The Mathematician Educator
2020, Vol 1, No. 2, 83-101

Nature and Perceptions of Pre-Class Tasks Used in a Flipped Linear Algebra Course for Pre-Service Teachers

NG Wee Leng, TEO Kok Ming, WONG Khoon Yoong, KWAN Kang Ling
National Institute of Education
Nanyang Technological University
Singapore

In recent years, computer-based technology has enabled university lecturers to teach their courses using non-traditional pedagogies. One such pedagogy is the flipped learning model. As flipped learning is being used more frequently to teach undergraduate mathematics, instructors need to collect data to identify practices that work well to promote student mathematics achievement and favourable perceptions toward this new learning mode. This paper describes six different types of pre-class tasks for a flipped Linear Algebra II course in a Singapore university, such as short videos narrated by the instructor, synopses, summary sheets, worksheets of problems and activities, and online quizzes. The sample comprised 15 pre-service teachers, who had adequate to good mathematics backgrounds, and their participation in this project would prepare them to implement flipped learning in school mathematics in the future. On average, they spent about one hour to complete these weekly pre-class tasks, but the stronger ones reported spending less time on these tasks than the other students. Almost all the students rated very highly these tasks in terms of helping them to learn and enjoyment at mid-semester and end-of-course surveys. These perceptions had weak correlations with the course grade. Suggestions for practice and future research are discussed.

Keywords: flipped learning, pre-service teachers, Linear Algebra, Calculus

Introduction

In recent years, university instructors in many countries have implemented “flipped” learning (also known as “flipped” classrooms, “inverted” classrooms, “inverted” instruction) as one form of “blended learning” to bring about more active learning of undergraduate courses, in particular Mathematics, Science, Engineering, Computer Science, and Information Technology. Under this model, the traditional pedagogy of using class time to lecture on new academic content followed by completing assignments outside class is “flipped” so that students learn some of the new content on their own and then come to class to resolve their confusions and to engage in extended problem solving. The details of flipped learning vary considerably, but the main claim, based on learning theories and research findings, is that under this learning model, supported by digital resources, students will enjoy this type of learning, master the academic contents, and develop intellectual skills, such as independent learning using technology and print materials, critical thinking, and collaboration (Bishop & Verleger, 2013; Brewley et al., 2017; Fulton, 2014; Hamdan, et al., 2013; Herreid & Schiller, 2013; Love, et al., 2014; O'Flaherty & Phillips, 2015).

The present study was undertaken to address three issues about flipped learning at the tertiary level. First, several recent studies on flipped learning for undergraduate level mathematics are short-term or small-scale. For example, Novak et al. (2017) focussed on only one flipped lecture because they did not wish to make a "flop" of their "flipped" attempt. Talbert (2014) discussed three flipped classroom designs for Linear Algebra classes: "as a one-time class design to teach a single topic, as a way to design a recurring series of workshops, and as a way of designing an entire linear algebra course" (p.361). Our study was of Talbert's third design, involving a semester-long, flipped Linear Algebra II course.

Second, some studies focus on mostly pre-recorded video-lectures with minimal supplementary support. For our flipped course, we designed six different types of pre-class tasks, including videos that could cater to the different learning preferences of the students.

Third, the students in our course were pre-service teachers rather than students from different disciplines (e.g., Brewley et al., 2017). They may have to design their own flipped lessons, as the Singapore Ministry of Education (Heng, 2013) has recently proposed flipped classroom as one innovation to enrich student learning in its schools in the near future. Thus, by participating in this study, it was hoped that these pre-service teachers would gain relevant direct experiences about flipped learning so that they are better prepared to conduct flipped mathematics lessons in the future.

Literature Review

Different definitions and implementations of flipped learning can be found in the literature (Bishop & Verleger, 2013; Brewley et al., 2017; Fulton, 2014; Hamdan et al., 2013; Herreid & Schiller, 2013; Love et al., 2014; Jungić, et al., 2015; Novak et al., 2017; O'Flaherty & Phillips, 2015; Talbert, 2014). These differences cover the four dimensions below:

- Content (what?): What academic content is to be learned by the students on their own and what to be addressed in class time?
- Activities (how?): How are the flipped learning activities organised and delivered to the students using online and print materials, in particular, the use of video-lectures, which is the most popular pre-class activity offered by tertiary instructors?
- Timing (when?): When do the students have to complete the flipped activities?
- Assessment (how well?): What outcome variables are investigated and how? How are the data on these outcome variables used to enhance learning, for instance, by giving feedback online or in face-to-face lessons?

As a consequence of these different definitions and implementations, mixed results are reported about student perceptions of flipped learning and academic achievement. This paper will report on both the aforementioned outcome variables.

With respect to achievement outcomes in tertiary mathematics, Lo et al. (2017) cited a quasi-experimental study about an undergraduate calculus course. Students in the flipped classes had similar scores as those in the traditional classes in both conceptual questions and computational questions. By contrast, in the study by Love et al. (2014) on Linear Algebra students in the flipped class saw greater improvement in test scores from one test to the next than the control section while the study by McGivney-Burelle and Xue (2013) on one unit of a calculus course found that students in the flipped classroom performed better in examinations than those in the traditional classes. Buch and Warren (2017) reported that a higher percentage of students in a flipped class passed a tertiary Pre-Calculus course compared to those in a lecture class, both classes taught by the same instructor. The effects were typically weak, and these seem to depend on the types of mathematics courses, flipped and in-class activities, and assessment tasks used in the research.

The literature has more reports on student perceptions than academic achievement. Student perceptions of flipped learning are often gathered from surveys and interviews of individual students or focussed groups. According to Fulton (2014), "[o]ne of the most widely accepted beliefs about flipping is that students love it" (p. 86), but other reviewers note that the findings about student perceptions or attitudes are mixed, even though positive results are reported more frequently than negative ones (e.g., Bishop & Verleger, 2013; Lo et al., 2017). Various factors can account for these findings. Positive factors include opportunities to re-watch the video-lectures at own pace, review video-lectures before examination, or more class time to go over difficult content. On the other hand, students do not like flipped learning when it is poorly organised, the video-lectures are too long, they are not interested in the subject, or they do not see the needs for flipped learning (Bishop & Verleger, 2013; Fulton, 2014; Hamdan et al., 2013; O'Flaherty & Phillips, 2015). To alleviate the issue of long video-lectures, Buch and Warren (2017) used videos of about 15 minutes each for two reasons: the students could "watch the lesson videos even when their time was limited" and they could "go back and re-watch videos on topics with which they struggled without having to watch the entire hour video" (p. 110). We have yet to find any report about the relationships between perceptions and achievement among mathematics students in tertiary flipped classes, and our study will explore these relationships.

Educational theories and research in school interventions consistently highlight the significant role of student prior knowledge in learning new contents. Ausubel's (1968) claim is now widely acknowledged:

> If I had to reduce all of educational psychology to one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.

Hattie (2015), in his well-known *Visible Learning* project, noted that teaching based on students' prior learning is one of six key factors that impact on student learning, and claimed that this applies at higher education too. His list also shows that the effects of prior achievement (effect size = 0.63) was much greater than that of attitudes towards content domains (effect size = 0.35). This study will examine the effects of students' prior performance in mathematics as a predictor of their perceptions of flipped learning.

Methodology

Participating students

Nineteen students were enrolled in this Linear Algebra II course, for the January 2018 semester (January to April), and 15 of them (9 females and 6 males) gave written consent for their course data to be used in this study. Only these 15 students were asked to respond to the weekly surveys, to be interviewed about their perceptions, and to be observed during class sessions. The course data of the other four students were excluded from the findings reported below. All the students, however, had to complete the learning and assessment tasks as part of the course requirements.

These students were in the second year of their Bachelor of Science (Education) programme, which integrates the study of Science and Education over a 4-year period. They were awarded teaching scholarships by the Singapore Ministry of Education to enrol in this pre-service teacher programme, and as awardees, they entered the programme with good mathematics backgrounds. Prior to Linear Algebra II, they had passed Calculus I, Finite Mathematics, Number Theory, Linear Algebra I, Calculus II, and Computational Mathematics.

The second author taught these students Calculus II in the August 2017 semester, also under the flipped model. The mathematical content of the two courses differed, but the flipped activities for each course were organised using the same design framework. Their Calculus II course overall scores ranged from 49 to 97 (out of 100), with median 78 and mean 78.7. Most of them, except for the student who had the lowest score of 49 and two others who had scores below 70, were mathematically competent and well prepared for this course. Concurrent to Linear Algebra II, they took Complex Analysis, Statistics I and Differential Equations. Differential Equations was taught by another lecturer, who also flipped his course by asking the students to watch his video-lectures before coming to class. No research data were collected from the students about these two other flipped courses, and our study aims to learn more about their flipped learning experiences.

Teaching schedule

The instructor of the Linear Algebra II course was also the second author (Teo) of this paper. As one of the researchers, he has provided an important insider perspective on many aspects of this study.

Teo had taught the Linear Algebra II course for several years under the traditional lecture plus tutorial model. In a mathematics course which adopts the lecture plus tutorial model, there are two hours of lecture and one hour of tutorial per week. During a lecture, the instructor will introduce and explain concepts, theorems and their proofs, and provide suitable examples to illustrate the concepts and theorems. The instructor may ask students to complete some exercises in class which involve the new concepts or applications of the theorems, and then discuss the answers to these exercises. For the tutorial sessions, the students are expected to solve 6 to 10 problems before coming to class. These problems are given to the students at least one week before the tutorial session. During a tutorial session, the instructor will ask some students to present their solutions to the problems to their peers. The instructor will discuss the solutions to problems which the students are unable to solve.

In planning this flipped course, Teo had modified past lecture materials as well as designed new ones, in order to cover similar amount of contents as the traditional version, so that the students were as prepared for the programme as past cohorts. In changing to the flipped model, he had spent about 120 hours on recording and editing 46 short videos, and preparing pop quizzes for these videos. This was a one-time effort and is comparable to what McGivney-Burdelle and Xue (2013) reported; they spent 1.5 hours to make one short video, and 45 minutes to prepare quiz and in-class problems.

The Linear Algebra II course comprised a total of 12 weeks of in-class sessions and 11 weeks of pre-class activities. There were a total of 12 two-hour and 12 one-hour in-class sessions (1 one-hour session and 1 two-hour session per week over 12 weeks). The course was conducted from January to April 2018, with a one-week mid-semester break after Week 7. In the previous semester the students took Linear Algebra I which covers functions and basic set theory, systems of linear equations and their solutions, matrices and matrix operations, invertible matrices, determinants, vectors in 2-space and 3-space, dot product and cross product, lines and planes in 3-space, and Euclidean n-space.

The learning activities were classified under three categories: pre-class tasks, in-class interactions, and post-class consolidation. One week prior to the first class, the students were notified through emails to complete the Week 1 pre-class tasks. At the first class, they were briefed on the structure of the flipped model, and their written consent to participate in this study was sought. For subsequent weeks, the videos and the supplementary materials for the upcoming week were uploaded to Blackboard, the online learning management system that all the students had to use for this course. They had one week to complete the pre-class tasks before coming to class. These learning activities are described in the following subsections.

Pre-class tasks

Pre-class tasks were systematically organised to prepare students for in-class discussion. The following six types of pre-class tasks were specially designed for this course.

(1) *Synopsis*. This gave an overview or advanced organiser of the weekly topic and videos, and it was typically half a page long. Some were in the form of a paragraph such as that shown in Figure 1 below.

> **Synopsis of Week 4 Lecture:**
>
> In the last lecture, we defined the important concept of spanning set of a vector space. In this lecture, we follow up by illustrating with examples how to determine whether a set of vectors is a spanning set for a vector space, and how to find a spanning set for a vector space. Next, we define another important concept: linear independence of a set of vectors in a vector space, and illustrate the definition with examples. We then state a necessary and sufficient condition for a set of vectors in a vector space to be linearly independent, and explain with an example how to apply the condition to show that a set of vectors is linearly independent.

Figure 1. The synopsis of linear independence (Week 4)

(2) *Videos.* The contents for each week were divided into several (three to five) videos, each running for 3 to 13 minutes long. This arrangement allowed students to watch and re-watch any number of the videos as time permitted, for the same two reasons mentioned by Buch and Warren (2017), as noted above. The videos were narrated by an instructor, captured by Camtasia, a screen-recording and video-editing software and uploaded to Blackboard. The students had to watch these videos online (instead of downloading them onto any device and watching them offline). The topics for the videos are given in Table 1 and covered standard definition theorems, simple examples, and routine procedures, leaving more elaborate proofs and challenging problems to in-class interactions. Each video was associated with an embedded pop quiz, a summary sheet, and a worksheet, also available via Blackboard. These supplementary tasks were meant to support self-learning via the videos.

Table 1.

Topics in Linear Algebra II covered in the videos

Week	**Topics**	**No. of videos**
1	Basic logic	5
2	Definition and examples of real vector spaces	4
3	Subspaces, linear combination and span	4
4	Spanning sets, linear independence	4
5	Linear independence, basis	4
6	Dimension	3
7	Row space, column space and null space of a matrix	4
	Mid-semester break	
8	Rank and nullity of a matrix, orthonormal set in Euclidean n-space, Gram-Schmidt process	5
9	Midterm test (no new content covered)	0
10	Orthogonal complement, least squares solutions	4
11	Eigenvectors and eigenvalues of matrices	5
12	Linear transformations	4

(3) *Pop quizzes embedded within the videos*. These were true/false items, and every video had one or two such items. An example of a pop quiz: *The set {(1, 1, 1)} is linearly independent; true or false?* These pop quizzes allowed the students to self-check their understanding of the basic concepts covered in the videos. They completed these pop quizzes anonymously and as many times as they wished. Their answers were checked immediately after submission online, but their responses were not graded. As students were encouraged to take a pop quiz again should they fail to answer all the questions correctly, no explanation was provided if an answer was wrong.

(4) *Summary sheet for every video*. It consisted of one or two questions that captured the main points of the video. For example, Figure 2 shows a screenshot of one summary question for linear independence. The rationale for writing down the answers manually is supported by Mueller and Oppenheimer (2014).

3. Video clip 4.3: Fill in the blanks.

Let V be a vector space, and let $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a set of vectors in V.

(a) Suppose $n > 1$. Then S is linearly dependent if ______________________________

______________________; S is linearly independent if ______________________________

______________________________.

(b) Suppose $n = 1$. Then S is linearly dependent if ________; S is linearly independent if

____________.

Figure 2. A summary question for a video on linear independence (Week 4).

(5) *Worksheet problems and activities*. These were routine and mostly computational problems, which are similar to the examples explained in the videos, and the students

were expected to solve the problems and complete the activities before coming to class. Figure 3 shows an example of such a problem.

Consider the vector space $V = \{(a, b, c): a, b, c \in \mathbb{R}, a = 2b + c\}$, which is a subspace of $\mathbb{R}^3$. For each of the following sets, determine whether it is a spanning set for V.

(a) $\{(1, 0, 1)\}$ (b) $\{(1, 0, 1), (1, 1, 0)\}$ (b) $\{(1, 0, 1), (1, 1, -1)\}$

Figure 3. A problem in the worksheet on linear independence (Week 4).

(6) *Weekly online quizzes*. The quiz items were of multiple-choice (some items had more than one correct answer) and true/false types, and there were 7 to 10 items per week. They assessed student understanding of the contents covered in all the videos for that week. Their answers were "marked" immediately as right or wrong, and the responses were captured on Blackboard. An "effort" grade (for participation, irrespective of right or wrong answers) was awarded, and this was one component of the "participation marks", contributing 10% toward the final course grade. This assessment is meant to motivate the students to take the pre-class tasks seriously. An example of a multiple-choice item in a weekly online quiz is shown in Figure 4. The student chose options A and C, but only C was correct. Option A was incorrect because the vectors 1 and *x* were not in the subspace *W*.

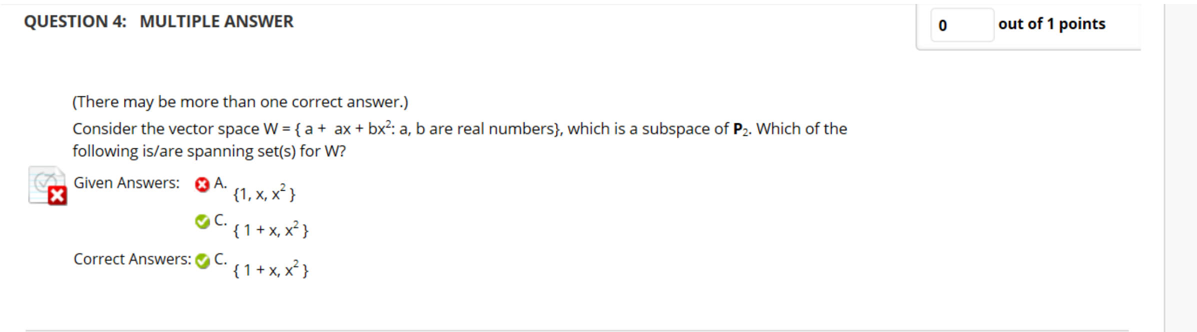

Figure 4. Screenshot of a completed online quiz.

Besides these six types of pre-class tasks, the students were expected to consult the textbook (*Linear Algebra: An Easy Introduction*) and other resources for more detailed solutions of examples not explained in the videos. They were also instructed to jot down any confusions to be raised at in-class sessions. Note that the course more or less followed the sequence in the textbook.

The above combination of video-lectures and supplementary materials was intended to make the course more comprehensive so that the students would engage in learning mathematics on their own through listening, writing, practising, and reviewing with immediate feedback to their attempts to solve problems, and ensure that their flipped learning experience was an enriching one.

In-class interactions

Over 12 weeks, the class met for a two-hour "lecture" and a one-hour "tutorial" per week. At these class sessions, more complex concepts, problems, and proofs were dealt with. Active learning was promoted through small-group discussion, whole-class discussion, student presentations at the whiteboard, and questions raised by the students themselves.

The two-hour "lecture" typically proceeded as follows:

(1) The instructor first reviewed specific points about the videos and explained corrections to the mistakes students made in the online quizzes. The students were encouraged to ask questions if they were still not clear about the materials covered in the videos.

(2) The instructor delivered mini-lectures on the more difficult contents, such as alternative solutions and proofs. These mini-lectures extended the contents covered in the videos.

(3) The students, in pairs or small groups, compared their solutions to the worksheet problems, while the instructor circulated the class to provide at-seat help and check on students' solutions.

(4) Some students were called upon to present their solutions on the whiteboard to the whole class. This was followed by brief exchanges with the presenter and comments by the instructor.

(5) Next, the students tackled the discussion questions in small groups. These discussion questions were conceptual and more difficult than the worksheet problems. They had to download these discussion questions from Blackboard, but they did not have to solve them before coming to class. Subsequently, the instructor engaged the whole class in discussing their solutions. This activity typically covered about one third of the session.

The one-hour "tutorials" were used to discuss the "challenging" problems in the problem sets; see item (2) in the following subsection. Students were asked to present their solutions on the whiteboard and to answer questions from their peers and the instructor. Finally, the instructor would guide the class to arrive at the correct solutions for problems that they still could not solve.

Post-class consolidation

The following activities were meant to assist students in organising their learning after the in-class sessions.

(1) Complete the online quiz again. Students who did not do well in the online quiz were encouraged to attempt the quiz again. This is one form of optional retrieval practice, since the answers had already been discussed in class. Their "new" scores were not used in the computation of the "effort" grade, as explained above.

(2) Post solutions of the "procedural" problems in the problem sets on Blackboard. There were altogether 9 problem sets for the course. Each problem set comprised around nine problems divided into two sections: section 1 with three to four "procedural" problems and section 2 with four to five "challenging" problems. These problems covered one to two weekly topics. The problem sets were uploaded to Blackboard one to two weeks before each tutorial session.
The 19 students were randomly divided into two groups (of 10 and 9 students respectively), and selected students in each group were assigned to post their solutions to the "procedural" problems in Blackboard. Most students scanned their handwritten solutions and uploaded them. A few students would type their solutions, and there is an equation editor for them to type mathematical symbols. They could read the solutions posted by their group members and comment on them online but not the solutions posted by the other group; this was to prevent students of one group from copying the solutions from the other group. During the course, every student would post solutions

to at least three problems online. The instructor would comment online on the posted solutions and suggest alternatives. Part of a posting by a student is shown in Figure 5. In this posting, the student answered the question of whether a given set of vectors in a vector space V is linearly independent and whether it spans V. The answer provided by the student is clear and correct, except for the incorrect use of dim(K) when K is not a vector space. The reason for asking students to post their solutions to the procedural problems in Blackboard was to free up more time for the discussion of the conceptual and challenging problems in the problem sets during tutorial sessions. It also enabled students to learn from and comment on each other's solution.

Question 1(b)

Let K denote the set given in Question 1(b).

We know that the dimension of K is 4, which is more than the dimension of the vector space V, as dim(V)=3.

From theorem 3, we know that since dim(K) > dim(V), K is linearly dependent.

Thus, the answers are as follows:

(i) K is not linearly independent (i.e. linearly dependent)

(ii) Let a vector in V be (a,b,c), where a,b,c are real numbers. If K spans V, then:

k(1,0,1)+ l (0,1,-1) + m (1,0,0) + n (1,2,1) = (a,b,c)

Equating the various terms, we have:

k+m+n =a

l+2n=b

k-l+n=c

Figure 5. An online posting of a solution by a student.

In addition to these two structured activities, the students were encouraged to seek help from the instructor at his office outside class time. However, students typically clarified their doubts during class and they would request consultation with the instructor just before the final examination.

Research questions, design, instruments, and procedure

This paper presents findings to answer the two research questions below:

(1) What were the students' perceptions of different types of pre-class tasks?
(2) Did their perceptions vary with mathematical backgrounds and gender?

This project was a case study in which "case" refers to the single class taking the Linear Algebra II course. There was no comparison group due to the small enrolment. A mixed-methods research design was used to collect quantitative and qualitative data over several occasions akin to the equivalent time series design for single group (Cohen et al., 2018). This intensive data collection was quite different from the typical one-off survey reported in the literature. It enabled us to make iterated changes to the course contents and delivery after considering the weekly surveys and online quizzes. With this possibility, we had also included some features of a design experiment (Sandoval, 2013).

The overarching framework for the design of the flipped Linear Algebra course is based on Brame's (2013) Four Key Elements of the Flipped Classroom as given below.

1. Provide an opportunity for students to gain first exposure to course material prior to class, whether through lecture videos or screencasts.

2. Give students an incentive to prepare for the day's activities by requiring them to complete a specific task before they come to class. This can be accomplished via automated quizzes, discussion board posts, or assignments to be reviewed in class.
3. Develop a mechanism to assess student understanding. Pre-class tasks can help the instructor tailor class content to match student need. Self-grading quizzes can provide students with self-knowledge as to where they need help. In-class activities can be structured so as to provide students with feedback both from their peers and the instructor.
4. Use in-class activities that focus on higher level learning objectives. As described above, class time should be used to promote deeper learning and to increase the skill with which students can apply and synthesize the knowledge they gained when preparing for class.

Furthermore, the design of the flipped Linear Algebra course was shaped and informed by constructivist learning theories, including Vygotsky's theory of Zone of Proximal Development (Vygotsky, 1978), in that participants of the study will engage in interactive, creative, and collaborative activities during knowledge construction. This feature is consistent with most studies on flipped learning as pointed out by Bishop and Verleger (2013). In addition, the design of the learning activities will be informed by Kolb's Experiential Learning Theory as is the case in a recent study by Lo and Hew (2017). Kolb (1984) modeled the learning process as a four-staged cycle which comprised of concrete experience, reflective observation, abstract conceptualisation, and active experimentation. The learning process is described as an idealised learning cycle if the learner goes through all the four stages.

The weekly surveys were used to capture student perceptions about the pre-class tasks, and it took only a few minutes to complete each survey. The students rated each task on a 6-point scale (1 to 6), 1 = strongly disagree; 6 = strongly agree, on three dimensions: "Easy to understand", "Help me learn", and "Useful" (Weeks 1 to 6; replaced by "Enjoyable" for Weeks 7, 8, 10, 11, 12). They also reported the amount of time they spent to watch each video and to complete the supplementary tasks. They submitted the hardcopy surveys to the instructor at the first in-class session of the following week.

A mid-semester survey was conducted at the end of Week 7 and an end-of-course survey on the last day of the course in Week 12. For these two surveys, the students indicated the workload for each activity (1 = not enough; 2 = just right; 3 = too much) and rated their perceptions on two dimensions ("helps me learn" and "enjoyable"). They also responded to three open-ended questions:

(1) Did you re-visit the videos after the in-class sessions? How many times? Why or why not?
(2) State any additional resources you have referred to (e.g., textbook, reference books, online maths videos) and how useful they were.
(3) State one major change you wish to see in the second half of the course. For the end-of-course survey, this question was replaced by: State one major change you think will help to improve the learning experience of similar flipped learning courses in tertiary mathematics.

At the end of the course and just before the final examination, 14 of the students were interviewed individually about their experiences of this course. These interviews were audio-taped and transcribed. Here are some of the interview questions:

(1) What is one key difference in the way you learned between this course and other mathematics courses that are not flipped?

(2) Did you manage to watch all of the video clips? Was there a lecture in which you had no time to watch the video clips at all? Why was that so? Did you read the textbook instead?

Students' mathematics achievement in the course was determined based on four components: in-class and online participation (10%), three 10-minute written quizzes conducted in Weeks 4, 7 and 12 (10%), one 1-hour mid-term test in Week 9 (20%), and one 2½-hour end of course examination, conducted 2 weeks after the end of the course (60%). The final examination comprised 5 constructed-response questions, requiring students to show detailed workings and explanations. Every student was allowed to bring in handwritten or typed notes on one A4 paper. The examination was conducted under invigilation.

Results

Time spent on pre-class tasks

Effective learning depends on the amount and quality of the times students spend engaging in the academic work. The weekly surveys collected the time spent on pre-class tasks as reported by the students. The weekly data were aggregated to give the overall time spent on the videos and the supplementary tasks, for the whole semester. Table 2 shows the highest, lowest and mean of these times and those of the total time spent on videos and supplementary tasks (to the nearest minutes). Note that in the table the numbers in the Total column are not sums of the corresponding numbers in the Video and Task columns. For instance, the student who had spent the longest amount of time on all the pre-class tasks was not the one who had spent the longest amounts of time on both videos and supplementary tasks.

Table 2.
Times (minutes) spent on videos, supplementary tasks, and both activities over 12 weeks.

Times (Minutes)	**Video**	**Task**	**Total**
Shortest	169	120	451
Longest	492	855	1255
Mean	349	415	764

The students spent vastly different amounts of time on these two types of tasks. On one end of the spectrum, one student spent only 30% as much time on the videos as the supplementary tasks, while on the other, a student spent 280% as much time on the videos as the supplementary tasks. On the other hand, about one-third of the class reported spending similar amounts of time on each type of tasks.

On a weekly basis, the students spent between half an hour to two hours (mean = 1.2 hours) on out-of-class activities. Despite this wide variation in out-of-class workload, almost all the students indicated in the mid-semester and end-of-course surveys that the amount of work for each pre-class task was "just right". Hence, flipped learning did not add drastic demands to their workload..

Figure 6 shows the mean time (minutes) spent on watching the weekly videos. In general, the students spent 20 to 50 minutes watching the videos each week. In Weeks 2, 5, 8, 11 and 12, three students or less failed to watch all of the videos. This suggests that the students were motivated to learn from these videos over the weeks, spending longer times in the later weeks of the course as the content became more difficult and the videos were slightly longer.

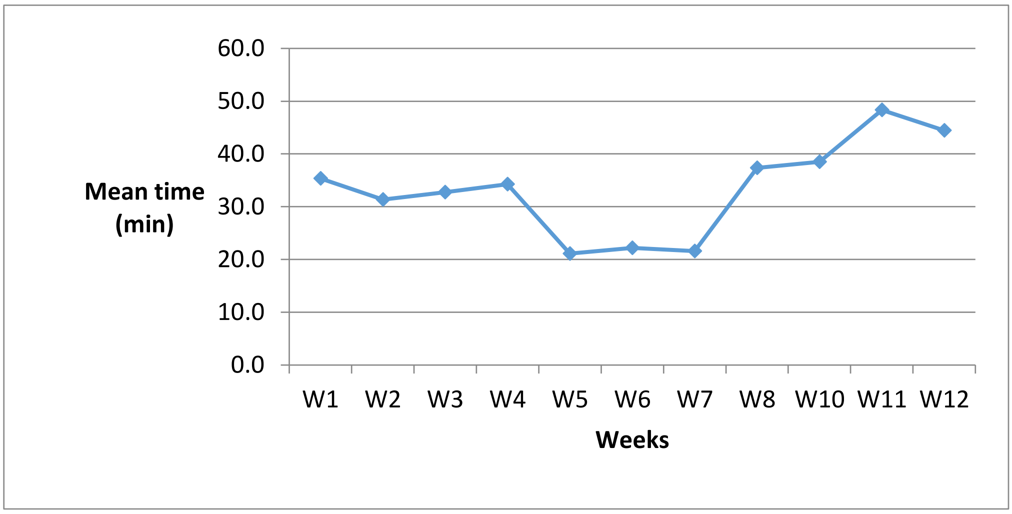

Figure 6. Mean time (minutes) spent on watching videos over the weeks.

An interesting issue was whether the reported time spent on pre-class tasks was related to student ability. Their Calculus II course grade in the previous semester was used as a predictor of the time spent; a negative but non-significant correlation was found (r = -.40, p = .143). The time spent was then used as a predictor of their Linear Algebra II course grade; again a similar, negative correlation was found (r = -.38, p = .167). These results suggest that stronger students could complete these pre-class tasks much quicker than the other students. For instance, Student X who obtained the highest overall scores for both courses reported spending about 600 minutes on pre-class tasks. At the end-of-course interview, he commented that he watched all the videos, found them easy to understand, and would prefer more questions to do, to "at least get to think".

Most of the students reported spending vastly different amounts of time on out-of-class activities. Student Y, who had the lowest overall scores for, reported spending 1003 minutes but found the workload acceptable because it was "structured, very organised .. Like you have week 1 to week 12 materials so you just study each week". During the interview, she mentioned that during the last few weeks when the deadlines from the all the courses were tight,

> I didn't get to watch all the videos. … Like the summary questions and see if I can look at the textbook instead of watching the videos.

Nevertheless, she liked the video lectures because she could pause and re-watch the videos and write her own notes.

Perceptions of pre-class tasks

The mean ratings of student perceptions of the pre-class tasks collected using the mid-semester and end-of-course surveys are given in Table 3.

Table 3.
Student ratings of pre-class tasks from mid-semester and end-of-course surveys (scale 1 – 6)

	Helpful		**Enjoyable**	
	Inter	End	Inter	End
Synopses	4.9	4.8	4.5	5.1
Videos	5.5	5.8	5.2	5.3
Pop quizzes in videos	4.7	4.9	4.7	5.0
Summary sheets	5.2	5.4	4.7	5.3
Worksheets: Problems and activities	5.7	5.9	5.3	5.3
Online quizzes	5.5	5.5	4.8	4.9

At these two stages of the course, the students consistently rated highly that these tasks helped them learn and they enjoyed completing them. Tackling the worksheet problems was most helpful as well as most enjoyable. At the slightly less favourable end, synopses (quite brief) and pop quizzes embedded in the videos (covering basic contents) were still rated quite highly but least helpful and enjoyable relative to the other tasks in the mid-semester survey, whereas online quizzes had the lowest rating in the end-of-course survey.

As the course progressed through the weeks, the students expressed slightly more positive perceptions towards some of these tasks. In particular, they appreciated more about the helpfulness of the videos (means: 5.5 to 5.8) and enjoyed more in completing the summary sheets (means: 4.7 to 5.3) and reading the synopses (means: 4.5 to 5.1). For example, Student Z, an average student based on Calculus II scores, changed her ratings for synopses from 2 (same for Helpful and Enjoyable) at mid-semester survey to 4 (same for Helpful and Enjoyable) at the end of the course. She explained her experience at the interview:

> No, I don't enjoy it. It's quite hard to understand the synopsis when you haven't finished the chapter …. It's only when you finished the chapter then you will understand "oh this means this this this". So I will actually only use it before test. But it's good for revision.

A plausible hypothesis to explore is that as the students gained more experience with these well-planned pre-class tasks, they appreciated how completing the tasks helped them master the content and some of them enjoyed the positive experiences more than they did in the earlier phase of the course.

Online quizzes were highly rated as helpful in the two surveys (mean: 5.5). As explained above, student scores in these quizzes were included in a small "effort" grade for the course, and this might explain why all the students submitted almost all the online quizzes. Figure 7 shows a plot of the mean percentage scores (score/maximum weekly score) on these online quizzes for the 11 weeks. The scores fluctuated over the weeks, reflecting the difficulty levels of the quiz items. However, the levels of the scores suggest that the students had learned the basic contents covered in the videos.

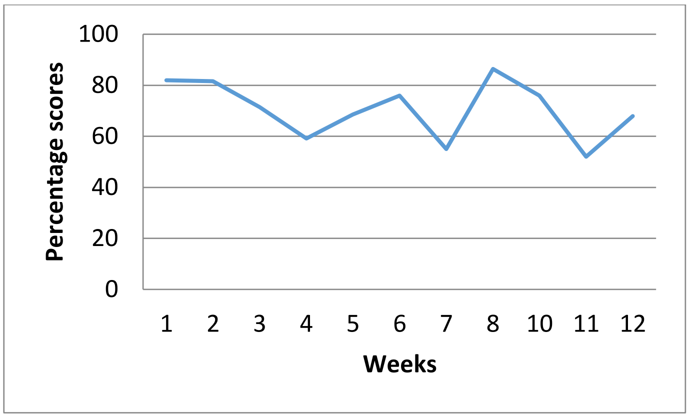

Figure 7. A plot of weekly percentage scores on online quizzes.

Student motivation was quite strong since all the students reported watching almost all the videos, missing only one or two videos in the last two weeks of the course. However, for any particular week, most of them watched almost all the videos only once. Only 3 students wrote in the mid-semester and end-of-course surveys that they re-watched some of the videos. The majority who did not re-watch the videos noted that, after the initial viewing, it was faster to refer to their own notes or the textbook. Almost all of them mentioned that the textbook was very useful, and they did not have to use other online resources.

Influences of previous performance on perceptions

The subsection entitled "Time spent on pre-class tasks" shows a weak relationship between students' performance in a previous course (Calculus II) and self-reported time spent on watching the videos. This subsection explores whether a similar relationship exists between previous performance and perceptions of the various pre-class tasks. The correlations of Calculus II course grade with perceptions are shown in Table 4. All but one of these correlations were statistically non-significant. However, the two larger negative correlations suggest that students strong in Calculus II found the synopses and summary sheets less helpful than the other students. On the other hand, the stronger students seemed to enjoy watching the videos and completing the worksheets and online quizzes more than the others.

Table 4.
*Correlations of Calculus II course grade with perceptions of pre-class tasks. (** p < .0)05)*

	Helpful		**Enjoyable**	
	Inter	End	Inter	End
Synopses	0.026	-0.278	0.083	0.293
Videos	0.251	0.285	0.340	0.305
Pop quizzes in videos	0.128	0.287	0.224	0.260
Summary sheets	-0.116	-0.228	0.164	0.193
Worksheets: Problems and activities	0.348	-0.050	.684**	0.369
Online quizzes	0.182	0.214	0.103	0.392

A similar pattern was obtained when these perceptions were correlated with Linear Algebra II course grade. Thus, these perceptions were very weak predictors of final mathematics achievement. A possible explanation of the low correlations was that these perception scores clustered at 5 or 6, and for a small sample (n = 15), this restriction of range might have reduced the correlations among the variables. In contrast, the strongest predictor for Linear Algebra II course grade was Calculus II course grade (r = .796, p < .001). This is consistent with the general finding about school learning that prior ability has much stronger effects than attitudes toward content domains on academic outcomes, as shown by the list of effects produced by Hattie (2015).

Gender differences in achievement and perceptions

There were 9 female and 6 male students in the sample. The female students were slightly weaker than the male students in Calculus II (means: 74 vs. 84), but the difference disappeared in the Linear Algebra II course grade (means: 83 vs. 84).

The male students spent about 15% less time on watching the videos and 14% less on completing the supplementary pre-class tasks. The differences in perception ratings between females and males were very small, ranging from -0.28 (female students rated pop quizzes more helpful at mid-semester survey) to 1.06 (male students rated synopses more helpful at mid-semester survey). Thus, these pre-class tasks did not favour any gender group, as suggested by how the students rated these tasks on helpfulness and enjoyment.

Discussion and Conclusion

The findings of this study may not be generalisable because it was a small one, it involved students who had strong mathematical backgrounds, and it did not include comparison groups. Furthermore, in this study, the students had prior experience of a flipped classroom, having attended a flipped Calculus II course in the preceding semester. Nevertheless, the following points may provide ideas for further practice and directions for future research about flipped learning.

The undergraduate mathematics course (Linear Algebra II) was delivered under flipped learning for one whole semester lasting 12 weeks. Engaging students over the whole semester, instead of just one lecture or a small section of a course, in flipped learning provides ample opportunities for them to meet the challenges of this new learning model and to develop the “culture” of “preparation required before attending class” (Novak et al., 2017, p. 656). Although students often express favourable perceptions of a new learning model because of novel effect, these perceptions may change under extended exposure. Our students consistently rated very highly their flipped experiences in terms of enjoyment and helpfulness throughout the semester-long course. At the end of the course, four students wrote about reducing the contents and lectures, one suggested using video-sharing platforms that work on all internet browsers, and one offered an insightful observation about self-learning:

> linkage between topics could be clear[er] as it may feel disjointed. Then again, own discovering of the links between topics may result in better retention of the information.

Further research can study the long-term perceptions and impacts of flipped learning, including the inculcation of fruitful learning strategies, such as be prepared for upcoming classes, writing own notes, engaging in active in-class discussions, and using resources to prepare for the final examination.

A plausible explanation for the students' consistently positive perceptions in this project relates to the well-designed structure of the flipped mode. The pre-class tasks catered to the different ways students from diverse mathematics backgrounds could learn mathematics, such as providing clear explanations in the videos, offering immediate feedback on pop quizzes and online quizzes and highlighting key content through synopses and summary sheets. Flipped learning must involve more than simply asking students to watch pre-recorded video lectures. Even though the instructor in this project spent 120 hours recording and editing the video lectures, this demand was a one-off that will not need to be repeated for subsequent runs of the course. The time he spent designing and implementing the three types of learning activities was also not burdensome, partly because he could convert previously used materials into the flipped mode, and partly because of his past experience of teaching flipped courses. Another advantage of flipped learning is that the materials can be used again in subsequent offerings, with revisions based on evidence collected from the students.

From the perspective of the students, our students reported spending, on average, slightly more than one hour per week to complete the pre-class tasks. Most of them found this additional out-of-class study load "just right", and the tasks embedded in this "acceptable" load were perceived to be very helpful and enjoyable. Thus, a fine balance of the benefits of self-directed learning and demands of time and effort on the students was obtained in our course. In the literature, one factor that might prevent tertiary instructors from experimenting with flipped learning is the extra workload on instructors and students. Our study shows that this additional demand is both manageable and beneficial, especially after repeated trials. Instructors from different institutes and teaching students from different disciplines might encounter challenges different from ours, and sharing their diverse experiences would help to the spread of flipped learning in tertiary institutes.

Education research has consistently found that prior cognitive achievement is a stronger predictor of academic performance than current perceptions, attitudes, or feelings toward the course. Our study also found that almost all the measures of perceptions had very small correlations with the final course grade. This should not imply that perceptions are not relevant to the successful flipped learning. Indeed, as noted earlier, students must be convinced of the needs to learn mathematics in non-traditional ways, such as flipped learning, and this factor must be carefully managed and assessed, for examples, using surveys and interviews.

As preservice teachers, the students were aware that the Education Ministry was mandating the use of flipped pedagogy in future K-12 classrooms. The multiple flipped experiences which the students had had, coupled with the expectations of the Education Ministry, might have some influence on their perceptions of the flipped activities, especially if they felt the need to support directives of the government. Indeed, some students did mention during the interviews that participating in this study might help them in implementing flipped learning as classroom teachers in the future.

This study found vast individual differences in the times spent on pre-class tasks, but they still reported consistently that these tasks were helpful and enjoyable. One implication is that students benefit from working in their own ways on different types of learning activities, and this variety supports self-learning of basic mathematics in preparation for class discussion. A slight improvement in the already favourable perceptions was found from mid-semester to the end of the course. This was a strong indication that long-term engagement with carefully planned learning activities can bring about more positive perceptions.

We believe that having students complete a full course under flipped learning might be especially relevant for our cohort of students because they were pre-service teachers who might be called upon to implement flipped learning for their future mathematics lessons. Together with this Linear Algebra II course, our students had completed three flipped mathematics courses, but this was the first course that required them to provide feedback and reflect on their flipped learning experiences. The combination of direct experiences and reflection would hopefully prepare them to empathise more with the needs and feelings of their future school students under this learning model. For example, they may be able to transfer how they had overcome the challenges of mastering new mathematics under flipped learning to the school situations. In this study, we did not ask about their experiences in terms of how they might influence their future teaching, and this is a potential area for further research on which we plan to pursue.

As noted above, our students entered the course with adequate to strong mathematical backgrounds. Future research can investigate flipped activities that work well for students of different mathematics backgrounds and from different disciplines, such as teacher education, STEM, economics, and the humanities as the literature has not reported impacts and perceptions of flipped mathematics courses on these diverse groups of students.

Acknowledgement

This work was funded by the Office of Education Research, National Institute of Education, Nanyang Technological University under Grant [OER 12/17 NWL; Flipped Linear Algebra and Calculus for Pre-Service Teachers (FLACPT)]

References

Ausubel, D. P. (1968). *Educational psychology: A cognitive view*. New York, NY: Holt, Rinehart and Winston.

Bishop, J. L., & Verleger, M. A. (2013). The flipped classroom: A survey of the research. *American Society for Engineering Education (ASEE) National Conference Proceedings*, *23*(1200), 1–18.

Brame, C. (2013). *Flipping the classroom. Center for Teaching, Vanderbilt University*. Retrieved from http://cft.vanderbilt.edu/guides-sub-pages/flipping-the-classroom/

Brewley, D. N., Boindala, P. S., & Sinclair, J. L. (2017). Ideation to execution: Flipping an undergraduate pre-Calculus course to create significant learning experiences. *IGI Global, Flipped instruction: Breakthroughs in research and practice*, 338–354.

Buch, G. R., & Warren, C. B. (2017). The flipped classroom: Implementing technology to aid in college mathematics student's success. *Contemporary Issues in Education Research*, *10*(2), 109–115.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education, 8th edition*. New York: Routledge.

Fulton, K. P. (2014). *Time for learning: Top 10 reasons why flipping the classroom can change education.* Thousand Oaks, CA: Corwin.

Hamdan, N., McKnight, P., McKnight, K., & Arfstrom, K. M. (2013). *A review of flipped learning. Flipped Learning Network.* Retrieved from https://flippedlearning.org/wp-content/uploads/2016/07/LitReview_FlippedLearning.pdf

Hattie, J. (2015). The applicability of visible learning to higher education. *Scholarship of Teaching and Learning in Psychology, 1*(1), 79–91.

Heng, S. K. (2013). *Student-centric, values-driven education: A broad and deep foundation for a lifelong journey*. Retrieved from https://www.moe.gov.sg/news/speeches/keynote-address-by-mr-heng-swee-keat--minister-for-education--at-the-ministry-of-education-work-plan-seminar-2013--on-wednesday--25-september-2013-at-915am-at-ngee-ann-polytechnic-convention-centre

Herreid, C. F., & Schiller, N. A. (2013). Case studies and the flipped classroom. *J Coll Sci Teach*, *42*(5), 62–66.

Jungić, V., Kaur, H., Mulholland, J., & Xin, C. (2015). On flipping the classroom in large first year calculus courses. *International Journal of Mathematical Education in Science and Technology, 46*(4), 508–520.

Kolb, D. A. (1984). *Experiential learning: Experience as the source of learning and development*. Englewood Cliffs, NJ: Prentice-Hall.

Lo, C. K., & Hew, K. F. (2017). Using “first principles of instruction” to design mathematics flipped classroom for underperforming students. *International Journal of Learning and Teaching, 3*(2), 82–89.

Lo, C. K., Hew, K. F., & Chen, G. W. (2017). Toward a set of design principles for mathematics flipped classrooms: A synthesis of research in mathematics education. *Educational Research Review, 22*, 50–73.

Love, B., Hodge, A., Grandgenett, N., & Swift, A. W. (2014). Student learning and perceptions in a flipped linear algebra course. *International Journal of Mathematical Education in Science and Technology*, *45*(3),317–324.

McGivney-Burelle, J., & Xue, F. (2013). Flipping calculus. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 23*(5), 477–486.

Mueller, P. A., & Oppenheimer, D. M. (2014). The pen is mightier than the keyboard: Advantages of longhand over laptop note taking. *Psychological Science, 29*(9), 1565–1568.

Novak, J., Kensington-Miller, B., & Evans, T. (2017). Flip or flop? Students’ perspectives of a flipped lecture in mathematics. *International Journal of Mathematical Education in Science and Technology*, *48*(5), 647–658.

O’Flaherty, J., & Phillips, C. (2015). The use of flipped classrooms in higher education: A scoping review. *The Internet and Higher Education, 25*, 85–95.

Sandoval, W. A. (2013). Educational design research in the 21st century. *Handbook of Design in Education Technology*, 388–396. Retrieved from https://www.researchgate.net/publication/259694116_Educational design_research_in_the_21st_century

Talbert, R. (2014). Inverting the linear algebra classroom. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 24*(5), 361–374.

Vygotsky, L. S. (1978). *Mind and society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.

Authors
Ng Wee Leng (corresponding author**:** weeleng.ng@nie.edu.sg),
Teo Kok Ming (kokming.teo@nie.edu.sg),
Wong Khoon Yoong (khoon.y.wong@gmail.com),
Kwan Kang Ling (kangling.kwan@nie.edu.sg)
National Institute of Education, Nanyang Technological University, 1 Nanyang Walk, Singapore 637616.