Hands, Head and Heart (3H) framework for curriculum review: Emergence and nesting phenomena

Tan, Da Yang
Singapore University of Technology and Design
<dayang_tan@sutd.edu.sg>

Tay, Eng Guan
National Institute of Education
Nanyang Technological University
<engguan.tay@nie.edu.sg>

Teo, Kok Ming
National Institute of Education
Nanyang Technological University
kokming.teo@nie.edu.sg

Shutler, Paul M. E.
National Institute of Education
Nanyang Technological University
<paul.shutler@nie.edu.sg>

Abstract. In this work, we report the emergence of the Hands, Head and Heart framework that arose within the curriculum review for Subject Knowledge courses for primary school pre-service teachers in the National Institute of Education, Singapore. Through an initial grounded analysis of a survey of pre-service teachers and faculty focus group meeting data, the responses were broadly categorised into hands, head, and heart domains and these formed an initial framework for discussions in the review committee meetings. By revisiting the data from the survey, an analysis through a "complexity" lens revealed the emergence of a characteristic nested self-similarity of the framework. During the course of several committee meetings, further self-similarity was discovered. We conjecture that Hands, Head and Heart framework and its self-similarity property provide a potential basis for a holistic approach to curriculum review. We used this framework to revise the learning objectives of the Subject Knowledge curriculum by resolving perspectives which previously seemed contradictory.

1. Introduction

There are numerous curriculum design and evaluation models, such as the CIPP Model (Stufflebeam, 2003), Kirkpatrick's' Model (Kirkpatrick & Kirkpatrick, 2009), Scriven's Goal Free Model (Scriven, 1991), Stake's Model (Stake, 1967), and Tyler's Model (Tyler, 1949). In particular, Tyler's Model was adopted in our department curriculum review for the mathematics content in the undergraduate degree programme at the National Institute of Education in Singapore (Tay & Ho, 2015). It was chosen by the mathematicians in the department because of its compact framework of learning objectives, learning experiences and assessment. In general, current curriculum models provide a systematic top-down conceptual framework for the design and evaluation of the curriculum based on their pre-specified objectives. The aim of this paper is not to critically examine the existing models, but to report a fortuitous emergence of a curriculum framework that resulted from another of our curriculum reviews that began with Tyler's model. We will compare Tyler's model with the new framework when it is more appropriate, such as when discussing its reductionist nature in the next section and its linear approach in the final discussion.

One may readily accept that curriculum design can be considered a highly interacting system involving many components, including both human and non-human elements. Nowotny (2013) adds a further insight when she explains that "the categories we normally use to neatly separate issues or problems fall far short of corresponding to the real world, with all its non-linear dynamical inter-linkages". While neat categorization and coding has served the purpose of providing a systematic lens to study various phenomenon in education, the complex interactions tend to be under-examined. Instead, given that many curriculum models adopt a top-down approach, complexity thinking provides an alternative bottom-up perspective which throws light on the non-linear interactions among components.

Complexity science hence represents a change in the paradigm in research, whereby instead of reductionism and breaking the system into smaller components, one now focuses on the whole system, the interactions among components and the emerging properties that arise as a result of these inter-component feedbacks and interactions. As noted by Davis and Sumara (2006), such interactions give rise to interesting phenomena such as self-organisation, nested structure, and bottom-up emergence. There is no commonly agreed upon definition of what constitutes "complexity", though various authors have provided perspectives to the description of complexity: Cochran-Smith, Ell, Ludlow, Grudnoff and Aitken (2014) summarise complexity theory as "a loose collection of scholarly work that takes up important questions about systems and how systems change, develop, learn, and evolve [which] emphasizes wholes, relationships, open systems, and environment" (p. 5), while Morrison (2008) explains it as "a theory of change, evolution, adaptation and development for change" (p. 19). Koopmans and Stamovlasis (2016) understand it as "a holistic view where the behaviour of the individuals is understood in its larger context" (p. 1). Castellani and Hafferty (2009) argue that in the social science context, complexity theory is "more a conceptual framework than a traditional theory"; it is closer to a scientific framework, i.e. it "provides researchers effective ways to organize the world, … a logical structure to arrange their topic of study, … [and] scaffolds to assemble the model they construct" (p. 34). However, what can be understood collectively from these arguments is that complexity provides a lens that transcends the usual means of dissection of data and focuses on their *interactions* for us to organise and acquire the perspicacity of the underlying order within the data. The use of the complexity lens in our development of our curriculum framework will be elaborated in the remainder of this paper.

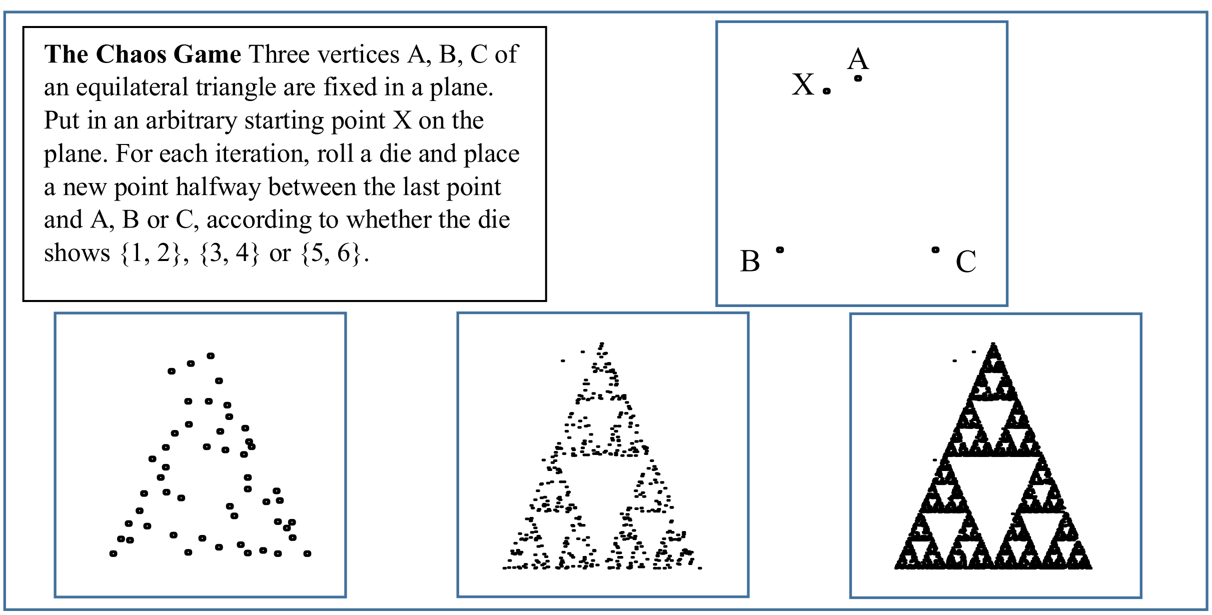

Figure 1: Four phases of the Chaos Game corresponding to 1, 50, 500, and 5000 points

Complexity science, foremost, has its origin in physical sciences, such as fractal geometry in mathematics and non-linear dynamics in physics. It overcomes some problems engendered by a typical reductionist and linear conceptualization by focusing on the emergence of order from the non-linear and recursive interactions of the components. Such emerging orders are common in many mathematical and physical systems, notably in Sierpinski triangles (see Figure 1), Conway's Game of Life, the normal modes of a double pendulum, etc. The key essence is that while the individual generating points appear to be random, when viewed in the appropriate frame and scale, the stable regularities will emerge.

Complexity theory is "transdisciplinary" (Davis & Sumara, 2006) and has overarching contributions in diverse disciplines such as healthcare (Brainard & Hunter, 2015; Braithwaite, Churruca, Long, Ellis, & Herkes, 2018; Kuziemsky, 2016), management (Perona & Miragliotta, 2004; Stacey, 2002), and social and natural networks (Ahn, Bagrow, & Lehmann, 2010; Barabási, 2016; Kitsak, et al., 2010; Wang, González, Hidalgo, & Barabási, 2009). In these works, the behaviour of these systems, that are viewed as complex, are studied. Similarly, in education research, there has also been a recent movement to view education and learning as a complex system (Davis & Dennis, 2006; Koopmans & Stamovlasis, 2016). Various works have explored the behaviour of such complex systems: For example , classroom dynamics (Burns & Knox, 2011; Larsen-Freeman, 2016), learning processes in classrooms (Steenbeek, Vondel, & Geert, 2017), and demographic educational modelling (Guevara, 2014). Given its extensive scope, complexity thinking should be an "important and appropriate attitude for educators and educational researchers" (Davis & Sumara, 2006, p. xi).

By examining the issues through the understanding of the interactions between components, one could look out for any potential features that similarly exist in other complex systems, such as nesting and attractors (which are particularly relevant to our work in this paper). Such interesting features have been found in the exploration of many other social systems (Albert & Barabási, 2002; Boccaletti, Latora, Moreno, Chavez, & Hwang, 2006; Song, Havlin, & Makse, 2005; Sugiarto, et al., 2017), hence these features are more common than one would anticipate. Therefore, it would be a worthwhile effort to seek and identify these emergence properties.

In this paper, we will demonstrate that when viewing through a complexity lens, based on an initial grounded analysis of our data obtained from a survey of pre-service teachers, focus group discussions for faculty and review committee meetings, we are able to observe the emergence of the Hands, Head and Heart domains. By treating these domains as the attractors, the nesting phenomenon is observed to emerge from our data. We then argue that such stable nesting properties serve as an ideal framework to drive the curriculum design.

We first provide the context in Section 2 with respect to the background of this work and the approach involved. We give a preliminary summary of the process that led to an initial framework, the emergence of a fractal framework through the use of a complexity lens, and the use of the framework in the ongoing curriculum review. In Section 3, we detail how the categorization of the three domains was obtained from the data. The preliminary framework suggested that the interactions among the domains needed to be further investigated. Thus, in Section 4, we revisited the data from the survey to understand these interactions. Through the use of a complexity lens again, we were able to see a self-similarity structure emerging within the framework. We provide an example on how the derived framework was utilized to drive the curriculum review process in Section 5. Finally, we give a summary and provide an outlook for future work in Section 6.

2. Background and Methodology, Initial and Emergent

As reported previously by Tay and Ho (2015), a curriculum review by the Mathematics and Mathematics Education Academic Group of the National Institute of Education was targeted to build up the mathematical capabilities of the undergraduates. In November 2017, the academic group began the curriculum review for Subject Knowledge (SK) courses. The SK courses are different from normal undergraduate mathematics courses and are a set of courses designed to help the primary school pre-service teachers (PTs) enhance their mathematical knowledge in preparation for the primary school mathematics syllabus. As it is not a requirement for the PTs to have a degree in mathematics in order to teach mathematics at primary level, the mathematical capability of the PTs varied widely; in extreme cases, the PTs may not have any further mathematics training beyond O-Level Elementary Mathematics. The SK courses are originally designed to complement the mathematics curriculum studies courses. These curriculum studies courses focused on the mathematics teaching pedagogies and assumed prior knowledge of relevant mathematics content of the PTs. This assumption turned out to be frequently untrue and so SK courses were developed to ensure that the PTs have adequate mathematical knowledge for the curriculum studies courses. PTs who are assigned to teach primary mathematics are required to take at least two out of the three SK courses: Number Topics, Geometry Topics, and Further Mathematics Topics (which comprise topics such as algebraic problem solving and some elementary statistics), depending on the programme they are enrolled in. Number Topics includes numeration systems, number operations, divisibility, and ratios, proportions and rates. Geometry Topics includes parallels and polygons, congruence, triangles and quadrilaterals, length, area and volume, and symmetry and tessellations.

The curriculum review began with the goal of studying the views of PTs, tutors, and faculty members about the SK courses to define appropriate Learning Objectives as per Tyler's Model. The committee formed for this purpose decided on the following methodology involving a survey, interviews for the tutors, focus group discussions for the whole department, and a number of meetings for the committee. The survey for the PTs was designed to understand their perceptions of the course with respect to the learning objectives, course content, materials, assessment, and the pedagogy. Taking the philosophy of a grounded approach, during the initial stage, the survey data was analysed with minimal expectations on the possible patterns that could emerge from the data. Open coding was adopted in the analysis of the survey data, and this was fed into the design of interview questions for the tutors, since "data collection and analysis are interrelated processes" (Strauss & Corbin, 1990). The collective results from the survey and interviews similarly guided the discussion during the faculty focus group meetings. We had expected the grounded approach to lead to obvious learning objectives within a few iterations.

However, upon interaction with the data, described later, the authors identified some non-trivial patterns, which were recursive and non-linear. The first set of recurring themes was identified during the analysis of the focus group minutes. Here, the open coding resulted in the grouping of the comments into three categories: teaching, mathematical thinking, and affect (further discussed in subsequent sections). To confirm the categorization, the tutor interview data and the survey data underwent a series of iterative open and axial coding until saturation.

By then, the first author, who was familiar with non-linear physical systems and its associated behaviours, such as self-organisation, recognised the valuable insights that the study of interactions could bring about. Similarly, the author was aware of the ongoing discourse on the applications of such ideas to the social sciences, including education. Concurrently, a more detailed analysis of the PT survey data pointed to a potential nesting of the recurring themes, which was, once again, a feature of a complex system. As such, we drew inspiration from the ground-up perspective in complexity sciences: instead of analysis and characterization of the common themes, an approach to study the *interactions* between each category was adopted, and new insights were synthesized, generated, and theorized. At this juncture, we contrast this approach to a reductionist one, such as in Tyler's model. Tyler's model specifically states that "we [should] turn to each of the sources in turn to consider briefly what kinds of information can be obtained from the source and how this information may suggest significant educational objectives" (Tyler, 1949, p.5). Nonetheless, the deliberations towards and the end-product of the learning objectives reduce completely to a student-centred perspective, namely the behavioural and content aspects of the objectives to be attained by the student (ibid. p.46-50). Such a reductionist approach seeks to neatly partition components and in the case of Tyler's model, separates the teacher's concerns as a *subsidiary* factor towards the *main* component of student learning. Instead, a complexity science perspective focuses on the collective behaviour and seeks to look out for any emergent behaviour that might arise from *interactions*. Thus, using a complexity science approach that values interactions between components, we would be compelled to look back at the teacher's concerns even after an initial formulation of the learning objectives for the student to see if these learning objectives would impact the teacher in return.

In this curriculum review, the symbiosis of the initial grounded approach and the emergence of order from the complexity lens allowed us to gain a unique perspective of the data through filtering data "noise" by imposing order on the data. Rather than an empirical theorizing based only on data points, as demanded from grounded approach, we performed additional analysis based on emerging theory, validated its stability and used that structure as a lens to look back and further analyse the collected data. In this sense, the data interacted non-linearly in order to derive the 3H framework (Hands, Head, Heart), which is the key contribution of this work. In other words, analogous to Sierpinski's triangle, with the overall nested structure, one would anticipate that any new data points would lie within the outline of the structure, even though the exact point where the data would land may not be fully deterministic. Such a stance is supported by Davis and Sumara (2006, p.34): "In a complex network, no part of the system has any meaning in isolation from the rest of the system …, and so one must take into account the structure of the whole system. In other words, … complexity is incompressible and ever expanding." Such symbiosis will be further highlighted in the remainder of this article.

To avoid confusion about how our framework emerged, we state here a preliminary summary as follows. Through an initial grounded analysis of a survey of pre-service teachers and faculty focus group meeting data, the responses were broadly categorised into hands, head, and heart domains and these formed an *initial* framework for discussions in the review committee meetings. By revisiting the data from the survey, an analysis through a "complexity" lens revealed the emergence of a characteristic nested self-similarity of the framework. During the course of several committee meetings, further self-similarity was discovered. We used this *emergent* framework to revise the learning objectives of the Subject Knowledge curriculum by resolving perspectives which previously seemed contradictory.

3. Emerging Theory from the Focus Group Discussions

27 faculty members of MME gathered in focus group discussions to discuss the SK courses. The two main items on the agenda were: (i) the learning objectives of the SK courses and (ii) the content that should be covered within the SK courses. The faculty broke into four focus groups. Three focus groups were tasked to discuss one each of the three SK courses, while the last group focused on the general learning objectives and whether problem solving, logic, and coding should be part of the content. Each focus group minutes was collected as data for analysis through iterative review as described in the following.

3.1. Preliminary Observations and Derivation of Theory

Fourteen distinctive responses from the four focus group discussions were recorded for the question "What should the learning objectives of SK be?" The following codes derived from a grounded theory approach are presented in Table 1. The codes were obtained by consensus from the six members of the curriculum review committee. Two of the members considered themselves hybrid mathematician-mathematician educators who had taught and researched in both content mathematics and mathematics education. Two of the members were pure mathematicians who taught undergraduate mathematics and were the mainstays in the SK programme. The sole mathematics educator was an expert in classroom research. The final member was a physicist by training and was the research associate for the curriculum review project.

Code #1: Teaching Driven
• Teachers should develop deeper conceptual understanding to avoid making errors in their own teaching • Sufficient knowledge is necessary so that they can, for example, critically evaluate the accuracy of mathematics questions
Code #2: Mathematical-Thinking Driven
• Application of ideas • Making connections • Develop mathematical thinking and logical thinking, e.g. geometrical proofs • Understanding of the different aspects of mathematical work • Develop the disciplinary thinking of a mathematician
Code #3: Affect Driven:

- Appreciation of mathematical knowledge
- Positive attitude towards mathematics as a discipline
- Appreciate the usefulness of mathematics
- Appreciate the beauty of mathematics
- Appreciate the power of mathematics
- Develop positive habits related to mathematics
- Build up confidence

Table 1: Coded responses for learning objectives

The codes are motivated in the following manner: #1 considers responses related to how SK can enhance a PT's teaching ability; #2 corresponds to a PT's identity as a mathematically trained person; #3 relates to desired emotions of PTs.

3.2. Hands, Head, and Heart (3H) Lens

From the preliminary analysis, we named the individual domains: Hands – Teaching Driven, Head – Mathematically Thinking Driven, and Heart – Affect Driven.[1] This was articulated during the first committee meeting after the focus group minutes were analysed.

In Transcript 1 below, the committee members, M1 and M2, were discussing the focus group data implications on learning objectives. In all our transcripts, square brackets [] are used to fill in words to enhance the clarity of the sentences, while parentheses () are used to denote our additional comments:

M1: Yeah, because the first step is: do the students like mathematics? It's like their feelings about the subject. Do they have got a positive attitude? Do they think it's interesting? That kind of thing, right? Because that's a very stable objective, and that's a firm basis on which to proceed. Now the second step, this thing about deeper conceptual understanding. What you call is the mind, you see. Do they really understand what's going on? Can they discern what's important, what's not important, you know that kind of thing. Okay so that's the mind thing right? And then the third bit is kind of a skill type [of] thing. It's like doing. Yeah so you like math, you understand it, but can you actually write the proof properly, you know, without making mistakes and structure it … Outline (referring to the codes) is a very stable, well-understood three component approach to a sort of a human system. So you got the how do people

[1] The 3H (Hands, Head and Heart) nomenclature appears independently in other writings. Easton (1997), describes Waldorf education as one that engages the whole child in the learning process. "Head, heart and hands" is used to emphasise the development of the whole child. Sipos, Battisti and Grimm (2007), advocate for the advancement of head, hands and heart as an organising principle for transformative sustainability learning. Here, head, hands and heart refer to the cognitive, psychomotor, and affective domains of the learner, respectively. The focus of both these 3Hs are the learner whereas our framework allows for 3H at multiple levels, including the PT, the PT's future students, the teacher educator, and more.

feel about it emotionally, how do people think about it intellectually, and can people actually do it and write stuff and, yeah interact with it with the outside world, effectively.

M2: Heart, mind and hand.

M1: Yeah, basically … Yeah, So I think those 3 points would be absolutely perfect as learning objectives. Very stable, very clear. And interestingly the content, what we currently testing in the exams - I think we agree the third one is probably not the most important. It's the first 2.

M2: Equally.

M1: Yeah, okay.

Transcript 1: 3H emerges

Transcript 1 represents the conception of the 3H nomenclature (with the Head domain initially called "mind"). We note also that by referring to codes as steps, M1 seems to acknowledge some connection between the codes.

To facilitate the discussion in the following sections, we have to bring forward here our definitions of the 3H which are put together from the ideas derived from the data that relate to each domain. The first sentence for each 'definition' is general. The following statements give examples in relation to a PT.

Hands: This is associated with the ability, competence and mastery to perform specific tasks. From a PT's perspective in an SK course, this is related to the idea of being able to literally write down the solutions to the mathematical problems in the course, for example, proving the congruency of two triangles (even though it is not taught in primary schools). As a result of our review, we have realised that Hands for the PT also includes the ability to solve the mathematical problems of their future students, for example, challenging problems in the high stakes national primary school examinations. Hands can be extended to include any activity that the PT would directly use to teach future primary school students. However, much of this extended use of the Hands is taught in the curriculum studies courses, for example, questioning techniques, use of manipulatives, use of technology, explaining of concepts. (Later, through the 3H lens, one can see that the PTs were not able to reconcile the Hands of 'doing mathematics' in the SK courses and the Hands of their practice as a teacher and this led to some tension between the PT's learning objectives and the lecturer's learning objectives.)

Head: This refers to acquisition of knowledge to think mathematically, logically, and to understand the inner workings of the underlying concepts. For the PT, whereas Hands is related to the acquisition of algorithmic skills in an SK course and pedagogical skills for functioning as a teacher in a curriculum studies course, Head suggests their ability to explain mathematical and problem-solving techniques. Using long division as illustration, Hands would correspond to a PT's ability to solve a long division problem and to demonstrate the algorithm, while Head would refer to the understanding of the underlying reasons why the long division algorithm works.

Heart: This domain relates to the confidence, disposition, affect, and the emotive feelings evoked when tackling the subject. For example, in the case of PTs, it refers to their appreciation of the beauty, history and usefulness (and possible misuse) of the mathematics they would learn.

We stress that the definitions have been distilled and refined from the many discussions among the committee members, which we omit for brevity. However, we maintain that these definitions are grounded in the data collected. As will be shown later, the 3H definitions are further validated in the analysis of the PT survey, and more interestingly, a nesting emerged from their responses.

3.3.Concerns in the SK courses – Analysis from the 3H Framework

The initial 3H lens provided an extra tool to analyse data from the focus group minutes. We will demonstrate this through a discussion on concerns in the SK courses. Similar analysis has been performed for other topics, such as on teaching of the SK courses.

The analysis of the focus group minutes led to the identification of the following concerns:

I. PTs look at the courses from a needs/pragmatic basis
II. Objectives must be clear before tweaking the content and delivery
III. PT understanding of mathematics is usually fragmented, i.e., topics are learnt in silos, resulting in the inability to explain concepts, make connections between topics, or appreciate mathematics as a whole
IV. Some PTs lack confidence and have anxiety about mathematics

I and II deal with the perception of the relevance of the course to the PT, and therefore can be considered as part of Heart according to the 3H framework. In this case, the PT cannot appreciate the reasons for taking SK which involve content that may not be directly related to the content that the PT would teach in primary mathematics. This forms the basis for the perceived lack of need or practicality in the SK courses. This problem is then compounded by the imprecise learning objectives, thus reducing the PT's Heart to appreciate the course. III exposes the causal links between Head, and Hands and Heart, where a fragmented understanding of mathematics (Head) leads to the inability to explain concepts or to make connections among topics (Hands), or to appreciate mathematics as a whole (Heart). One can then infer further that the inadequacy of Head and Hands then leads to IV (Heart). One can continue to argue that the shortcomings in Heart would therefore fuel the deficit in Head and Hands, resulting in a cyclic process that impedes the PTs learning.

As we have briefly demonstrated, viewing the data alongside their interactions through the 3H lens allowed us to understand the dynamics of these concerns. This led us to our first framework for the curriculum review as illustrated in Figure 2. The framework includes not only the domains of Hands, Head, and Heart, but alludes also to the interaction among these domains.

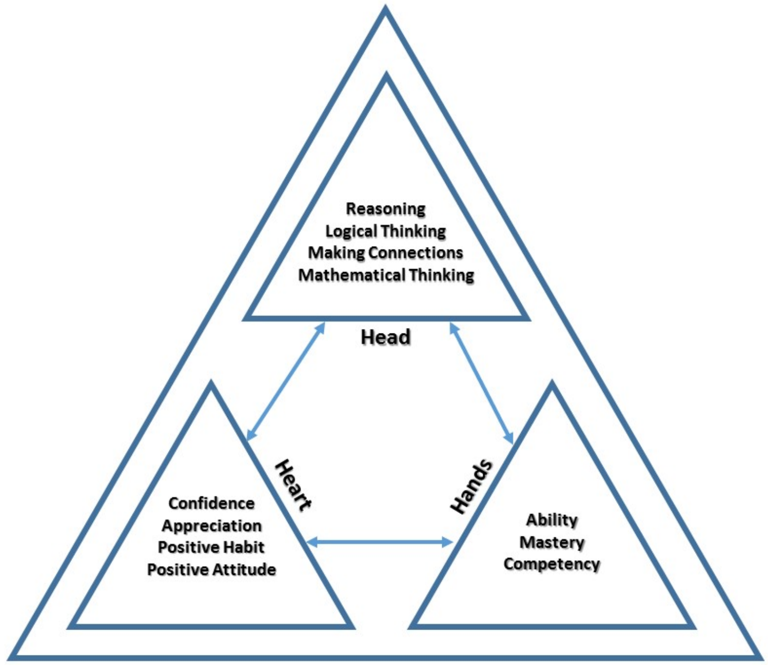

Figure 2: Preliminary structure of the Hands, Head, and Heart (3H) framework

The identified concerns were cross-referenced with the survey and interviews for further verification and validation, hence rendering these key points credible. Commonly, one would attempt to directly tackle the concerns by providing a direct solution to each concern. However, the 3H framework cautioned against rushing into a linear causation paradigm. Instead, we were led back to acknowledge the inherent complexity of an educational landscape as articulated by Jörg, Davis and Nickmans (2007, p. 145):

"Present paradigms in the field of education are based on physicalism or linearity thinking, they neglect the inherent complexity of educational reality and therefore are not able to develop an in-depth understanding of this reality."

Indeed, to consider the curriculum review in a holistic manner, one must acknowledge that components within the curriculum setting would interact with each other in a highly non-linear and recursive manner.

4. Development and Iterations of the Theory – Revisiting the Survey Results

Prior to the focus group discussions, PTs who took SK courses in the preceding semester were invited to participate in a survey consisting of open-ended and Likert scale questions (see Appendix A). Of the 69 who responded, 36 were PTs from the Postgraduate Diploma in Education (PGDE) programme, while 33 were PTs in the 4th year of their undergraduate programme. The PGDE PTs had taken Number Topics and Geometry Topics and the undergraduates had taken Further Mathematics Topics. The following section articulates the revisiting of the survey data after the focus group discussions, to see if the responses by the PTs could be reframed through the 3H lens, and if one could derive any systematic relational behaviour between the responses.

For the subsequent discussion, we shall focus on responses to Item 2 in the survey: "I think it is necessary to have a deeper understanding of the mathematical concepts (beyond the level covered within Curriculum Studies courses) in Primary Mathematics." This item seems, to us, to address the outstanding dread of the course, i.e., the unhappiness felt by the apparent disjunction between the SK content and primary school mathematics.

We assumed that the PTs had been made aware that SK was content-based and meant to build understanding of mathematics related to the teaching of primary school mathematics. We thus interpreted the responses as their desired learning outcomes from SK. Through the 3H lens, we gained a more holistic insight into the PTs' responses (see Appendix B). As a first step, we categorised the responses into their respective 3H categorisations.

4.1. PTs' Perception of SK

The majority (17 out of the 29 meaningful responses) of the PTs who answered gave reasons that can be associated with-their ability to be better teachers and equipped with better pedagogy. Within this group, responses such as the ability to deliver and simplify explanations, to gain longitudinal coherence, and to work with high ability students, are some mathematical tasks that are distinctively associated with teaching, and hence may be associated with the Hands domain.

The understanding of the difficulties of the topics, the ability to make connections and the conceptual view of understanding formulas or methods could be characterised as Head. In this case, the PTs perceived that the acquisition of the knowledge from SK could allow them to synthesize and expand their knowledge base.

Responses, such as enabling PTs to feel prepared for the teaching practicum (a high stakes programme assessment) and gaining general appreciation of the mathematics, relate to Heart.

4.2. Emergence of Self-similarity

Where PT responses indicate a lack of appreciation for the necessity of SK, one may attribute it to the perception of the lack of relevance to their future teaching. It seems that the Heart is affecting the Hands. One also sees a nested Heart within the Head domain when the PTs indicated that having a deeper understanding would enable them to gain affective sensitivity to their future students. Through the 3H lens, we see the emergence of nesting. The elaboration of the PT responses regarding their future students can be interpreted as the nested structure within each of the domains, i.e. nested self-similar structures pertaining to their future primary school students. This gives rise to a more detailed 3H framework as shown in Figure 3.

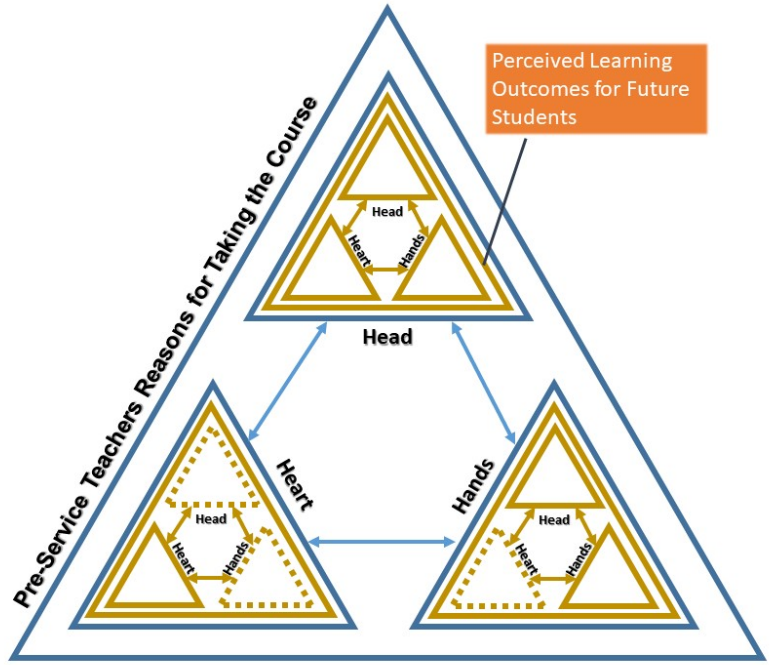

Figure 3: Self-similarity of 3H with respect to PT perception for taking the course. Solid triangles denote nested domains that are supported by the survey data and dotted triangles denote the absence of supporting data.

To validate this new framework, we identify responses to Item 2 that mention the PT's future students and then study if the statements can satisfy (i) the first layer of 3H with respect to the PT's learning outcome and (ii) the nested layer pertaining to the 3H of their future students. 16 of the responses mentione both themselves and the students and thus satisfies both conditions (i) and (ii). Of the three domains, Head displays the most prominent nesting. The phrase "deeper understanding" naturally prod PT responses towards the Head. An example of such a response is "Before teachers teach about the basic concept of mathematics, they should understand what is beyond it [in order to] help their students better when they break it down to chunks for them". Nonetheless, we do have responses that indicate other possible nesting within Head. Examples are (a) that deeper understanding would help the PT understand why students would "struggle" – Heart is nested in Head; and (b) that it is useful to have deeper understanding so as to help the students in "heuristics and problem solving" – Hands are nested in Head.

Similar nesting for other domains is also partially identified from responses to the other items in the survey. Our findings are summarized pictorially in Figure 3 for brevity. The solid triangles indicate that there is data that are available to support the existence of the nested domain. We think that we have sufficient data supporting nested domains to give us a strong reason to hypothesise the existence of the self-similar fractal structure when it comes to understanding the learning of the PTs, in particular their motivation for taking the course, and how this can in turn affect the learning of their future students.

Interestingly, one can indeed further hypothesise the existence of another outer layer for the SK tutors. Certainly, their own Hands, Head and Heart are domains of consideration for each 3H domain at the PT level. Indeed, this level surfaced in one of the committee meetings (see Transcript

4). Thus, the layers correspond to the different stakeholders and their interactions would have interesting effects on the system.

5. Using the 3H Framework to Drive Curriculum Review

With the 3H domains and their nesting features derived, the stability of the overarching system means that one can use this framework to drive the curriculum review. This framework allows one to focus on the key concerns within the curriculum and to identify the domains (i.e., 3H) and their relationships among each other and at different nested levels. Observe that the framework does not seek to be prescriptive from the top down. Instead, the layers of the 3H framework allow for the curriculum developers to have insights even from the bottom-up.

The committee that was responsible for the curriculum review met several times to discuss the course of action for the review to move forward. Each meeting lasted for about an hour and the members would discuss plans to move the curriculum review forward, initially grounded on the data collected from the survey and focus group minutes. Meetings were audio-recorded and transcribed, thereby serving as artefacts for analysis in this section. We report in the following how the framework that emerged from the discussions in previous sections was harnessed to drive the curriculum review.

5.1. Uneven emphases in learning objectives

Figure 4 illustrates the SK learning objectives prior to the review, which are to build up knowledge, develop understanding, and improve confidence in the teaching of primary mathematics. While well intended, the objectives are generic and not sufficiently well-defined to guide the implementation of the curriculum. From the 3H perspective, the objectives lack emphasis on the Hands, since building up of knowledge and developing understanding would be Head, while improving confidence would be Heart. Furthermore, the model in Figure 4 is linear in nature, and hence unable to reflect the non-trivial recursive properties of learning in terms of 3H.

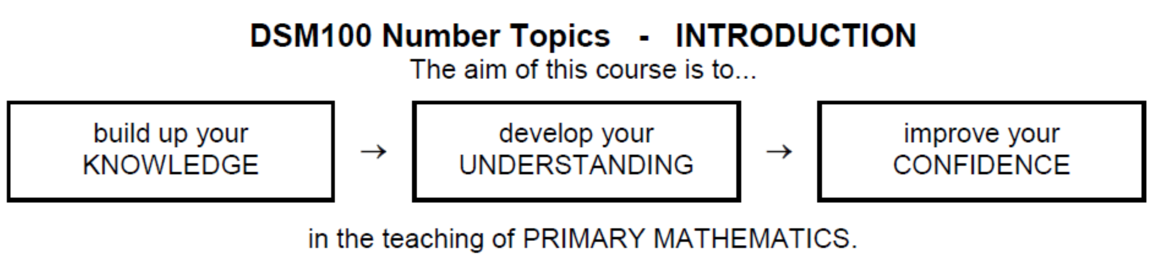

Figure 4: Learning objectives of the course before the curriculum review, as stated in the course notes.

Ironically, the committee observed that the prevailing mode of instruction and assessment placed an over-emphasis on only the Hands and Head, while neglecting the Heart Domain. M1 then used the 3H framework to comment on the prevailing state of the SK courses in two separate incidences.

M1: What seemed to be true is that the problem comes about being one of these three things (referring to 3H) being dominant, and the other two being too weak, and it is not always the same one, different ones for different things (i.e., content), so certain rebalancing of the learning objectives is what would kill the problem.

M1: Mostly what is going wrong with the courses are that one of the three [domains] gets over-emphasised, and it's not just always Hand, [for example] in congruency and similarity, we over-emphasised the why (here M1 refers to the Head), they end up not being able to do the Hand, they can't solve problems, they mess it up every time. I think Heart is particularly weak, [which was why] initially I would think that it might be a flow, where Heart is the problem we are trying to solve, we believe it is worthwhile because it is something we want. The Head is how you solve it, how you generate new math to solve this kind of problem. And then Hand is like correctness, like you can solve it correctly, and that feeds into the Heart, because you feel good, you got some skills, some competence.

Transcript 2: 3H drives the curriculum review

The second comment was made when considering the topic of congruency and similarity in geometry. M1 found that some PTs both could not translate the content to the solving of questions, nor were they able to gain confidence in handling mathematical tasks from this topic. Ironically, PTs have themselves learnt this topic while they are in secondary school and so, to some of them, this is secondary school material and is irrelevant to the primary syllabus.

The comments also show that the 3H framework was being perceived as cyclic and/or resonant in structure. This led naturally to discussions that acknowledges the non-linear interactions of the 3H domains. The cyclic process from Heart to Heart via Head and Hands alluded to in the second comment would correspond to a positive feedback loop within the 3H framework.

Transcript 3 shows how 3H was used in a discussion on topics to be removed from the prevailing syllabus:

M3: Now it seems that before we chop anything, we have to relook our learning objectives, see whether we have to do that (referring to the sections of content) or not. How the guiding framework helps us in our discussions, otherwise we would never get anywhere

M1: The flipside is that using this (referring to the framework) certain things must change. It's not like we are feeling unhappy about it but we feel powerful [because we are] not having a good reason to change. [With the framework], now we have good reason [to change]. If one of the things is being emphasized too much at the cost of the others…

M3: We have gone beyond this, because now we are trying to kill all these sacred cows, (he listed the topics), people say cannot. But based on the learning objectives, it becomes why not, they don't fit the learning objectives.

Transcript 3: 3H drives content reduction

In this exchange, one observes the use of the 3H framework as a justification to make difficult decisions, such as in choosing parts of the content to retain or remove.

5.2. Changing the learning objectives

The committee decided that the learning objectives of SK ought to encompass the elements in 3H. Each of these dimensions of learning was consciously given an equal emphasis. The learning objectives of the SK courses were hence redefined as the following.

Head: "PTs should be able to know why, not just how, the concepts underlying primary mathematics work."

Heart: "PTs should be able to appreciate the beauty, history, and usefulness of the mathematics underlying the primary school content."

Hands: "PTs should be able to solve any primary school mathematics problems."

At first glance, it appears that the scope of the learning objectives is too narrowly defined by restricting them to primary school mathematics. However, guided by the 3H framework, the committee was led to support the PT's teaching (Hands) by ensuring he or she would be able to tackle any primary school mathematics syllabus questions from their future students and responding emphatically to the relevance that the PT is desiring (Heart).

The committee also explored the potential recursive relationships among these domains and derived the following useful relationships connecting them:

Head to Hands: Understanding underlying concepts will help with mastery of tasks and problem solving in mathematics.
Head to Heart: By understanding underlying concepts, one can appreciate and see connections, rather than just see everything as a set of rules.
Heart to Head: By appreciating the history of a piece of mathematics, one will be able to understand why certain mathematics were done, for instance, to solve some kind of practical problem.
Hands to Head: By gaining enough practice, one can eventually understand the mathematics behind it.
Hands to Heart: By gaining mastery in practice, one will feel confident in the subject.
Heart to Hands: By being convinced that a piece of mathematics works and is relevant, one will be willing to spend time to work on it.

5.3. 3H nesting expands scope of concern

The emergence of self-similarity provides an insight that not only the needs of the PT but also those of the SK lecturers need to be considered. This is illustrated in Transcript 4 which records a discussion on whether negative numbers should be taught with mathematical rigour or symbols which are extensions of known operations with natural numbers.

M2: So even while they're (the PTs) doing the Hand work, if their Head just doesn't get it or their Heart isn't into it, they can't do it well. So it's a fractal quality. Now I noticed another fractal quality of Head, Hand, Heart which is not the student but the lecturers. If the lecturers, in this case, are not happy with what they are teaching, they can't teach well. I think that's another matter to bring up.

M1: mhmm…(agreeing)

M2: So now what you're bringing up is you're suggesting the change because is it the students don't like it. They say it's a tautology, why are you teaching like this? But the lecturers have to be convinced. Otherwise the head, hand, heart also…, they're not going to teach it properly. They're not going to… [the lecturer] will just keep banging, banging and banging and insisting his way. That's something to bring up. It's among the lecturers. So [we] have to consider this head heart hand thing… because like you said it doesn't square with secondary school [curriculum], then we have all these notation, concepts thing… I think modern algebra or (inaudible) was a means to pass the symbolic stage to give it some grounding which is rigorous mathematics. I can think of another example in JC math which is what we call the position vector and the direction vector and they look exactly the same [to the students]. [proceeded to give another example on position and direction vectors]…I think for some lecturers, is… to them, say "I don't want to teach them (their students) this because in the end they don't really understand what's the truth [mathematically]."

[voices agreeing]

M2: That's the thing… the self-similarity within the lecturers is also quite important. So while the students (referring to PTs) don't understand, you see, to them it's tautology or it's obvious because they don't understand.

M4: …because you learn in secondary school you learnt what is 3 minus 5, what's 3 plus 5, you drill into their head so that's why it's obvious.

M3: It's the same as proving the base angles of isosceles triangles are congruent. They've been using it since P5, [so the PTs ask] why are we proving this thing. That we discuss the last time. That's an issue we need to address under the Heart side.

M2: So the question now is, if we look at this (3H) as a guiding principle, the students think as tautology. Do we want to change the lecturers? Or do we want to change the students?

Transcript 4: 3H nesting expands scope of concern to lecturers

Nesting within the 3H framework prompts for issues to be seen at different levels. We think that this will be a key contribution for curriculum review. For example, M2 pointed out that students were not happy to prove what seemed to them to be tautologies: "… their Head just doesn't get it, or their Heart just isn't into it, they can't do it well." The solution would then be obvious, that is to remove the topic of negative numbers from the SK syllabus since this topic is not taught at the primary school level. The fractal aspect of the framework suggested a nesting perspective which prodded a relook at a possible 3H sub-frame at the lecturer's level: "Now I noticed another fractal quality of Head, Hand, Heart which is not the student's but the lecturer's."

6. Summary and future work

The 3H framework emerged from an initial grounded theory approach followed by a complexity approach upon interaction with the data. A complexity paradigm challenges the linear approach to curriculum design. By only viewing learning experiences and learning outcomes as a linear input and output process, one loses sight of the interconnectivity between the different learning outcomes and their corresponding interactions.

We briefly contrast our curriculum design model with Tyler's model, as an exemplar of linear models. Tyler's model was originally adopted in our curriculum review with its key merit of being an objective and consistent model that clearly articulated the stepwise process of deciding learning objectives, achieving them through learning experiences, and checking efficacy through assessments. In deciding if a particular topic should be added to a curriculum, Tyler's model would consider in a linear manner how the topic can achieve the learning objectives within the curriculum, how that topic should be organised, what resources should be provided and how it can be measured. In our SK curriculum review, this approach may fail to pick up the unhappiness of the lecturers because lecturers are merely a stakeholder group in Tyler's model, and not a layer of consideration as in the 3H framework. Even if the unhappiness had been picked up, Tyler's model would find it hard to justify certain actions to the lecturers. This is unlike the 3H framework, which regards the interactions among domains and layers as crucial. It thus was able to explicitly pick up the lecturers' concerns, put them together with the concerns of the PTs and argue for a solution. The nesting feature of the 3H framework allows for the consideration of the key factors (Head, Heart, and Hand) from layers of stakeholders, their relationships, both among factors and among stakeholders.

In this work, we have documented the initial phase of the curriculum review that we undertook to review the SK courses. Through this review, we have found that by considering the responses and data from a bottom-up approach, one observes a natural emergence of the common themes that are mentioned, i.e. the Hands, Head, and Heart domains. Furthermore, such domains present a nested self-similar structure, a feature that is common in many complex systems. By considering the domains as highly interacting and non-linear systems, the committee could discuss with insight the concerns that need to be addressed for the new curriculum to work.

From a mathematics education research perspective, it would be interesting to use the 3H framework to understand why curriculums are often either static or reactive, rather than proactive to changes to societal needs and demands. As an example, perspectives from the 3H lens would focus on how the lack of buy-in from teachers or faculty would hinder such review efforts, how addressing their Head, Heart, and Hands would allow for greater support, and how that support would then permeate to the other layers of stakeholders. Even at pre-tertiary level, one can further harness the 3H lens in the following way. Given that school curricula or initiatives in general are prescribed from top-down (starting, for example, from the education ministry), the 3H lens can then provide an insight into how the teachers at school level react to the prescribed curriculum with 3H, and how this contributes to the 3H of their own students. One can further propose a school-based curriculum review based on 3H bounded by the prescribed curriculum from the top and study if that will result in a better enactment of the curriculum.

As a continuation of the curriculum review, the 3H framework is currently being used in the review of the content within components of the SK courses. Though not reported in this work, the framework has been used to provide comprehensive perspectives of the merits and issues with the current content and a whole set of new materials based on reduced content are being developed for the new SK curriculum.

Acknowledgement

This work was supported by RS 2/17 TEG of National Institute of Education, Nanyang Technological University, Singapore. Ethics approval was granted in IRB-2017-03-041.

References

Ahn, Y.-Y., Bagrow, J. P., & Lehmann, S. (2010). Link communities reveal multiscale complexity in networks. *Nature, 466*(7307), 761.

Albert, R., & Barabási, A.-L. (2002). Statistical Mechanics of Complex Networks. *Reviews of Modern Physics, 74*(1), 47.

Barabási, A.-L. (2016). *Network Science.* Cambridge University Press.

Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., & Hwang, D.-U. (2006). Complex networks: Structure and dynamics. *Physics Reports, 424*(4-5), 175-308.

Brainard, J., & Hunter, P. R. (2015). Do complexity-informed health interventions work? A scoping review. *Implementation Science, 11*(1), 127.

Braithwaite, J., Churruca, K., Long, J. C., Ellis, L. A., & Herkes, J. (2018). When complexity science meets implementation science: A theoretical and empirical analysis of systems change. *BMC medicine, 16*(1), 63.

Burns, A., & Knox, J. S. (2011). *The Electronic Journal for English as a Second Language, 15*(1).

Castellani, B., & Hafferty, F. W. (2009). SACS Toolkit - Theoretical Framework. In B. Castellani, & F. W. Hafferty, *Sociology and Complexity Science - A New Field of Inquiry* (p. 34). Springer.

Cochran-Smith, M., Ell, F., Ludlow, L., Grudnoff, L., & Aitken, G. (2014). The challenge and promise of complexity theory for teacher education research. *Teacher College Record, 116*, 050302.

Davis, B., & Sumara, D. (2006). *Complexity and Education.* Routledge.

Easton, F. (1997). Educating the whole child,"head, heart, and hands": Learning from the Waldorf experience. *Theory into practice*, *36*(2), 87-94.

Guevara, P. (2014). Toward a Common Structure in Demographic Educational Modeling and Simulation: A Complex Systems Approach. *Complicity: An International Journal of Complexity and Education, 11*(2), 86-101.

Jörg, T., Davis, B., & Nickmans, G. (2007). Towards a new, complexity science of learning and education. *Educational Research Review, 2*(2), 145-156.

Kirkpatrick, D. L., & Kirkpatrick, J. D. (2009). *Evaluating Training Programs: The Four Levels.* Berrett-Koehler Publishers.

Kitsak, M., Gallos, L. K., Havlin, S., Liljeros, F., Muchnik, L., Stanley, H. E., & Makse, H. A. (2010). Identification of influential spreaders in complex networks. *Nature Physics, 6*(11), 888.

Koopmans, M., & Stamovlasis, D. (2016). Introduction to Education as a Complex Dynamical System. In M. Koopmans, & D. Stamovlasis, *Complex Dynamical Systems in Education: Concepts, Methods and Applications* (p. 1). Springer International Publishing Switzerland.

Kuziemsky, C. (2016). Decision-making in healthcare as a complex adaptive system. *Healthcare management forum, 29*(1), 4-7.

Larsen-Freeman, D. (2016). Classroom-oriented research from a complex systems perspective. *Studeies in Second Language Learning and Teaching, 6*(3), 377-393.

Morrison, K. (2008). Educational Philosophy and the Challenge of Complexity Theory. *Educational Philosophy and Theory, 40*(1), 19-34.

Nowotny, H. (2013, November 14-15). Retrieved April 11, 2019, from Helga Nowotny: http://helga-nowotny.eu/downloads/helga_nowotny_b143.pdf

Perona, M., & Miragliotta, G. (2004). Complexity management and supply chain performance assessment. A field study and a conceptual framework. *International journal of production economics, 90*(1), 103-115.

Scriven, M. (1991). Prose and Cons about Goal-Free Evaluation. *American Journal of Evaluation, 3*(4), 1-4.

Song, C., Havlin, S., & Makse, H. A. (2005). Self-similarity of complex networks. *Nature, 433*(7024), 392.

Stacey, R. D. (2002). *Complexity and management.* Routledge.

Stake, R. E. (1967). *The counterance of educational evaluation.* Department for Exceptional Children, Gifted Child Section.

Steenbeek, H., Vondel, S. V., & Geert, P. V. (2017). The Socially Situated Dynamics of Children's Learning Processes in Classrooms: What Do We Learn from a Complex Dynamics Systems Approach? *Complicity: An International Journal of Complexity and Education, 14*(2), 60-77.

Stufflebeam, D. L. (2003). The CIPP Model for Evaluation. In T. Kellaghan, & D. L. Stufflebeam, *International Handbook of Educational Evaluation* (pp. 31-62). Dordrecht: Springer.

Sugiarto, H. S., Lansing, J. S., Chung, N. N., Lai, C. H., Cheong, S. A., & Chew, L. Y. (2017). Social cooperation and disharmony in communities mediated through common pool resource exploitation. *Physical Review Letters, 118*(20), 208301.

Tay, E. G., & Ho, W. K. (2015). Teaching undergraduate mathematics – reflections on Imre Leader's observations. In R. Göller, R. Biehler, R. Hochmuth, & H. Rück, *Didactics of Mathematics in Higher Education as a Scientific Discipline* (pp. 87-90). Kassel, Germany: Kompetenzzentrum Hochschuldidatik Matematik.

Tyler, R. W. (1949). *Basic Principles of Currciulum and Instruction.* Chicago: University of Chicago Press.

Wang, P., González, M. C., Hidalgo, C. A., & Barabási, A.-L. (2009). Understanding the spreading patterns of mobile phone viruses. *Science, 324*(5930), 1071-1076.

Sipos, Y., Battisti, B., & Grimm, K. (2008). Achieving transformative sustainability learning: engaging head, hands and heart. *International Journal of Sustainability in Higher Education*.

Appendix A: Survey questions for the PTs

This section provides the survey questions for the Further Mathematics course that were administered to the PTs (questions for other courses were identical except for the course name). The survey was on a voluntary basis and had full anonymity in their online submission. They were also allowed to choose not to answer any question if they did not wish to, although in practice all the 69 respondents at least answered all the Linkert scale questions, but were more selective in answering the open-ended questions.

Statement/Question	1 – strongly disagree, 5 – strongly agree
The Subject Knowledge course, ASM40A: Further Mathematics Topics, builds up my confidence in teaching Primary Mathematics. I think it is necessary to have a deeper understanding of the mathematical concepts (beyond the level covered within Curriculum Studies courses) in Primary Mathematics.	1 2 3 4 5 1 2 3 4 5
Why do you think that it is necessary/not necessary?	
Further Mathematics Topics help me to understand the mathematics taught in Primary Mathematics at a deeper level. I find that the contents in Further Mathematics Topics are relevant to my teaching in Primary Mathematics.	1 2 3 4 5 1 2 3 4 5
Which topics in Further Mathematics Topics do you find relevant to your teaching in Primary Mathematics? Why do you think they are relevant? (N.A. if not applicable) Which topics in Further Mathematics Topics do you find irrelevant to your teaching in Primary Mathematics? Why do you think they are irrelevant? (N.A. if not applicable)	
There is too much content to cover in the Further Mathematics Topics course for one semester.	1 2 3 4 5
The course notes for Further Mathematics Topics: • are enjoyable to read; • enhance my understanding of concepts taught in class. The exercises for Further Mathematics Topics: • are manageable; • enhance my understanding of the topics. The further exercises for the Further Mathematics Topics are: • interesting to work with; • challenging. The tests conducted in ASM40A: Further Mathematics Topics: • are fair ways to assess my performance in the course; • determine how I study for the course; • require a lot of memory work.	 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5

If you are given a choice for alternative forms of assessment, what would you suggest? Please give a reason for your suggestion(s).	
I find that the Subject Knowledge courses (ASM Series) have helped me to better understand my Curriculum Studies (ACM Series: Teaching and Learning of Primary Mathematics) courses.	
Please elaborate, perhaps with an example, your answer above.	
In your own words, recall the stated objectives of Subject Knowledge (ASM40A) courses. (What did the tutors say the course objectives were?)	
What would be your desired objectives of Subject Knowledge (ASM40A) courses? (You may write down the stated objectives above if you agree with them.)	
What aspect(s) of the courses do you like and why?	
What aspect(s) of the courses do you think may be further improved and how may they be improved?	
Any additional feedback and/or suggestions on the courses?	
If you have taken courses from Subject Knowledge English and Subject Knowledge Science, are there any good practices from the SK English and SK Science that may be applied to SK Math courses?	

Appendix B: PT responses on perceived learning objectives

The following were the responses of the PTs to the question of why it was "necessary to have a deeper understanding of the mathematical concepts (beyond the level covered within the Curriculum Studies courses) in Primary Mathematics". In total, there were 48 responses to this question. 29 responses were considered to be in direct response to the question and they are listed below.

1. **Viewing the course as a step towards being a better teacher and/or improve their pedagogy**

As the PTs had a distinct career path as a teacher in their immediate future, it was obvious that their identity as a teacher-to-be would largely influence their consideration as to whether a particular activity would be relevant to their professional development. As a result, there were a total of 17 responses that were of this kind. The paragraphs in *italics* were the actual responses by the PTs. For easy reading, we categorised the results based on our initial coding scheme, which are ***italics and bold***. We have further categorised the responses through the 3H lens.

Hands:

Delivery of explanations. Four of the responses indicated that it would be necessary for them to have deeper knowledge so that they would be able to explain to their future students better.

"Deeper understanding allows us to deliver lessons with more clarity and it reaches the pupils better too."

"One has to have a deeper and broader understanding of concepts one intends to teach, in order to teach it well, such as being able to explain and provide alternative solution methods."

"Necessary because having deeper content knowledge will help in my explanation beyond instructional understanding which I think would help students develop deeper conceptual understanding of the subject. If we only know the steps, and if students were to ask how certain formulae or steps are derived, we will not be able to answer them."

"It is necessary because this is a degree course. Furthermore, only with knowledge beyond the curr[iculum] studies in primary math[ematics] would one be able to explain mathematical concepts."

Simplifying explanations. While the previous category was only concerned about the ability to present mathematical ideas and concepts, this category involves the decomposition of the mathematical idea, identification of the correct representation for the students and/or to develop useable definitions for the students.

"I believe that before teachers teach about the basic concepts of mathematics, they should understand what is beyond it. That way, they could help their students understand better when they break it down to chunks for them."

"You need a much deeper understanding to explain a certain concept as simple as possible to students and relate it to something they can connect to."

"We need to know more in depth in order to find the right method to simplify explanations to our students."

Longitudinal coherence. *"It allows us to understand the pre-requisites better and what is the next higher learning for my students which will facilitate the prior knowledge I can tap on and what level of scaffolding I need to be mindful of."*

Working with high ability students. *"For our own knowledge and also improves our ability to help/challenge the higher ability students."*

Effective teachers. Two of the responses used the word "effective", but they were not further elaborated.

"In order to be effective in teaching Primary Mathematics, I feel that it is important to have a firm foundation in content knowledge"

"So that we can be more effective in teaching Maths"

Head:

Making connections. Here, the responses illustrate how PTs associate deeper knowledge with connections.

"You need a much deeper understanding to explain a certain concept as simple as possible to students and relate it to something they can connect to."

"A deeper understanding provides us the basis to help students to make connections and real world applications."

There were also diverse reasons PTs have in viewing why a deeper understanding was necessary.

Understanding heuristics and problem solving. *"Since primary school students learn heuristics which require deeper thinking, it may be useful to have a deeper understanding of mathematical concepts which may be useful in helping students to understand heuristics and problem solving."*

Understanding difficulties of the topic. *"In order to teach, the teacher will need to know the topic and understand the possible difficulties of the topic to better help the students."*

Heart:

Emotional preparedness. *"The courses help me to understand and upgrade my content knowledge on Mathematics which made me feel prepared for practicum."*

General appreciation. *"Helping students think like a mathematician, i.e. use abstract thinking, helps them appreciate better why they are learning it (not for exams)."*

2. Gaining affective sensitivity to their perspective students

Another set of interesting responses showed that a few of the PTs viewed having deeper understanding as a means to understand their future students' viewpoint. While the responses above have already hinted at some level of nesting by considering their future students in their responses, the below responses were explicit about it:

"Similar to how we deliver according to the ZPD of our children, it should be of interest to us educators to be a +1 to be able to sustain interest."

"Great in helping me understand why students would struggle in understanding certain concepts."

"Helping students think like a mathematician, i.e. use abstract thinking, helps them appreciate better why they are learning it (not for exams)."

3. To understand formula or method

A few responses took an instrumental view of having a deeper mathematical knowledge, whereby they see it as a means to understand the formula or method. One would have expected that such responses to be overwhelming, but only three responded in this manner.

"If we only know the steps, and if students were to ask how certain formulae or steps are derived, we will not be able to answer them."

"We learn better when we understand the concept behind certain formulas we have to use."

"So that I can understand how this method came about instead of just memorizing and doing the questions"

4. Other positive responses

There were a number of other responses that, while they could not be classified directly, gave us a hint on what the PTs thought about why a deeper understanding would be necessary.

"Especially important for non-math majors like myself... one module right before final posting did not feel like it was sufficient at all."

"To be more equipped with subject content"

"So that you would be equipped with more maths knowledge"

The above responses highlighted a possibility of how the PTs perceived themselves as requiring more Subject Knowledge to fill the inadequacy in their knowledge.

5. **Negative responses**

On the other hand, of the responses that explained why they do not find having a deeper understanding a necessary endeavour, they all pointed to the fact that the content was not taught in primary mathematics:

“We will not be teaching it to our students in the future.”

“Would rather have a course which focuses on primary level pedagogies”

“Don’t see the relevance between how to teach well and knowing such knowledge”

6. **Responses that are considered not meaningful**

Here we will list a sample of the responses that are not considered to be directly relevant for completeness. While they may be useful to understand their sentiment (usually frustration), such responses do not directly respond to the question that was asked.

“I can’t even do those questions on my own. I don’t even know how to solve them without help. How am I supposed to teach upper primary mathematics?”

“It’s not the content, it’s the way the tests and exams are done. We are doing PGDE.”

“I would rather have a course which focuses on primary level pedagogies; I found it difficult to catch up with the material on the course and the content seemed to be skewed towards people with a mathematics background.”

“Define deeper. Sometimes, at different levels, there is no need to go into such a deep level that it confuses the trainee teacher who have no 'DEEP' mathematical background. Think engineering for example, some things can just be 'taught' without going through the grind... what are these things?”