% of the cohort, minority ethnic groups are over-represented in the system’s low-progress Normal Academic and Normal Technical courses (track) – the latter being the least prestigious track at the secondary level (Albright, Heng, & Harris, 2006; Anderson, 2015; Ng, 2013). Students in the lower track in Singapore education system are often imbued with certain naturalized characteristics, such as “weak”, “lower progress”, that draw from widely available categories of ability as well as how the ability is measured and defined in local educational institutions (Anderson, 2015).

Given this concentration of students from a less advantaged background in non-elite schools, it is urgent for researchers to examine how teachers perceive these children and help
them to learn, especially in the Singapore school system characterized by the high-stake examinations. Aiming at high academic performance makes conversations about students’ work commonplace in schools. However, with limited studies delving into such conversations, we are yet to know how Singapore teachers learn professionally through the core teaching practices to work with diverse students (Author, in press). In this particular study, we aimed to contribute an understanding of how schools and policies can support teacher learning to understand and teach diverse students via one core teaching practice--deliberate discussion of student learning data--and develop adaptive expertise in teaching.

**Methodology**

As mentioned earlier, this study sought to understand teacher learning about teaching diverse learners when participating in discussions on analysing students’ work during lesson study. In addition to the individual interviews, the main data we present and analyse here is the recording of a 60-minute episode of a post-lesson discussion session by eight Gladstone Elementary School (GES, pseudonym) mathematics teachers (two male teachers and six female teachers, Table 1) engaged in lesson study in the 2016–17 school year, as well as relevant interview data with the participants. These teachers were trained to teach mathematics during their pre-service teacher education programs and they have had at least 3 years of teaching experiences. The discussion episode was recorded as part of an ongoing study into lesson study to engage diverse students, in which the research team observed and recorded bi-weekly discussions involving two teams of teachers in two schools (grades 4–6), collected samples of pupil work, interviewed teachers, and observed research lessons. We selected the schools from areas with student populations from diverse socio-economic backgrounds, after obtaining consent from the principals and teachers concerned. The case we present in this study is situated
in GES. The school serves lower-income neighbourhoods in the east of Singapore and has an enrolment of about 900 pupils and 60 teachers/staff. A large percentage of the pupils come from families in the lower strata of social and economic status and lack support in their academic work. Ms. T, the teacher teaching the research lesson, has been teaching for more than seven years at GES. At the time of the lesson, Ms. T was the homeroom teacher for the fourth-grade class and had been teaching them since the beginning of the school year. She had 20 students in her class, comprising nine boys and 11 girls. The students were from diverse ethnic backgrounds, with three Chinese students, 15 Malay students, and two international students.

<table>
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<tr>
<th>Name</th>
<th>Gender</th>
<th>Years of teaching</th>
<th>Ethnicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. C.</td>
<td>Female</td>
<td>5 years</td>
<td>Chinese</td>
</tr>
<tr>
<td>Ms. P.</td>
<td>Female</td>
<td>12 years</td>
<td>Chinese</td>
</tr>
<tr>
<td>Ms. F.</td>
<td>Female</td>
<td>7 years</td>
<td>Malay</td>
</tr>
<tr>
<td>Mr. A.</td>
<td>Male</td>
<td>9 years</td>
<td>Malay</td>
</tr>
<tr>
<td>Ms. M.</td>
<td>Female</td>
<td>3 years</td>
<td>Chinese</td>
</tr>
<tr>
<td>Ms. B.</td>
<td>Female</td>
<td>21 years</td>
<td>Chinese</td>
</tr>
<tr>
<td>Mr. S.</td>
<td>Male</td>
<td>14 years</td>
<td>Malay</td>
</tr>
<tr>
<td>Ms. T.</td>
<td>Female</td>
<td>6 years</td>
<td>Chinese</td>
</tr>
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</table>

The main episode analysed in this article was selected for discussion for a number of reasons. First, it runs approximately sixty minutes, which we have found is about the average duration for teacher discussions. Second, the episode represents a complete, well-bounded post-lesson discussion session, with a beginning, middle and end and related to the research lesson.
Third, it incorporates a range of important pedagogical hypotheses when analysing students’ work and considerations related to teaching a diverse range of students. It is also a relatively routine segment; like all the discussion sessions in our corpus, this session was not prepared specifically for us. While the episode is dense with interesting interactions and issues, it generally reflects the post-lesson discussion activity and discourse that we have witnessed in other discussion sessions focusing on analysing student work at GES. We make no claims to its representativeness, only that it is not in any way a rare sample of the practices we have encountered.

This focal discussion episode occurred on 23 February 2016, in our second of six observations of Ms. T and her team in discussion for one research lesson. Their discussions were usually facilitated by the Mathematics Subject Head, Ms. P, and a university researcher, Dr C. The teachers sat in a circle to discuss the selected students’ work, which was projected to the screen in the front. Their discussions were mainly about understanding students’ typical errors and underlying misconceptions in learning about the concept of fractions. The errors were selected to show the common struggles that students encountered when learning key mathematical concepts. Such discussions on students’ work and selected typical errors went throughout the lesson study cycle from lesson planning, observing the research lesson, and post-lesson discussions.

The episode was fully transcribed before analysis, and we engaged in some analytic steps, including:

a) Close reading and micro-analysis of the transcript, in which we slowly moved through the transcribed episode, moment-by-moment and line-by-line, asking at each instant what was happening and why, and what possibilities it opened up for participants.
b) We compiled a running list of the important issues the participants raised for us in the episode, among them the evidence of attempts to elicit student thinking, understand student errors, the various ways the teacher clears the students’ misconceptions, follow-up instructions, teachers’ roles, and the perceptions of student learning. We selected from among these issues three themes for deeper exploration. Our selection was guided by three considerations: we sought moments that resonated with the rest of our data set, that involved the hypotheses and thinking that teachers generated, and that appeared to afford opportunities for teachers to adapt their prior knowledge and conceptions.

c) Focusing on each theme, we returned to a micro-analytic investigation of the transcribed episode asking at key points: What were teachers’ prior knowledge and conceptions here? What are the considerations in favour or against each thought? Why did the participant select to discuss or follow up certain hypothesis? How did they respond?

d) We discussed the episode, triangulated with relevant data we had with other discussion sessions and earlier interviews, and exchanged interpretations with a number of the participants in the individual follow-up interviews.

Finally, we should note that while our interpretations were informed by the literature on lesson study, adaptive expertise, and practice-based teacher learning, we have also attempted to approach the data with an open mind, disciplining our gaze to generate additional understanding and test our ideas against the available data.

Findings

We found that the discussions revolving around the analysis of the students’ work directed the focal topics for lesson study and instructional improvement in GES. The teachers adapted the standardized test items and developed a worksheet for students to complete before
the lesson study cycle. After marking the students’ worksheets, teachers discussed and analysed the pattern of the mistakes students had made, and identified “typical errors” that signal the common conceptual misconceptions of the students. Some typical errors seemed to be surprising for teachers to unpack students’ understanding, as what is represented in the discussion episode below. The discussion went on to hypothesize possible causes for the students’ errors and strategies to correct students’ misconceptions. Teachers also discussed the design of a follow-up research lesson to address the identified typical errors. Worksheets with similar items were then administered in class to test if students’ misconceptions had been cleared up, which elicited the focus of the post-lesson discussion to decide how to follow-up with future teaching.

The findings of this study show that the teachers benefitted from the professional discussion of the information collected about student learning. Firstly, they began to focus on students’ thinking and posited various suggested ways to reveal the gaps between the student thinking and the teacher thinking, rather than focusing on teaching procedures that seemed best to each teacher. Secondly, some of the teachers realized that their understanding of the curriculum content was limited. In particular, they noticed that small changes in teaching could help or hinder students’ understanding of the mathematical concepts. Thirdly, the availability of a “knowledgeable other” facilitated a more in-depth discussion of students’ work to improve teacher learning. Last but not least, lesson study provided opportunities for teachers to articulate and share their thinking which expand their repertoire of expertise for adaptation in their practices.

**Focusing on students’ thinking**

In the beginning of the semester, teachers attributed students’ mistakes to their lack of language proficiency. However, as the discussions progressed over the next few sessions,
teachers questioned their assumption that students (especially lower-progress students) had limited reading comprehension skills, and began to examine closely the wording of the mathematics problem itself. They then became more aware of teachers’ use of language, and how these factors may impact students’ conceptual understanding.

The first turning point occurred when teachers brainstormed what they considered “key attributes of an effective problem-solver.” Some teachers said that the students lacked logical thinking. The students “may not apply thinking skills and strategies to solve math questions” (Ms. B, 16 February 2016). Some mentioned that the students were not exposed to word problems yet. Most teachers suggested students had poor language skills to “comprehend a problem and manipulate data,” and they seemed to speak in a “market language” (Ms. F, 16 February 2016). However, one teacher, Ms. C, questioned why students from China, with English as their second language, could solve the math problems in her class. The issue was then re-examined, and somehow the assumption about language proficiency changed once the teachers closely examined the typical mistakes in the students’ classroom assessment tasks.

Teachers’ awareness of students’ thinking was heightened during the lesson study cycle when the teachers met to analyse students’ typical errors to develop a lesson to refine students’ understanding of fractions, as represented in the selected episode on 23 February 2016. The teachers examined the following problem during the discussion session:

Yilang had a sum of money. He spends 3/10 of his money on books, 2/5 of his money on stationery and the remaining amount of $36 on a T-shirt.

A. What fraction of Yilang’s money was spent on the books and stationery?
B. Find the sum of money Yilang had at first.

The teachers then selected two typical errors students made for solving this math problem to discuss and tried to identify students’ misconceptions with fraction. As shown below, one
example of a typical student error was projected onto the screen for discussion (the original picture included).

Students’ work reproduced:

\[
\text{A)} \quad \frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10} \\
\text{B)} \quad \text{\underline{\text{\$36}}}
\]

1 unit \quad \$36 \\
4 units \quad \$36 \times 4 = \$144 \\
3 units \quad \$36 \times 3 = \$108 \\
Total \quad \$144 + \$108 = \$252 \\
\$252 + \$36 = \$288 \\
Answer: \text{A)} \quad \frac{7}{10} \; ; \; \text{B)} \quad \$288

The teachers quickly found that this student did the section (A) correctly but failed to solve the second section. The “remaining amount” was misunderstood by the student as one unit of a whole equally divided by eight, equal to 36. Below is an excerpt from the teachers’ discussion:

Ms. P: Part A is actually intentional, to scaffold the addition of these two to get uh, seven-tenths, and for the child to see that in one whole there is ten out of ten, and if this is the amount that is spent on the books and stationary, then the remaining would be… three out of ten. Three units. And we need them to equate three units out of ten to be thirty-six. …Yeah. So the children are not able to see that the total sum of money is represented by ten. Ten units… and there is an assumption that uh, what’s left is one part. (long pause). So this is one example…They thought thirty-six dollars was one unit. … Ms. M, is that your group?

Ms. M: My- my group.
Ms. T: Ok. Did the children express why they had- why they equated one unit with 36 dollars?
Ms. M: He’s not able… He got stuck on this question for very long. So in the end… He just wanted an answer. So he just- he just times according to the- he times seven is because of A, so he thinks that it’s actually connected as well. Seven-ten. So the understanding is not there when I ask him why did you times, what is it that you’re doing. So he- because he can’t link the three-ten to actually the remainder. So he actually just takes [took] one. He takes [took] one t-shirt to assume that it’s one unit. … So I have a few that did this method [branching], then after doing the branching they left the branching there, but- but he still goes and takes three-tenth plus two-fifth… Instead of- so they are actually quite confused, partly is because they have been learning the branching method, and they forgot that- but they realize three-tenth of the money- and I- because I’ve been telling them, I think I’ve been drilling it into them.
Ms. P: So they are not able to apply the branching strategy...
Ms. F: Maybe you recently taught this, so they thought that it is related to your recent lesson on branching method.
Ms. T: I believe it is uh- yeah I thought that was the case as well
Ms. M: So I assume some of them, they do not even know what is branching- when you do branching, what is the meaning of the branching.
Ms. F: What kind of question need to use branching…
Ms. T: Maybe it’s the keywords, remaining confusing for them…consider revising the wording.
…
Ms. F: Yeah. And, ok, he- perhaps he- he understood about the three units, but he couldn’t see that the entire whole is… is ten units. Yeah.

(Transcript of Discussion, 23 February 2016)

These teachers who have had three or more years of teaching experience, have developed their understanding of the students and correlated such understanding with specific explanations and teaching tactics. Their spontaneous responses to the identified typical error seemed to be drawn from the repertoire of their established expertise of working with their students. But can they expand such repertoire and incorporate additional interpretations on student work and learning?

As shown in the excerpt above, the teachers recognized the language issue again: students’ understanding of the word “remaining.” However, , they also proposed a more nuanced understanding of students’ thinking with alternative hypotheses: teaching methods, the wording
of the problem, or students’ (mis)understanding of the mathematical concept. Through the
discussion on a student’s error that seemed typical of some students, they realized that the
student’s “understanding [of the concept] is not there.” It may be due to the misapplication of a
newly learned problem-solving strategy - “branching,” or it could be that the wording of the
problem was “confusing.” It may also be due to the students’ conformity to a certain way of
thinking by being drilled into the procedure without proper conceptual understanding. In the end,
they zoomed in to discuss how this typical error reflects students’ misunderstanding of not just
the word “remaining,” but what conceptually constitutes as a “whole” for a fraction. In this way,
the teachers seemed to move away from the vague assumptions about students’ low language
capacity, generated alternative hypotheses, and tried to get close to students’ thinking and
understanding.

After they collectively explored their initial assumptions about why students could not
understand the concept, they seemed to be more open to exploring alternative ways to get closer
to students’ thinking, including re-examining the content knowledge the teachers themselves had
understood and the approach through which they conveyed the content.

“Small things overlooked (by teachers), a big jump for them (students)”

By examining closely and discussing students’ work, the teachers began to use alternative
ways to understand students’ thinking behind the errors made in solving word problems. The
following excerpt shows that when discussing students’ typical errors with a mathematical
expert’s prompts, teachers were pushed to reflect upon their own thinking and understanding of
the taught content. It was only when teachers were clear about what they were teaching that they
could detect the students’ misconceptions and their thinking.
When teachers discussed another student’s answer (which also typifies the errors that many students had made) to the fraction problem above, they kept wondering why the student could not identify the parts of the whole:

Ms. F: Yeah, and ok he- perhaps he- he understood about the three units, but he couldn’t see that the entire whole is… is ten units. Yeah.
Ms. P: Ok, but because the- the system was given here.
Ms. C: No, but I don’t think he understands the three units. He got the three units; he took ten, the denominator is ten minus seven numerator. That’s how he got the three.
Ms. F: But if he…
Ms. C: It so happened- coincidentally-
Ms. F: But he understood there’s like… But it does, ok. It’s a bit confusing for me to listen to him as well. It is like he knew that there are three units left, but then he didn’t-understand that there are ten units in total.
Dr C: There are a few problems here. One is- you know in fractions right? Yeah… especially when it comes to fractions of a set. Now your whole is a collection of discrete items. And uh, you remember how they learned fractions in the beginning?
Ms. T: Fraction of the whole?
Dr C: Ah, a fraction of a whole. Now when you do a fraction of a whole, what is the important point that you were trying to drill into the students?
Ms. F: Equal parts.

Then Dr C drew these diagrams on the whiteboard (Figure 1.), comparing two squares of different sizes with half of each shaded. He asked if the shaded areas are equal and whether they can be represented by the same fraction. The teacher hesitated when giving their responses.

(Figure 1.)
Dr C: If this is the whole, what part of the whole is shaded? Half. No problem right?
(Group laughter)
Mr. S: No, very scared we get it wrong [crosstalk] (laughter)
Dr C: So if this is the whole, what part of the whole is shaded?
Ms. T: Half.
Ms. F: Still half what.
Dr C: So uh, as a student, what do you think is the first kind of confusion they have?
Ms. M: That one is one square, this one got four squares?
Dr C: Ah. Why? Why is the fraction still?
Ms. T: Half.
Dr C: Half. Now, when you explain to the student, fractions are part of the whole, now what do you think the student associate the fraction with?
Ms. M: [inaudible] Part of the related whole? (long pause)
Dr C: And? This is the part of the whole right? So this part is…?
Ms. F: Half.
Dr C: Half… So what’s wrong?
Ms. T: But they’re not the same-
Mr. S: The whole is different.
Dr C: Ah. Now you know that the whole is different right? … [D]oes it mean that these two halves are different? That means this number half, is it equal to half?
Ms. F: It’s about the- [crosstalk]
Dr C: It’s about the whole.
Ms. F: So that one, that one we need to explain to the kids, because it’s like you take a bar of chocolate, or you take a pizza, you have different sizes of pizza. … And you can’t compare it to the whole. Even the parts that you break them into, they’re not the same size [but the fractions are the same].

(The teachers hesitated to admit that these two halves equal to each other in fraction although they have different wholes. Then they were reminded to re-examine the definition of the fraction in the teachers’ guide provided by the Ministry of Education.)

Ms. F: In the guide, it is said that it is [a fraction is] a number, that shows the relationship between-
Dr C: - that is used to show the relationship between the part, and it’s whole.
Ms. C: That is used to?
Ms. F: Represent the relationship between a part and a whole.

(Transcript of Discussion, 23 February 2016)

The teachers examined this definition of the fraction as a number which expresses the “relationship between a part and a whole” together. They found that they may have over-simplified and expressed a different meaning of fraction to students. More specifically, they had been using fractions as “equal parts of a whole”, which may have led to many students not knowing fractions are numbers to indicate a relationship. They started to wonder about the importance of strengthening the concept of “whole” when clarifying students’ misconceptions about fractions manifested in the typical mistakes in students’ work. And they realized that this concept of “whole”, and the relationship between the parts and the whole were somehow missing in their first draft of the lesson plan. Ms. P commented, “All through this there are small little bits that we overlook and we assume……It’s a very huge gap that they have to jump over.” It seems that this reflective practice initiated by examining students’ work may have helped teachers to review their content mastery and teaching practices rather than just examining
students’ performances and stopping there. These two aspects are equally important for teaching, teacher’s thinking and student thinking; and the collective effort of bridging both ends may help teachers pinpoint what to start with to improve student learning and their teaching.

**Others’ perspectives: Seeing classroom teaching in a new light**

As shown in the excerpts shown, Dr C played an important role in eliciting the teachers’ thinking, providing alternative perspectives, and giving advice/resources for teachers during the discussion. Our findings suggest that teachers gained some insights into the teaching of mathematics from his comments. But he is not the only resource person for teachers to learn about the content, the students, and their teaching. They learned much from each other as well. Ms. P, as the Subject Head, sorted out the materials for discussion and supported the team members during the process. Mr. S and all the other colleagues also offered help when Ms. T had to figure out students’ common misconceptions to tackle within the research lesson: “So it’s actually very beneficial to hear from the other more experienced teachers – you know – what are the common pitfalls that the students might fall into, and how we can actually work around it to reach out to them.” Such insights also helped her to see her teaching in a new light and centre her focus on students’ understanding of the concept. As she said, this whole process of working with colleagues to discuss students’ work in preparation for teaching “…brings me ... to this very conscious – uh – perspective to make sure that the children understood what they’re doing, instead of just focusing on the algorithm.” (Interview with Ms. T., 19 April, 2016)

With alternative perspectives provided by others, this very consciousness about student learning later results in a deliberate “forward thinking,” as put by Ms. B, “to imagine the possible thinking, different ways of thinking that the children may have...it helps to prepare for the lesson
when you want to map student thinking with what the content requires them to think.” (Interview with Ms. B, 17 November 2016)

Hence, such a fruitful discussion seems to open windows for these professionals to examine their own teaching and learn from each other’s insights.

4.3. Learning affordances for adaptive expertise embedded in lesson study

Our findings revealed that lesson study has provided a useful platform for the teachers to examine their beliefs on student learning and their teaching. The teachers were asked about their perceptions of how lesson study helped them in this process of learning. Their responses can be grouped into four aspects: (1) eliciting hypotheses in dialogue; (2) creating space for alternative perspectives; (3) collaboratively scrutinizing student learning evidence for follow-up teaching; and (4) identifying problems for further discussion. These four aspects can work in sequence to form an inquiry process delineated in Figure 2 below, and they can also work together holistically to help teachers to reflect and refine their professional practices in the classroom.
Teachers tend to plan their teaching with “forward thinking,” in Teacher H’s words, to predict what students may think and what misconceptions they may have when encountering content. Such “forward thinking” is usually based on what teachers’ past experiences and what they believe to be useful or accurate. It needs to be tested in the practice of teaching. In the dialogue with colleagues about refining a research lesson, some teachers seemed to be able to eliciting their own understanding of students and teaching:

*Ms. T: Yah. So it’s through this lesson study, it brings me - uh - to this very conscious perspective to make sure that the children actually understood what they’re doing, instead of just focusing on the algorithm.*

This awareness of one’s own beliefs can be further enriched by the alternative perspectives provided by the colleagues during lesson study. No matter whether it is a resonance or dissonance approach, the teachers appreciated the various ideas provided by the colleagues.
However, the knowledgeable other seemed to play a significant role in convincing the participants to take an alternative perspective. Without a knowledgeable other to participate in the lesson study process, the participants seemed to be content with what they had presented as a good lesson. Ms. B, Head of the Department of mathematics, described how Dr. C, the knowledgeable other influenced their lesson study cycle:

So every time when we say something, he will add on and then we will learn more. And we can help us to think with him to improve on our lesson plan ... because he is the knowledgeable other. Maybe we feel that this is the way, but then he would give us suggestions and then point out you know uh you know better maybe we assumed that this was the correct way of teaching, but he may point out our mistakes you know so we can improve.

It may be because the assumed authoritative status of a university lecturer that the teachers felt obligated to conform to his suggestions. However, if the knowledgeable other provided a sound suggestion to re-examine the teachers’ understanding of the concept of “fraction,” the teachers seemed to benefit from an alternative perspective. As Mr. A stated:

In the past, I’d think that – ah - I’d just explain to the students that, “Ok, now, instead of looking at a fraction of a whole, now we’re looking at a fraction of a set, and we can have, umm, items in the group. That’s very much like teacher-led, a teacher telling the students, “This is the fact, take it.” Whereas, this year – because we have Dr. C. here explaining to us that it may not be so straight-forward as what the teachers perceive it. So I’d say that this year is actually a very enriching year for me to look at the different perspective on how we can approach the same topic.
To examine whether an alternative perspective works or not, teachers in GES situated their learning in a specific task of **collaboratively scrutinizing student learning evidence** by analysing the typical errors in students’ work. This has been shown in the vignette shared in the earlier section. The discussions seemed to be solid and the new perspectives convincing when situated in the evidence of student learning. Mr. S termed this evidence as “learning data”:

*Learning data from, what are the, what are the clear objectives of certain lessons you need to teach them, right. So how to use data to assess student understanding, you know that could be one thing. The other thing is sharing about the lessons. The other thing is sharing of ideas, how to improve a lesson, how to get feedback from teachers, uh, getting feedback from observers and all that…even from the knowledgeable others.*

The advice from others can only make sense in analysing the “data” of student learning. And the problems identified while examining evidences of students’ learning seemed to motivate teachers to further discuss and improve on their teaching.

*Ms. T: I guess we also need to know the class uh, learning profile, we have to know what works, to what wouldn’t work with the children, but sometimes we still need to test out. There, there are times where we are not sure, but we think maybe it would work, then we should always try it out, in our lesson. Then uh, after that, um, we will have to, as a, as a group, what we’d do is that every lesson we discuss lesson ideas for, um, of coming lessons. At the same time, we also give feedback on what works and what didn’t quite work based on student learning. So that will be useful as well, because what work, what might work and not work for me, might be otherwise for another class.*
This quote from the interview with Ms. T illustrates how teachers can learn from a collaborative analysis of student work and raise issues for planning future lessons. This process can be done on a daily basis and can be elaborated during the process of lesson study as shown in our data.

**Discussion/Implications**

This qualitative study builds on previous studies that show how teachers’ established expertise can guide teachers to notice the essential situation-specific details that demand instructional adaptations (Allen et al. 2013; Corno 2008). Further, it illustrates how open discussions on a particular core instructional practice—analysing students’ work—can guide teachers on becoming more aware of their routinized understanding and responses to student learning, and how to develop and enrich their repertoire of expertise.

For a productive teacher discussion of students’ errors to take place, it may be helpful for teachers to move away from a deficit view by assuming that students do not have sufficient language skills. Instead, teachers became more aware of what students were thinking, how they expressed their understanding, and how teachers could make teaching language “clearer” to students when presenting mathematical concepts. This finding resonates with what Bertrand and Marsh (2015) indicated as the importance of teachers’ sense-making of students’ work when they learned to closely examine student thinking rather than attributing student performance to the supposedly stable student characteristics. By doing so, our teacher participants benefitted from the professional discussions on students’ work analysis during lesson study, developed new knowledge about mathematical content, clarified their understanding of the concept of fraction, and became more willing to adopt various teaching strategies to help students acquire conceptual
understanding. All this learning happened in an active professional discussion among colleagues about students’ work facilitated by external and internal resource persons during a lesson study cycle.

Teachers have long been criticized for not being able to analyse students’ work for assessment (Campbell 2013). However, many prior studies ignore the real function of analysing students’ work in the classroom – providing formative feedback for student learning and instruction rather than crafting a perfect instrument to evaluate the learning and teaching in the classroom as if it is fixed. If this instance of students’ work analysis is not valid, or relevant, the follow-up adjustments can always help to correct it (Shepard 2000). Such adjustments, useful for a more accurate understanding of learning and instruction, probably can only be heartily adopted by teachers when they emerge from the process of collecting information with their own assessment approach, closely examining such information to understand student thinking and challenging each other’s thoughts in a professional discussion. Thus, this study positions the teachers as agents to not only devise and implement classroom practices, but also revise their understanding of students, content, and teaching to develop adaptive expertise.

In the meantime, it is important to note that the facilitator, Dr C, and Mrs. P., led more in-depth discussions linking the analysis of students’ work to student thinking and understanding of mathematical concepts. In the study conducted by Little et al. (2003), it was found that, although the teachers deemed it helpful to discuss students’ work, they were often uncertain about what to say about the students’ work given their own multiple interests and diverse backgrounds. Although we admit that the assumed authority by a university researcher may have catalyzed the adoption of new insights on student learning, this study may demonstrate one approach for
teachers to work with researchers and curriculum specialists to discuss selected students’ work for understanding student thinking.

Also, this research further develops the idea of core teaching practices (Ball, Ben-Peretz, and Cohen 2014; Grossman 2008) for teacher learning in the Singapore context, and shows that core practices (such as working with student-learning data) do not naturally help teachers learn. As discussed earlier, although routine practices help to build teaching expertise, they also post challenges for teachers to be aware of or critique their current understandings and teaching practices. Focusing on student learning evidence in a professional discussion provides a multitude of alternative perspectives that teachers can consider, based on which teachers could notice the effects and problems of their teaching. The teachers’ hypotheses can together form a framework for them to make such connections in the future, even if only one of them is more appropriate to interpret teaching and to learn in a particular situation. The discussion sessions supported with selected students’ work, and the presence of Dr C, pushed the teachers to examine their prior assumptions and practices and exchange ideas about what to do with them.

The study is limited in at least two aspects. First, although analysing student work is a major instructional activity for individual teachers on a daily basis, an open discussion on students’ work in a lesson study cycle only started as a new practice. As such, participant teachers might have been hesitant in articulating and sharing their views especially with an external expert present. Further follow-up observations of teacher discussions over time and in-depth interviews with teachers is needed to tease out their reasoning and judgments in context. Secondly, in interpreting the findings on how discussions about student-learning data can help teachers learn, the school culture needs to be taken into consideration. Gladstone Elementary School has a long tradition of collegiality to support teachers with time and structure, and a
conducive culture for teachers to discuss students’ work in depth. If such practices are transplanted into different school cultures, the teachers may have different patterns of learning with each other when discussing students’ work.

**Conclusion**

This study explores how teachers undertake professional discussions using information collected about student learning in the process of lesson study, and how such practice can help teachers understand student learning and their own teaching from alternative and novel viewpoints. It is found that teachers learn from the in-depth dialogue on the misconceptions behind students’ typical errors in solving problems. In this way, teachers do not confine themselves to the stereotypes of how (well) diverse students can learn mathematics. Rather, they try to understand how their students’ work manifests student thinking and addresses students’ misconceptions. Thus, the study provides one perspective for examining discussion on students’ work as a tool to (re)search students’ thinking and investigate how teachers make sense of students’ thinking develop their adaptive expertise. Further studies are needed for us to understand the kinds of classroom strategies teachers can use to collect useful and valid data for in-depth discussions, and the follow-up practices needed to connect awareness and actions in teaching.

**References:**


Author, 2015 [details removed for peer review].

Author, in press [details removed for peer review].


