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THE TEACHING OF MULTIPLICATION FACTS

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A child can perform multiplication in many ways and some of the strategies are picked up by himself when he is solving more difficult facts. According to Rathwell (78), three types of activities can help children learn the basic facts of multiplication. These are

- (1) concept development,
 - (2) strategies as aids for learning facts
- and (3) mastery of facts.

In the first, students will learn to conceptualise multiplication whereas in the other two, students will learn to form associations and internalise the multiplication facts. This article describes some of the useful strategies in teaching multiplication facts.

Strategies for Multiplication Facts

Three types of strategies can be classified in the teaching of multiplication facts. They are

- (1) the structured approach,
- (2) the pattern approach and
- (3) finger multiplication.

Structured Approach

One approach in the teaching of multiplication is the emphasis on the structure of the number system with respect to multiplication. This would help a pupil to reduce the number of facts he needs to learn. According to Bruner, the basic element in learning is the importance of exposing the learner to the structure of a subject which stresses relationships between the parts rather than presenting the subject as a string of unrelated concepts or skills. Seeing relationships between the parts help the child to comprehend, remember and apply to a new situation. Ausubel's

meaningful learning also emphasizes the teaching of the structure of a discipline. Hence, in the teaching of multiplication facts, properties which should be emphasized are commutativity, associativity, identity element, zero element and distributivity.

The commutative property states that the order or sequence in which two whole numbers are multiplied has no effect on their product. For example, $3 \times 8 = 8 \times 3 = 24$. When a child knows the multiplication fact of 3×8 , he should obtain 8×3 using the commutative property.

The associative property states that when three numbers are multiplied, they can be multiplied in any order. For example,

$$\begin{array}{l} 3 \times 4 = 3 \times (2 \times 2) \\ \quad = (3 \times 2) \times 2 \\ \quad = 6 \times 2 \\ \quad = 12 \end{array} \qquad \begin{array}{l} 7 \times 4 = 7 \times (2 \times 2) \\ \quad = (7 \times 2) \times 2 \\ \quad = 14 \times 2 \\ \quad = 28 \end{array}$$

The above examples show that when a student encounters difficulty in working out 3×4 and 7×4 , the associative property can be used to reduce the problem to one which can be solved with knowledge of the two times table, that is, $3 \times 4 = 6 \times 2$ and $7 \times 4 = 14 \times 2$. The student uses a simpler fact to work out a harder fact.

The distributive property involves both addition and multiplication. For example, 3×8 can be written as

$$\begin{array}{l} 3 \times (2 + 6) = (3 \times 2) + (3 \times 6) \\ \quad \quad \quad = 6 + 18 = 24 \\ \text{or } 3 \times 8 \quad = 3 \times (4 + 4) \\ \quad \quad \quad = (3 \times 4) + (3 \times 4) \\ \quad \quad \quad = 12 + 12 = 24 \end{array}$$

This example also demonstrates that hard facts can be broken down using simpler facts without actually memorising the harder facts.

Using the properties of 0 and 1, students should be able to recognise that any number multiplied by 0 gives zero and any number, other than zero, multiplied by 1 will be equal to that number.

Pattern

Until the sixties, students in primary schools were expected to master the times tables by rote. This approach seems to be out of fashion nowadays although it is presumed that some schools are still hanging on to it. Being able to recognise patterns in learning multiplication facts will help students to understand not only the multiplication concept but also to recall the multiplication facts more readily. For example, students can readily recall the 5 times table if they notice the pattern of the ones' digits in the products. The ones' digits of the 5 times table are 5 and 0 alternately.

In multiplication, the even and odd patterns can be detected and generalised. Notice that in the table below, when both factors are even numbers, the product is always even. For example, $2 \times 6 = 12$, $4 \times 8 = 32$ and $8 \times 8 = 64$.

x	0	1	2	3	4	5	6	7	8	9
0			0		0		0		0	
1			2		4		6		8	
2	0	2	4	6	8	10	12	14	16	18
3			6		12		18		24	
4	0	4	8	12	16	20	24	28	32	36
5			10		20		30		40	
6	0	6	12	18	24	30	36	42	48	54
7			14		28		42		56	
8	0	8	16	24	32	40	48	56	64	72
9			18		36		54		72	

Table 1. Patterns For Even Factors

When both factors of a multiplication sentence are odd, then the result is odd. For example, $3 \times 5 = 15$, $7 \times 9 = 63$ and $9 \times 5 = 45$.

When one factor is an even number and the other factor is an odd number, then the product is even. For example, $2 \times 5 = 10$, $4 \times 9 = 36$ and $6 \times 7 = 42$.

In observing this, one notices that the products of the 2, 4, 6 and 8 times facts are all even irrespective of the multiplicands being even or odd. Another pattern is that the ones' digits of the products of the 2, 4, 6 and 8 times tables are in cyclic order as shown in the table below:

	Ones' Digits In Cyclic Order									
2 'x' Facts	0	2	4	6	8	0	2	4	6	8
4 'x' Facts	0	4	8	2	6	0	4	8	2	6
6 'x' Facts	0	6	2	8	4	0	6	2	8	4
8 'x' Facts	0	8	6	4	2	0	8	6	4	2

Table 2. Patterns For 2, 4, 6 and 8 Times Tables

Another set of multiplication facts which can be learned and remembered with ease is the 9 times table. They have a nice pattern as the sum of the ones' and the tens' digits of each product is always equal to 9. For example,

$$9 \times 4 = 36, \text{ and } 3 + 6 = 9,$$

$$9 \times 8 = 72, \text{ and } 7 + 2 = 9.$$

Besides this, the tens' digit of the product is always one less than the multiplicand. For example, in $9 \times 4 = 36$, 3 (the tens' digit of the product) is one less than 4 (the multiplicand). Using these two patterns, a student should be able to obtain and master all the facts of the 9 times table.

Referring to the 3 and 6 times facts, a student can notice that the sum of the ones' digit and the tens' digit in each product is 3, 6 or 9. The sums of 3, 6 and 9 which appear in the 3 times facts are in cyclic order. Similarly, the sums of 6, 3 and 9 which appear in the 6 times facts are also in cyclic order as shown below:

x	0	1	2	3	4	5	6	7	8	9
	Sums of Digits in Cyclic Order									
3 'x' Facts	0	3	6	9	3	6	9	3	6	9
6 'x' Facts	0	6	3	9	6	3	9	6	3	9

Table 3. Patterns For 3 and 6 Times Tables

In Bolduc's article, "The Monsters in Multiplication", which appeared in the *Arithmetic Teacher* (1981), ten types of multiplication facts were identified as difficult facts. Two of them are 3×4 and 7×8 . However, if a student can recognise patterns, these two facts can be remembered easily. Consider the two sets of consecutive numbers,

1, 2, 3, 4 and 5, 6, 7, 8.

We have $12 = 3 \times 4$ and $56 = 7 \times 8$.

With this aid, the two facts can be more easily remembered by the students.

Finger Multiplication

It is possible to use fingers to help children practise multiplication facts. Teachers have found that methods using finger multiplication particularly help the weaker pupils who tend to forget the facts they have learned. The simplest use of finger multiplication is for the nine times facts. The figure below shows the different numbers represented by each of the fingers of both palms (facing upwards).

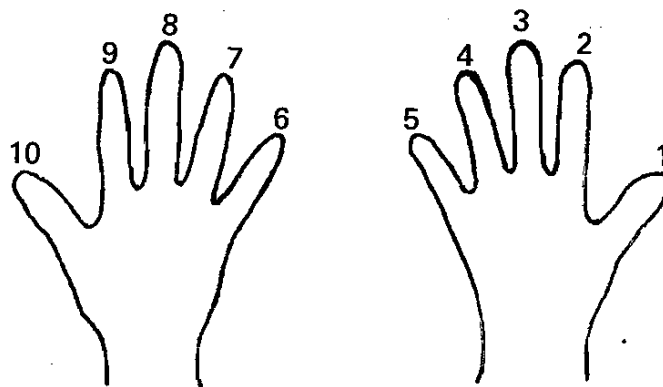


Figure 1. Finger Representation Of Numbers 1 to 10.

To obtain 9×4 , bend the fourth finger from the right as shown in Figure 2.

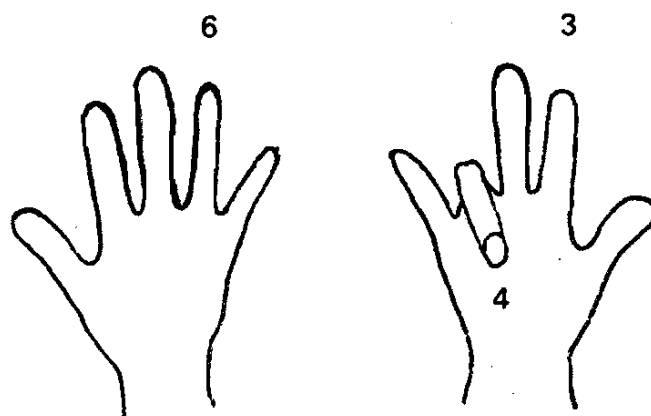


Figure 2. Finger Representation of 9×4

Fingers raised to the right of the bent finger represent the tens' digit, i.e., 3 and fingers to the left represent the ones' digit, i.e., 6. Thus $9 \times 4 = 36$.

This method can be used for multiplicands from 1 to 10 but not for zero.

Some other strategies which are not included here but commonly used by students are reciting by rote, counting and skip counting. These are considered to be immature strategies.

Helping children develop thinking strategies is an important step between concept development and drill for mastery. Using thinking strategies as an intermediate aid can help children learn multiplication facts and to commit them to memory.