Excellence in mathematics education: Multiple confluences

Ban Heng Choy
National Institute of Education, Nanyang Technological University
<banheng.choy@nie.edu.sg>

Excellence in mathematics education is often linked with high performance in international achievement tests such as TIMSS. In this short paper, I broaden the notion of excellence by considering how the different aspects of mathematics education come together instead of only focusing on what these aspects are. Using confluence as a metaphor to describe excellence, I examine Singapore's excellence in mathematics education by showing how the "big things" of education such as societal expectations, policy formulation and implementation, and how the "small things" of classroom practices—scheme of work, tasks (especially typical problems), and examinations—flow together towards the same vision of ambitious teaching articulated by the Singapore Mathematics Curriculum Framework.

Excellence—from the Latin word *excellere*, meaning surpass—is multi-faceted. In mathematics education, excellence is often associated with high performance in international achievement tests such as TIMSS and PISA. Achieving top performance in these tests has been likened to obtaining medals in the "Olympics" of education (Leung, 2014) and declining performance over the years in these achievement tests has triggered calls in various countries to reform mathematics education (Gerritsen, 2021). However, I believe most mathematics educators would see performance in these international benchmark tests as a very narrow interpretation of excellence. Examining the notion of excellence in mathematics education may require us to investigate a myriad of educational components operating together in diverse contexts. In this paper, I use the metaphor of *confluences*—where two or more rivers, each with their own flow and paths, meet to form a bigger river—to characterise excellence. I view the notion of excellence in mathematics education as the coming together or flowing together of different educational aspects at a single purpose: to *provide all* our students with *quality mathematical learning experiences* so that they are supported to *achieve* the *desired learning outcomes*.

Having high expectations and providing strong support to all students relates to the notion of equity, a necessary ingredient for achieving excellence in mathematics education (NCTM, 2000). There are two aspects of confluences here. First, there is a directed flow of policies, initiatives, and practices towards the same goal of providing high quality learning experiences for all. Second, there is a coming together of different understandings about the main elements of an excellent mathematics education, namely curriculum, teaching, learning, assessment, and technology. The idea is not to have a single understanding about what or how to teach. Rather, the aim is to achieve a balance point in which our different understandings about mathematics teaching and learning are compatible. In practical terms, this means that the educational policies, initiatives, and practices are in sync with the purpose of providing high quality learning experiences for all. Hence, finding the balance point and getting the policies, initiatives, and practices to "flow" in sync are the key levers to excellence. Seeing excellence in mathematics education as confluences therefore positions excellence as a journey and not merely a destination. In the rest of the paper, I will illustrate this idea of seeing excellence as confluences through the Singapore experience in mathematics education.

2021. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 53-56. Singapore: MERGA.

Confluences of 'Big Things'

I begin by looking at the confluences of key elements of an excellent mathematics education. To that end, the principles for school mathematics, as proposed by the National Council of Teachers of Mathematics (NCTM), serves as a good reference point. According to NCTM (2000), the following six principles are fundamental to achieving excellence in mathematics education: equity, curriculum, teaching, learning, assessment, and technology (NCTM, 2000, pp. 12–24). On the surface, it is hard to imagine why anyone would have issues with these principles but the "math wars" in the US suggests otherwise. On one side, traditional mathematics advocates emphasise the importance of mastering procedures (back to basics) and use of more teacher-directed teaching approaches such as direct instruction; on the other side, reform mathematics advocates emphasise the importance of developing conceptual understanding via the use of more student-centric approaches such as inquiry-based teaching. These "wars" are not unique to the US and different versions of these wars are still "fought" in various countries (Chernoff, 2019; Yoon et al., 2021). I find these wars unproductive because the polarising language used in these discourses promotes a "winner takes all" notion of what excellence in mathematics education means.

Avoiding these extreme positions, excellence in mathematics education can be characterised by the confluences of societal expectations, policy formulation, and implementation. In other words, the actions of the policy makers, school leaders, teachers, students, parents, and mathematics educators should flow together towards a clearly articulated vision of mathematics education. Flowing together towards a common vision does not necessarily mean having a one-size-fits-all approach to teaching and learning. Rather, the idea is that different policies, initiatives, and practices, which may differ in their epistemological foundations, are directed at achieving the same vision. Such a notion allows for a balancing of different pedagogical and curricular positions. Singapore, widely acknowledged for its excellence in mathematics education, is an example of this confluence.

In Singapore, we place a high premium on education and there is a high expectation for every child to do their best in education. All schools are well-funded and there is a high expectation for the professionalism of teachers and their quality of teaching. The Ministry of Education in Singapore, the governing body responsible for policy formulation and implementation, are largely made up of teachers. There is one teacher training institute responsible for pre-service teacher education to ensure consistently high-quality teacher education. All these environmental factors come together to lay the groundwork for Singapore's excellence in mathematics education.

Singapore's mathematics education and assessment, from primary school to pre-university, is guided by the Singapore Mathematics Curriculum Framework (SMCF) since 1990. This framework focuses on developing students' competencies in mathematical problem solving, supported by five-interrelated components (Ministry of Education-Singapore, 2018): understanding concepts, proficiency in skills, competencies in processes, positive attitudes for mathematics, and metacognition (p. 10). It is interesting to note that most, if not all, of Singapore's curricular policies and initiatives, including the SMCF, take ideas from all over the world to be adapted to the Singapore's context. Perhaps, it is Singapore's pragmatic approach that has enabled these different ideas to come together as a coherent curricular intent (Tay et al., 2019).

As detailed by Lee et al. (2019), the SMCF guides how different national policies such as National Education, ICT Masterplan, and more recently, 21st Century Competencies are implemented through the intended mathematics curriculum. Changes in policies are appropriately integrated within the mathematics curriculum while keeping an eye on the goals articulated by the framework. Hence, changes to the national curriculum, pedagogical approaches, assessment emphases, textbooks, curricular materials, and even school-based curricular innovations are all introduced in reference to this framework. In addition, communication on these changes is carefully orchestrated to ensure consistent and coherent messaging and schools have some autonomy to implement these ideas in different ways. This ensures that the curriculum goes beyond a collection of activities and initiatives to a more connected and coherent focus on mathematics and its implementation, which may be uneven at times, is moving in the same direction. These confluences of different policies, initiatives, and practices at the ambitious goals of mathematics teaching have improved the state of Singapore's mathematics education over the years.

Confluences of 'Small Things'

Despite the seemingly eclectic mesh of ideas for our intended curriculum, one of the keys to Singapore's excellence in mathematics education lies in the recognition that effective teaching can take a variety of forms (Kilpatrick et al., 2001). This is evident from how mathematics teachers *comprehend* and *transform* the intended curriculum into instruction (Shulman, 1987). Each school interprets the curriculum documents and translates the intended curriculum into implementable schemes of work, detailing the selection and sequencing of content as well as the pedagogical approaches tailored to their students.

Singapore teachers use a variety of teacher-centric and student-centric approaches in their teaching while juggling the balance between developing procedural fluency and conceptual understanding (Leong & Kaur, 2019). For example, the prevalent use of typical problems or textbook-type questions in mathematics classroom in Singapore, particularly how these problems are selected, adapted, and implemented deserves more attention (Cheng et al., 2021; Choy & Dindyal, 2018, 2021). In particular, Choy and Dindyal (2021) described how a competent secondary school teacher in Singapore noticed the affordances of typical problems and orchestrated a productive discussion around them, similar to the five practices proposed by Smith and Stein (2011). While Smith and Stein (2011) highlights the importance of using a rich task to orchestrate such discussions, Choy and Dindyal highlights the possibility of using typical problems for mathematically productive discussions.

Similarly, Choy (2020) described how a beginning primary mathematics teacher orchestrated a discussion around the seemingly simple question: 0.8×4. These examples amongst others (see Cheng et al., 2021) suggest there is something interesting going on at the classroom level. These teachers' practices cannot be simply classified as traditional teaching or reform-based teaching because these labels do not capture the complexity of their practices (Leong & Kaur, 2019). Instead, what these teachers have done is to create high-quality mathematical learning experiences for their students in ways that honour both conceptual and procedural fluency (Choy & Dindyal, 2021). More importantly, these practices are not unusual in Singapore. Based on a large-scale study on the enactment of the Singapore mathematics curriculum (Kaur et al., 2019), the researchers highlight that there is a prevalent and skilful use of such problems both for mastery and concept development, with many of these classrooms said to be mathematically productive.

This is despite the commonly held perception that our mathematics education is predominantly focused on high-stake examinations. What is often neglected is that these examinations do not simply test students on their procedural fluency, but they are designed to assess whether students understand and apply mathematical concepts to different problems in different contexts. Hence, teachers tend to maintain a strategic approach to teaching mathematics, balancing the need for conceptual and procedural fluency as stipulated by the SMCF.

In this short paper, I have tried to paint a landscape of Singapore mathematics education by showing how the “big things” of education, such as societal expectations, policy formulation and implementation, and how the “small things” of classroom practices—scheme of work, tasks (especially typical problems), and examinations—flow together towards the same vision of ambitious teaching articulated by the SMCF. The picture is one of many different rivers, both big and small, coming together at different points to flow towards the sea, which forms part of the larger water cycle. It is not so much the features of mathematics education that makes it excellent. Rather, it is the confluences of these big and small pieces of mathematics education that generate the supportive environment to empower teachers in their work to enhance students’ learning experiences and achievements.

References

Cheng, L. P., Leong, Y. H., & Toh, W. Y. K. (2021). Singapore Secondary School Mathematics Teachers’ Selection and Modification of Instructional Materials for Classroom Use. In B. Kaur & Y. H. Leong (Eds.), *Mathematics Instructional Practices in Singapore Secondary Schools* (pp. 205-230). Singapore: Springer Singapore.

Chernoff, E. J. (2019). The Canadian Math Wars. *Canadian Journal of Science, Mathematics and Technology Education, 19*(1), 73-76. https://doi.org/10.1007/s42330-018-0037-9

Choy, B. H. (2020). Where to put the decimal point? Noticing opportunities to learn through typical problems. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Interim Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (pp. 107 - 115). Khon Kaen, Thailand: PME.

Choy, B. H., & Dindyal, J. (2018). An approach to teach with variations: using typical problems. *Avances de Investigación en Educación Matemática*(13), 21-38. Retrieved from http://www.aiem.es/index.php/aiem/article/view/227

Choy, B. H., & Dindyal, J. (2021). Productive teacher noticing and affordances of typical problems. *ZDM – Mathematics Education, 53*(1), 195-213. https://doi.org/10.1007/s11858-020-01203-4

Gerritsen, J. (2021). Royal Society experts to inform ‘refresh’ of maths teaching. Retrieved from https://www.rnz.co.nz/news/national/435667/royal-society-experts-to-inform-refresh-of-maths-teaching

Kaur, B., Toh, T. L., Lee, N. H., Leong, Y. H., Cheng, L. P., Ng, K. E. D., Yeo, K. K. J., Yeo, B. W. J., Wong, L. F., Tong, C. L., Toh, W. Y. K., & Safii, L. (2019). *Twelve Questions on Mathematics Teaching*. Singapore: National Institute of Education, Nanyang Technological University.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Lee, N. H., Ng, W. L., & Lim, L. G. P. (2019). The Intended School Mathematics Curriculum. In T. L. Toh, B. Kaur, & E. G. Tay (Eds.), *Mathematics Education in Singapore* (pp. 35-53). Singapore: Springer Singapore.

Leong, Y. H., & Kaur, B. (2019). The Enacted School Mathematics Curriculum. In T. L. Toh, B. Kaur, & E. G. Tay (Eds.), *Mathematics Education in Singapore* (pp. 55-65). Singapore: Springer Singapore.

Leung, F. K. S. (2014). What can and should we learn from international studies of mathematics achievement? *Mathematics Education Research Journal, 26*(3), 579-605. doi:10.1007/s13394-013-0109-0

Ministry of Education-Singapore. (2018). *2020 Secondary Mathematics Syllabus*. Singapore: Curriculum Planning and Development Division.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics Inc.

Tay, E. G., Toh, T. L., & Kaur, B. (2019). Surprising Singapore. In T. L. Toh, B. Kaur, & E. G. Tay (Eds.), *Mathematics Education in Singapore* (pp. 1-9). Singapore: Springer Singapore.

Yoon, H., Bae, Y., Lim, W., & Kwon, O. N. (2021). A story of the national calculus curriculum: how culture, research, and policy compete and compromise in shaping the calculus curriculum in South Korea. *ZDM – Mathematics Education*. https://doi.org/10.1007/s11858-020-01219-w