Title	Contours of self-efficacy across nested mathematical domains: A case of a Singapore student with a history of low performance in mathematics
Author(s)	Yew Hoong Leong

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Abstract

Self-efficacy is a subject of ongoing intense research in cognitive psychology. Studies within this tradition are focused on how this agentic aspect of human functioning fits within and contributes to a network of other sociocognitive functionalities. From a mathematics education perspective, we seek a theoretical re-framing of self-efficacy that accounts for and advances thinking in students' (lack of) learning of mathematics in the classroom. This study takes this perspective by starting with a student's actual experiences in switches of self-efficacy states. Through a case study of a student who has a profile that matches one with low mathematics self-efficacy, I examine the contours of self-efficacy across nested mathematical domains. This nuanced view provides an alternative to static presumptions of self-efficacy models common in research reports in this area, and is in keeping with the dynamic ebb and flow of actual classroom experiences.

Keywords Mathematics self-efficacy · Trigonometry · Low performance

Introduction

As part of a bigger project to study the instructional practices of Singapore mathematics teachers, I had the opportunity to observe up close some students' classroom experiences. I was particularly drawn to one student — henceforth referred to as Amalina — because she would express her disposition towards tasks in binary terms: "I can do it" or "I can't do it." These terms — or their equivalents — were used frequently throughout the post-lesson interview session with her. The use of these terms to describe the belief of her own ability to perform the tasks set before her reminded me of a construct commonly used in cognitive psychology: self-efficacy.

Interestingly, within the same lesson, on the tasks where she felt she "can do it," she displayed the characteristics of high self-efficacy: she persevered with the problems, she was unperturbed by her wrong answers but remained confident she "got it" when attempting subsequent similar questions, but on the tasks where she felt that she "can't do it," her response followed a textbook description of low self-efficacy: she gave up easily and quickly before trying to engage with the problem, and displayed signs of avoidance. I was intrigued at the mixed picture of the same person within the same lesson. What accounted for the frequent switches in her efficacy states? Was it a case of generally high efficacy state with occasional dips, or one where it was overall low but with occasional peaks? In this paper, through the examination of Amalina, I present a case study of a student's experiences with switches in her efficacy states, with a view of uncovering the contextual interaction of mathematical content, instruction, and student thinking surrounding the phenomena.

I first turn to some extant literature on self-efficacy. As the literature on self-efficacy is copious and multi-faceted, I begin with broad definitional and foundational insights from the seminal authors before narrowing the focus to *mathematics* self-efficacy and the issues that relate directly to the current study.

Self-efficacy

According to Bandura (2003), who is widely regarded as one who did seminal work in this field, "perceived self-efficacy refers to beliefs in one's capabilities to organise and execute the courses of action required to produce certain attainments" (p. 3). To underscore the significance of self-efficacy in shaping personal agency, he added, "Among the mechanisms of agency, *none is more central or pervasive* than beliefs of personal efficacy. Unless people believe they can produce desired effects by their actions, they have little incentive to act" (pp. 2–3, emphases added). In empirical studies, self-efficacy was found to mediate in a significant way in a number of important processes that determine educational success, including cognitive processes such as mental construction of outcome scenarios and metacognitive strategies (e.g., Hoffman & Spatariu, 2008; Krueger & Dickson, 1994; Schunk & Rice, 1991), and motivational processes such as causal attributions of retrospective events and perceived outcomes of future events (e.g., Fast et al., 2010; Ganley & Lubienski, 2016; Pajares, 1996; Pintrich & Schunk, 2002). That self-efficacy plays a key role in many of these critical processes related to successful learning indicates that we cannot simply disregard our students' personal efficacy when we plan classroom instruction.

From an education perspective, this concern to help students build positive self-efficacy leads to interest in another area of study in this domain: *sources* of self-efficacy. As Bandura's (1986) theoretical propositions in this area have become sites of continuing further research, including recent ones (e.g., Chan & Lam, 2010; Gao, 2020; Lau et al., 2018; Yıldız et al., 2019), I provide here a summary of the four principal sources of self-efficacy, as explicated in the updated version from Bandura

(2003): (i) Enactive mastery experiences. This refers to personal experiences of actually working on tasks. (ii) Vicarious experiences, which refer to the perceived experiences of the same task that is performed by others. (iii) Verbal persuasion. This refers to social recognition or evaluation of one's capabilities. (iv) Physiological and affective states. This refers to one's somatic state — the judgment of one's physiological condition and one's mood — as affecting one's capabilities. How these sources interact to form the individual's sense of personal efficacy is not straightforward. According to Bandura (2003), "There has been little research on how people process multi-dimensional efficacy information. … The factors that carry efficacy value vary in their informativeness and degree of interrelatedness" (p. 114). In short, it is hard to predict, for a given context and a given person, how the various factors interact to provide information — including the strength of this input — to affect the individual's self-efficacy. Nevertheless, it is widely accepted that these mentioned sources are significant to students' formation of self-efficacy.

Mathematics self-efficacy

As mathematics educators, our interest is more specifically in students' beliefs of their capabilities in *mathematics*. Moreover, it is unclear to us how a more general sense of self-efficacy — if there is any usefulness in such a construct — would help predict or explain one's sense of efficacy in mathematics. A casual observation of students (and introspection) would lead one to see that beliefs of capabilities can be quite different in different domains. In fact, Bandura (2012) warned that "people's beliefs in their capabilities vary across activity domains and situational conditions rather than manifest uniformly across tasks and contexts in the likeness of a general trait" (p. 13). This is similarly attested by other researchers within the field. In particular, Usher and Pajares (2008) stated that "it makes little sense to compare … the sources of general academic self-efficacy with students' mathematics-specific efficacy judgments" (p. 763).

There have indeed been efforts to assess students' mathematics self-efficacy, and there have been consistent reports of its strong prediction of mathematics achievement (e.g., Fast et al., 2010; Ganley & Lubienski, 2016; Pantziara, 2016; Zhu & Chiu, 2019). Numerous studies that follow a quantitative approach to measuring mathematics self-efficacy (e.g., Anderson & Betz, 2001; Britner & Pajares, 2006; Smith, 2001; Usher & Pajares, 2006a, b) focused on the correlational strengths of each of the sources to mathematics self-efficacy. The results that emerged from these studies, as synthesized by Usher and Pajares (2008), were that apart from establishing a strong influence of personal mastery experience to mathematics self-efficacy, the contributions from other sources — vicarious experiences, social persuasion, physiological, and affective states — were mixed across these studies. In reality, contextual factors vary and these can largely account for the different results reported across these studies.

Some researchers turn to qualitative methods to study the more nuanced contributions of each of these sources of self-efficacy, especially taking into consideration contextual and cultural factors. As an example, in Usher's (2009) study of eight Year 8 students through interviews, she confirmed that personal mastery experiences provided strong influence over the students' mathematics self-efficacy judgments, but it was "*authentic* mastery experiences" — a term coined by Pajares (2006) — that mattered more. That is, teachers should not pare down the cognitive demand of tasks to "fake" successful mastery for these students — they may interpret such moves as reinforcing of their low levels of performance and thus contribute to lowered confidence. On vicarious experiences, she also found that the influence on one's self-efficacy was not always straightforward; in one subject, the unsuccessful experiences of the significant member in his family — usually taken to be correlated to low self-efficacy in quantitative studies — were a spur to him to be more competitive in mathematics, clearly revealing that there were stronger factors at play in shaping his mathematics self-efficacy.

Self-efficacy across mathematical tasks

While the current literature — such as briefly surveyed in the preceding sections — provides much empirically supported findings in confirming the sources of mathematics self-efficacy, and the variations due to broad contextual and cultural factors, I am surprised that despite decades of research in this field, few of these studies specifically provided a helpful theoretical lens through which to explain the phenomenon of Amalina which I described in the introduction of this paper.

In particular, although many of the authors that I mentioned in the review even warned about the complex interplay of factors in shaping one's self-efficacy, and that contextual (including domain-specific) considerations are significant too, which would imply that personal efficacy is likely to vary in different situations — right down to different tasks in the case of Amalina! — it seems they nonetheless made implicit methodological assumptions of the *stability* of one's self-efficacy in a particular domain. For example, many studies about "mathematics self-efficacy" treat this construct as a fixed trait possessed by the individual as if one's efficacy is uniform across all types of mathematics tasks. Likewise, interpretations and educational implications were offered based on this assumption that an assessment of a student's mathematics self-efficacy — usually based on general statements of one's capabilities of doing mathematics, such as in Girnat (2018) and Gao (2020) — is generalizable to all instances when that student comes into contact with mathematics. This assumption is challenged by the case of Amalina.

Nevertheless, there are some researchers who acknowledge explicitly the changes in efficacy states even within the domain of mathematics. Zimmerman and Cleary (2006) mentioned these qualifiers of self-efficacy: level, generality, and strength. Level refers to the degree of complexity of tasks. This means that a student's sense of efficacy towards two mathematical tasks of different levels of complexity can be different. Generality refers to the degree of transferability of self-efficacy beliefs within or across domains. It addresses questions of whether personal efficacy in relation to a task is tied particularly to it or affects more generally — and the extent of its generality — to a broader set of tasks in which the former is representative. This aspect of efficacy is also emphasized by Lennon (2010). Strength refers to the degree of certainty in which to perform a task successfully. A weaker or stronger sense of efficacy may significantly influence one's resilience in the engagement of a task. Steel et al. (2017) provided empirical support for these constructs as they found that in students' personal efficacy judgments, they differentiate between levels of perceived difficulty as well as generality of domains.

I consider the work that is reported in this paper as continuing in this commitment of maintaining the more subtle differentiations in mathematics self-efficacy. In particular, I take the view that mathematics self-efficacy of a person can vary significantly across mathematics tasks (as shown in the case of Amalina) and at different "grain sizes": the "zoomed-in" picture of one's efficacy with respect to a particular task can differ from the "zoomed-out" view of the same person's efficacy towards the subject of mathematics as a whole. This grain size view is also shared by Tirosh et al. (2013, p. 311): "Self-efficacy beliefs are not only domain-specific (e.g., mathematics, history, science) and content-specific (e.g., within the domain of mathematics there is numeracy, patterns, geometry, etc.), but may well be task-specific (e.g., what is the child asked to do) and situation-specific (e.g., is the task implemented in class, outside, individually, in a group)." The contribution of this study is not merely in responding to the scarcity of research in this area; it attempts to go beyond showing the existence of finer qualifiers of mathematics self-efficacy to the investigation of how these various fine-grained portraits of efficacy interact with each other. It responds to these questions: How do we account for personal efficacy switches across mathematics tasks that appear similar? How do self-efficacy beliefs at different grain sizes relate to each other?

This study is intended to draw out the not-so-apparent contextual, mathematics-specific, and cognitive confluence of factors that would lead to each of Amalina's "states," as well as the switch between these states across different tasks and task-types, of personal efficacy.

Method

Context

At the point of data collection, Amalina's mathematics teacher, Meng Tin, was teaching the topic of Trigonometry at Year 9 Normal (Academic) level. In Singapore, mathematics taught at the secondary levels (Year 7 to Year 10/11) are offered to students at largely three bands depending on the assessed abilities of the students at the end of Year 6: Express, Normal (Academic), and Normal (Technical). The percentages of Singapore students studying mathematics at each of these bands are roughly 60, 25, and 15, respectively. According to Meng Tin, out of the 4 times in which she taught Normal (Academic) mathematics, Amalina's mathematics class was "the most challenging Normal (Academic) class I had [taught]" (pre-module interview of Meng Tin). She explained that, out of 23 students in the class, 8 of them were classified by her as under the "low progress" category. Amalina was included in this category. Her mathematics grade in the Year 6 National examination was "E." This represented a score below the 20th national percentile. Based on this profile, one can expect Amalina to face immense challenges with respect to her mathematics self-efficacy throughout her secondary school years.

Meng Tin taught the Trigonometry module over a number of lessons that span roughly 6 h in duration. The content coverage in brief was (i) introduction and definition of sine, cosine, and tangent within the context of right-angled triangles; (ii) solving problems involving right-angled triangles using a combination of sine, cosine, tangent; and (iii) application of these in "real-world" scenarios. The focus of inquiry for this study — content-wise — is trained on Amalina's experience with (ii) and (iii). These were also sections which Meng Tin identified as more challenging for the class.

Data

As the focus was on Amalina's personal sense of efficacy in mathematical tasks, the first data source was her verbal self-disclosure of how she felt towards mathematical tasks that were presented to her. These were extracted from a semi-structured interview with Amalina that lasted 30 min. This instrumental mode to derive subjects' sense of capabilities is prevalent in qualitative studies on self-efficacy (e.g., Gao, 2020; Usher, 2009). The interview began with a task (see Fig. 1) that was presented to her. It was chosen based on its similarity to the tasks that Meng Tin gave the students in Lesson 6 (which was the last lesson of the module) — henceforth known as the Lesson — immediately prior to the interview. The role of the interviewer during Amalina's task engagement was that of observer. She also encouraged Amalina to continue engagement with the task until it became clear that Amalina would not want to continue. The rationale for presenting recently taught tasks to Amalina are (a) I was interested in her personal efficacy towards specific mathematical tasks, not "general" mathematics efficacy. The requirement for this specificity means she has to be presented with an example of such a task, not merely descriptions of it; (b) as she initially engages (or disengages) with the task, we are able to interpret her stance towards it — for example, her immediate engagement reveals something of a positive efficacy towards it, while delays and body language can reveal avoidance and

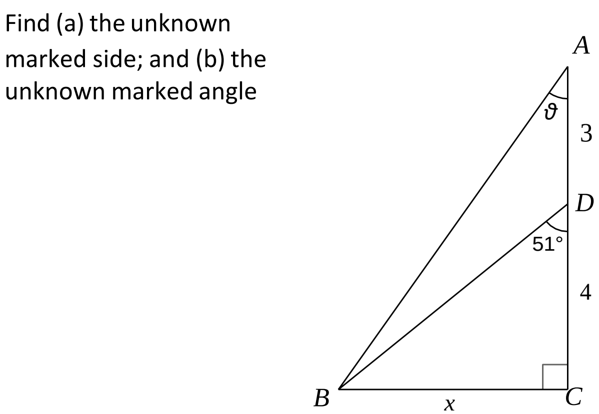


Fig. 1 The task given to Amalina at the beginning of the interview

hence signs of negative efficacy; (c) the recently taught task similarity allows us to capture the dynamics of switches in efficacy states (if any) that may reflect the instabilities of the corresponding mathematical ideas in her mind — a feature that this study set up to explore; (d) based on her response to the tasks, the interviewer can then follow-up with suitable prompts to elicit her beliefs.

After the task attempts, the interview questions to Amalina included the following: What are the things you learnt from today's lesson? What help you most in learning mathematics? What do you find challenging in today's lesson? How do you feel about today's lesson? Where is a high point in today's lesson? Where is a low point in today's lesson? Clearly, in line with the objective of examining contextual influences of her personal efficacy judgments, these questions were meant to draw out Amalina's description about how she *personally experiences* the mathematics class, rather than merely providing a description of events. Consistent to the use of semi-structured interviews, these questions were meant to prompt Amalina's line of thought; based on her initial responses, the interviewer drilled increasingly deeper to greater specificity. For example, on the question of what she learnt, Amalina initially referred vaguely to some notions related to Trigonometry. The interviewer followed up by asking Amalina to point out specific portions in the teacher-given notes (hereafter referred to simply as "notes") and that led her to identify more precisely the content points she learnt.

To situate Amalina's comments during the interview within the particular context of learning Trigonometry, I collected data of her work during the lesson prior to the interview through three video feeds trained on: (A) the things she wrote on the notes, (B) her and the two classmates seated beside her, and (C) Meng Tin. These combined video data sources helped us piece together and pinpoint the classroom references she made during the interview. As an example, during the interview, she made reference to an item in the notes which she struggled with; it was not a cursory mention — she went into the specifics her "confusion." (A) allowed us to see the exact points in the working of the task in which she paused — a proxy of uncertainty and struggle — and the errors she committed; (B) provided us with her actions, facial expressions, and gestures to confirm experiences of struggles; moreover, it showed the interactions with her neighboring classmates and the helps she obtained from them; (C) gave the teacher's instructional context around the task in which to interpret her classroom behavior.

I also obtained a set of all the materials that were distributed by the teacher to the class for this module, which included the notes, the "exit tasks" given at the end of each lesson to help students in their self-assessment, and homework assignments. For the notes, in Singapore, it is common for secondary mathematics teachers to design a full set of notes for a whole module prior to the in-class instruction (Leong et al., 2019; Leong et al., 2021). These notes usually contain structured content as well as spaces for students to write their working into written tasks. With the notes and the other distributed materials, I could make more accurate references to the trigonometric tasks done in class and the chronological development of the module intended by Meng Tin. I also had copies of Amalina's homework submissions for this module which I used to cross-reference her proficiency or deficiency on certain tasks.

Analysis

The first step in the analysis process is to look for "switches" of self-efficacy states because they are viewed as sites whereby the complex confluence of factors comes into play. This in-depth examination of the nuances underlying shifts in self-efficacy beliefs is of interest not just because it plugs a methodological gap in extant literature; it has the potential to uncover specific insights about the sensitivities of students' self-efficacy in a way that presents useful educational implications.

The initial search for these switches is in the primary data source of the interview with Amalina. Since she gave extensive descriptions of the trigonometric task she did during the interview as well as the tasks she was given in the lesson, the most obvious switch is the shift in the language she employed when expressing how she thought of the task (either before she engaged with it or during the engagement with it) — for example, between "I can do it" at one point and "I am confused" at another later point, or vice-versa. Apart from these direct verbal expressions of personal efficacy judgments, there were also more subtle cues of changes in her view towards tasks. Here, I am helped by the sources of self-efficacy reviewed in the earlier sections of this paper. For example, when Amalina made reference to "her friends" in the interview, she did so within the context of deriving confidence to make advance with the task she felt stuck with. Interpreted through "vicarious experiences" as a source of self-efficacy, this can be viewed as potential enhancement of her self-efficacy by virtue of seeing how "her friends" of comparable ability were able to positively engage with the same task.

These evidences of switches as mentioned by Amalina in the interview were checked against the other sources of data for confirmation. An illustration of how this checking was done: At the beginning of the interview, Amalina was given the task as shown in Fig. 1. She spent 5 min on Part (a) of the task and wrote out the solution in full. When she proceeded to Part (b) of the task, she looked at the question without writing anything down for about 1 min, and then gave up. The contrast in her disposition towards the two parts of the task pointed to a potential switch of states. I conjecture that the switch had to do with the difference in the nature of trigonometric content required in each part — the first was about finding length of sides; and the second was about finding angle which required the use of *inverse* trigonometric functions. She indeed mentioned at the later part of the interview that it was so.

At the end of this first step of the analysis process, I identified these switches (arranged chronologically):

(1) Switch Section I/Section II — this refers to the switch across the type of contents in these sections of the notes. The content in the former is about "find side," and content in the latter includes "find angle";
(2) Switch Item 11a/b — the label refers to the switch in relation to her attempt of Item 11 of the notes;
(3) Switch Item 14a/b — similarly, switch in her attempt of Item 14 of the notes;
(4) Switch post-lesson — this refers to her after-lesson personal efficacy with respect to trigonometric tasks engaged in the lesson.

Following Lampert (2001), I adopt "flexible frame sizes" when analyzing these switches instead of using conventional fixed units of analyses. A cursory look at these switches reveals that they cover regions of Amalina's experiences of different grain sizes, ranging from her engagement with one task to her impression of an entire lesson. Insisting on using a fixed frame size would miss out on the relevant contextual richness which contributed to efficacy switches in each case. The suitable level of "zoom" to examine the switch is dependent on fit of purpose rather than adherence to a pre-fixed frame. This "zoom" metaphor was extended by Leong et al. (2015) methodologically to provide a progressive widening of analytical lens: each of these switches is first analyzed separately; but subsequently, I widen the "zoom" to bring these analyses together into a coherent picture by taking into consideration the emerging common themes. This purpose-driven analytical approach will be made clearer as I discuss the findings in the next section.

Findings

I shall first report the analyses surrounding each of the switches in order before widening the lens to analyze them together at the end of this section. The analyses will uncover the contours of Amalina's mathematics self-efficacy. They are "contours" in a geographical metaphorical sense in that they cut across self-efficacy in the various terrains of grain size and contexts.

Switch Section I/Section II

Section I and Section II refer to the sections in the notes. Their written section titles are "Apply trigonometric ratios to find unknown sides of right-angled triangles" and "Apply trigonometric ratios to find unknown angles in right-angled triangles," respectively. Meng Tin devoted much of the previous three lessons on the coverage of Section I. Towards the second half of the previous lesson, she started on Section II and she continued with it in this lesson under consideration.

During the first part of the interview, Amalina was presented with a task (Fig. 1) that reflected the span of coverage across Section I and Section II — the first part of the task was on "find side" and second part was on "find angle." Amalina's working on the task is presented in Fig. 2. There was an obvious difference in the confidence with which she approached the two parts of the task. She readily attempted — and

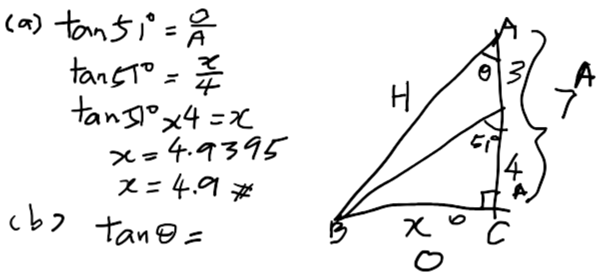


Fig. 2 Amalina's written working on the interview task

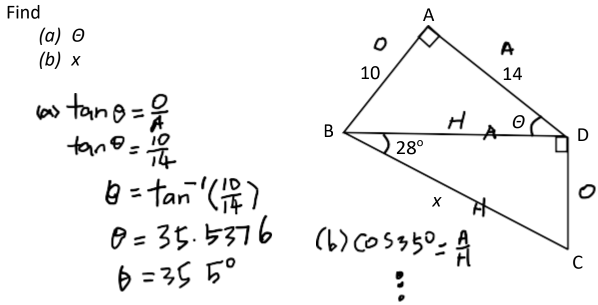
correctly — the first part on obtaining “x” but paused for about a minute for the second part and gave up after writing “tan θ = .” Her actual words during the interview were, “Actually I’m not sure how to do the angle. … I remember how to find x …. But then for the angle, I tend to forget.” There was a clear distinction in her personal efficacy across the divide of Section I (“find side”) and Section II (“find angle”).

What could be the reason behind the “switch” in her efficacy judgments across the two types of task? One obvious explanation is the amount of time she spent on both types of tasks: Up to that point in time, she had had significantly more classroom time on Section I than on Section II. Moreover, she completed homework tasks on Section I but not yet on Section II. She completed 12 similar “find side” tasks in the Section I homework and were completely correct for 8 of them. For the remaining 4 tasks, two of them involved rounding inaccuracies, one a misread of a given value, and the last has to do with mistakes in the identification of “adjacent side” and “hypotenuse” due likely to fixation to prototypes of right-angled triangle (which will be discussed in more detail in the analysis of the next switch). In other words, she was generally successful with the contents of Section I and this would have contributed to her sense of efficacy for “find side” tasks. This explanation is coherent with the findings as reported in the literature on mastery experiences being a strong influence on efficacy judgments.

But this does not mean that she had no sense of efficacy at all with Section II tasks during the lesson. Towards the end of the lesson, the class was given an “exit task.” It was Meng Tin’s practice to give students an exit task to attempt at the end of a lesson as a way to reinforce the content taught in the lessons; it was also meant to provide students an opportunity for self-assessment. Amalina’s solution for the “find θ” part of the task is shown in Fig. 3. [Her solution for Part (b) was also correct but as it is not relevant to the current analysis is removed.]

She exhibited no signs of avoidance of the task during the lesson. In fact, in her written solution, there was hardly a pause — she completed the solution in 64 s. She was so intent on getting it right that in the middle of it, she told her noisy neighboring classmates, “Can you keep quiet? I need to focus.” But what was hardly noticeable initially was that the first part of these 64 s — 14 s of it — was spent on looking intently at the notes of Section II. That her accessibility to her written work on the notes bolster her sense of efficacy for Section II tasks is supported by her comments during the interview: “I get mixed up with find x and y and find the angle. … *I need*

Fig. 3 Amalina’s written working on the exit task



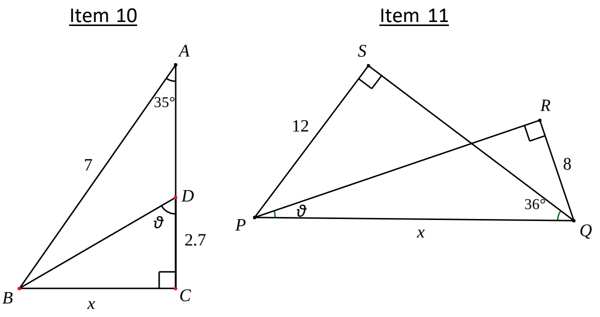
to refer to my notes" (emphases added). That her notes were not available to her when she attempted the interview task contributed to her lowered efficacy towards the second part of the task. The availability/unavailability of the notes explains the distinct difference in her approach towards "find angle" task between the exit task and the interview task. At a later part of the interview when Amalina was asked about resources that she found most useful in the learning of mathematics, she said, "For me, notes. 'Cause if I am doing the problem again, I will refer to the notes. When I refer to the notes, then the steps will—like—naturally come back to me again."

This is an interesting finding because there is hardly any literature that links teacher-given notes to students' self-efficacy. In the case of Amalina, the notes seemed to serve as a kind of buttress to prevent a collapse of self-efficacy (for Section II) until sufficient mastery experiences "take over" as a more permanent source of efficacy (as in Section I).

Switch Item 11a/b

An extract of Item 10 and Item 11 from the notes is shown in Fig. 4. This type of questions which involve composite triangles was also given as homework to students in the previous lessons under Section I. When discussing Item 10 and Item 11 with the class (time line 19.01–27.09), Meng Tin asked (19.08), [with her arm raised to signal that she expected students to respond by raising their hands] "How many of you managed to solve [Item] 10 on your own?" Amalina raised her hand. This was the first indication that she entered the discussion of Item 11 — which is very similar to Item 10 — with positive personal efficacy. Meng Tin then asked two students to come to the board to present their solutions for respectively. While they did so, Meng Tin went from table to table to check on students' work. At 21.57, she came to Amalina, took a look at her solution for Item 10, and said to her, "Very good. You manage to solve it on your own." There was a smile on Amalina's face. That was a boost to her self-efficacy. At 22.33, Amalina took a look at her classmate's solution of Item 10 on the board, nodded her head, and placed a tick against her own written solution of Item 10.

Fig. 4 Item 10 and Item 11 from the notes



In fact, the confidence she had in her ability to attempt this type of questions was so high that something unexpected took place in the lesson. But before I describe the surprising episode in class, I point out the context of surprise by referring to Amalina's view of rendering her mathematical workings public in class: "… [For] the whiteboard right, if I [present my work] on the whiteboard, it's just like – how to say – my classmates will see. Then if I get it wrong, then after that – paiseh [a Malay word for embarrassment]" (interview of Amalina). In other words, Amalina would not normally want to expose her mathematical work in the public domain when she was unsure of the solution for fear of public embarrassment. But this was exactly what she did in 25.50 during the lesson. The teacher was explaining the board's solution of Item 11(b) on "find angle." The critical step was $\sin \theta = QR\,/\,PQ$. Amalina had a different working, but she must be so positive about her steps that she overcame the fear of public embarrassment to ask the teacher publicly, "'cher, but I used tangent leh." What followed was a discourse between the teacher and Amalina in full view by the whole class. Meng Tin pointed out to her that her use of "tangent" was incorrect; she also went on briefly to recapitulate that "opposite side" was opposite to the "reference angle" and "hypotenuse" was the side opposite to the right angle. That Amalina's personal efficacy was affected by this in-class experience was revealed in her response to the interviewer after the lesson about the "low point" in the lesson. Amalina made specific reference to this episode in class. To confirm, the interviewer played a portion around 25.50 of the class video to show it to Amalina. She was visibly sad when she answered, "Yea." That this lowered self-efficacy persisted was substantiated by her response to the interview task that is reported in the preceding section on Switch Section I/Section II. This became even more conspicuous when I observe the close visual similarity between the interview task (Fig. 1) and Item 10 (Fig. 4): we would have expected that since Amalina was successful with Item 10 that she would also find no difficulty with the interview task. That it was not so is further evidence of her affected efficacy.

It is interesting that Amalina re-interpreted Item 11(b) experience through the dichotomous classification of "Section I find sides" (positive efficacy) versus "Section II find angles" (negative efficacy) — see the analysis of Switch Section I /Section II. But recall that Amalina's mistake with Item 11(b) was not with "find angles" per se, but a wrong identification of what constitutes "opposite side," "adjacent side," and "hypotenuse" with respect to a reference angle. That the orientation of the right-angled triangle (see Fig. 4) was unconventional might have thrown her off as she might have a mental prototype of adjacent and opposite sides as being roughly horizontal/vertical — thus predisposing her to consideration of "tangent" instead of "sine." This disposition is also known as the "prototype phenomenon" in the literature (e.g., Hershkowitz, 1990; Okazaki & Fujita, 2007). That this was likely the cognitive trace of her error is supported by a similar error made in one of her homework submission for Section I (see Fig. 5). However, instead of taking this analytical route to pinpoint her own error, she chose to attribute her low self-efficacy to this broad class of problems on "find angle." This way of attributing efficacy as a response to negative experience with a task will be a point I will return to later. For now, I conjecture that this (to us) simplistic dichotomous divide served a dual heuristic function efficacy-wise for her: It confirmed to her (as analyzed In the Section I/Section

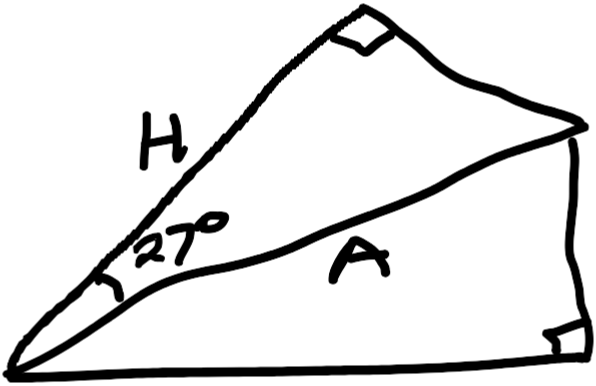


Fig. 5 Amalina's assignment of "Adjacent side" and "Hypotenuse" in her homework submission

II switch) that "I can't do Section II find angle"; but "I can do Section I find side" — and thus preserving as resilient positive efficacy for this reserve.

Amalina entered into the contextual domain of this specific type of composite triangles trigonometric problems with positive self-efficacy. It was buttressed by multiple sources of positive efficacy beliefs — the effort she put into the homework (mastery experiences), the compliment of her teacher (social persuasion), and the confirming answers on the board (vicarious experiences). Her positive efficacy was particularly evidenced by her willingness to put her work under scrutiny in the public domain. But this efficacy was adversely affected by a "switching experience" — the embarrassment of being shown her error publicly. But this switching experience did not result in a total collapse of her efficacy; rather, her re-conceptualization of her domain of mathematical experience allowed her to admit (and confirm) weakness and low efficacy with respect to some task-types while keeping resilient her efficacy for other task-types.

As the method in which I proceeded with the analytical process is amply illustrated in the details of the first two switches, I will be deliberately brief in my description in the last two switches.

Switch Item 14a/b

Item 14 (Fig. 6) was located within Section III of the notes entitled "Problems involving the use of trigonometric ratios." Amalina was intrigued by the contents of this section. It was likely that this positive disposition (affective state) would contribute to her sense of efficacy for this section. This was supported by her behavior in class towards Item 14. When Meng Tin asked her to present her solution on the board later — recall the efficacy issues in relation to public discourse as discussed in the earlier analysis of Switch Item 11a/b — she was smiling and ready to do so! Note that Item 14a is a "find side" task. Interpreted thus, it is clear that 14a is perceived as yet another element in her set of "find side" type of questions which she "can do" — and further reinforced through enactive mastery experience her capability for such tasks. She said, "For the distance [pointing to the requirement of 14a] *I can do* it

AB and *CD* are two buildings on the level ground *BD*. A wire *AC* connecting their tops is 30m long and makes an angle 200 with the horizontal. The height of building *CD* is 40m. Find

(a) the distance between the two buildings
(b) the height of *AB*

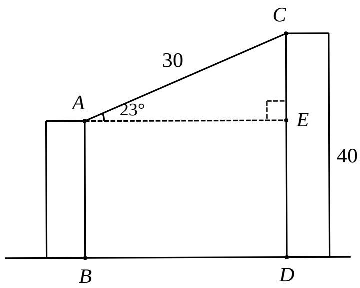


Fig. 6 Item 14 in the notes

because you only have [to use] the TOA CAH SOH[1]—just find the adjacent [side]" (interview with Amalina, emphases added). But the switch in her efficacy states took place when she turned to 14b. This is confirmed by Amalina's response to teacher Meng Tin, "a I can but b I cannot [with a sheepish smile]."

Amalina was unfamiliar with the analytical strategies for this problem. But she did not give up altogether. She turned to her neighboring classmates (whom she called "her friends") for help. She was able to follow the strategy outlined by "her friends" to perform the steps required to obtain the correct answer. She remembered the steps of this 14b solution so well that she was able to articulate them again during the interview. That she had owned the learning for herself can be seen from her successful attempt of the next task in the notes which is similar to Item 14.

Efficacy-wise, there was a dip when she proceeded to 14b. But it did not result in her abandonment of the task. Rather, she turned to "friends" as a way to buttress her efficacy for the task which in the end translated into successful completion of the task which in turn further strengthens her efficacy for such tasks. This may be interpreted as a case of vicarious experiences (of "her friends") as a source of her own efficacy. But it was not merely a case of "since my peers can do it, I can do it." Her friends' superior ability did not in itself demotivate her; rather, it drove her to learn from them to catch up in her efficacy-deficits.

Switch post-lesson

As mentioned in the earlier analysis of "Switch Section I/Section II," Amalina displayed clear signs of low efficacy during the first part of the interview when presented with a Section II-type "find angle" task. We might have easily concluded that she finished the lesson (and the interview) in a negative-efficacy state. This was not the case. Further in the interview, she repeatedly mentioned that she "feels good" about the lesson because she "can do" most of the tasks.

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[1] TOA CAH SOH is a commonly used mnemonic device among Singapore secondary mathematics teachers to help students relate the trigonometric function to the ratio of the sides — for example, TOA: Tangent of angle as the ratio of Opposite side to the Adjacent side. TOA CAH SOH is roughly translated as large-legged woman in a local dialect.

It is an indication of her overall efficacy judgment when reflecting on the entire lesson as a whole. When reflecting on the lesson, she appeared to glossed over those specific points — as reviewed in the earlier analyses of switches — of low sense of efficacy, even the point as recent as the start of the interview where she abandoned the second part of the task altogether. She chose to focus on those portions of the lesson where she "can do it." I surmise that were two factors contributing to this take on her personal efficacy: on the whole, the positive experiences of success with tasks overwhelmingly outweighed the negative momentary junctures; and even at junctures where she had low sense of efficacy, she was able to find help — from notes and from friends — to eventually overcome the difficulties with the tasks and thus ended the encounter with them on a positive stance.

It is also interesting to me that when Amalina described the lesson in positive terms that she also mentioned the teacher's name specifically in the middle of it. If we look back at the analysis of "Switch Item 11a/b," we will see that Meng Tin played a part in triggering the dip in Amalina's efficacy for "find angle" task. We might have expected that encounter to linger in her consciousness and result in a negative view towards the teacher. But that Amalina associated Meng Tin with positive experiences of self-efficacy indicates that her perception of Meng Tin's positive contribution to her learning of mathematics was not so easily negated by even a rather embarrassing encounter. Indeed, at other portions of the interview, Meng Tin was always mentioned as helpful to her learning. It is particularly telling that when asked by the interviewer on a "high point" in the lesson, she talked specifically about Meng Tin — how "cool" she was. It seems reasonable to assume that Meng Tin had established a culture in the classroom that supported the development of the students' mathematics self-efficacy.

Zoom-out to view the switches

From the foregoing analyses, there were at least three levels of nested mathematical contexts to which Amalina expressed efficacy judgments: task (e.g., Item 11), type of tasks (e.g., Section II "find angles" type), and content within a lesson (e.g., mathematical content taken in its entirety in the Lesson). I represent their relationships in the form of a Venn Diagram as shown in Fig. 7. Each of the tasks (Item 10, Item 11, and Item 14) are depicted as having substantial overlap with the respective types (Section I, Section II, and Section III) but not all completely subsumed within the type because there are content regions in some of these types that are not directly part of the content of the types.

In the case of Item 10, 10a is part of "find side" type but 10b is not; but in terms of efficacy links, it is only between the "find side" similarities between Item 10 and Section I, even though she was also successful with the "find angles" part of the task. That is, efficacy-wise, there is a bidirectional support between the "find side" part of Item 10 and the "find side" of Section I: Her overall positive efficacy for "find side" type of tasks provided a positive disposition in the engagement of Question 10; and her positive experience with Item 10 further strengthens her efficacy for "find side" type of tasks. Clearly, this picture is an over-simplification. In reality,

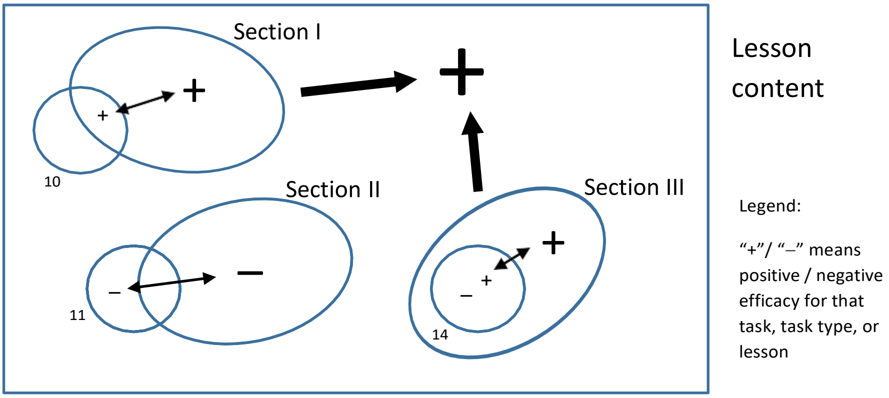

Fig. 7 A Venn Diagram summarizing the interactions of Amalina's efficacy judgments in various mathematical domains

there were numerous tasks like Item 10 adjoined to Section I. Through a series of such multiple task-type bidirectional reinforcements, the efficacy resilience of Section I type of tasks was built up, and protected — as in, subsequent failures at tasks would be interpreted in a way that would shield the efficacy of Section I from being affected negatively.

The relationship of Item 11 to Section II is similar to that of Item 10 to Section I, so I will only highlight the differences. In Item 11, the situation was about negative self-efficacy. The negative part of the task does not actually overlap with the contents of "find angle" in Section II but was interpreted by Amalina to reinforce her sense of low efficacy for this type of tasks. Again, the relationship between task and type is bidirectional: coming into Item 11, she had a rather fragile sense of efficacy for "find angle" tasks — one that has to be propped up by aids such as reference to notes; this has an effect on how she approached the task. The negative experience with the task was readily interpreted as confirmation for the low efficacy with the type. Interestingly, and as I will reflect later by way of educational implications, this association of negative reinforcement between task and type need not require a direct subset relationship, but merely simplistic subjective surface similarity — as in, "Item 11b is about finding angles, and since I can't do 11b, I can't do find angle."

There are yet some differences in the relationship between Item 14 and Section III. The content of Item 14 falls completely within the domain of "Applying trigonometric ratios to solve problems" in Section III. As shown in the analyses, Amalina had both positive and negative efficacy judgments on different parts of Item 14 but in the end, the positive efficacy links "trumped" the transient negative sentiment — which was quickly set aside with the help of "friends." That this might have largely to do with the "carry over" effect of resilient "find side" type efficacy from Section I should be acknowledged but I have resisted adding more arrows which will conflate Fig. 7.

When Amalina looked back at her experiences with the Lesson, there were sufficient "reservoirs" of positive efficacy recollections — while overlooking the negative ones — of types of trigonometric tasks to give her an overall sense of efficacy for the mathematical contents covered in the Lesson (hence the arrows from Section I and Section III to the superset).

Discussion

The findings of this study can be read — from one angle — as being in support of the extensive research results on the sources of efficacy reviewed in the earlier sections of this paper: Most obvious in Amalina's resilient personal efficacy with respect to Section I "find side" type of tasks, there were strong influences from (and continual contribution to) personal mastery experiences of this type of tasks; the lack of mastery experiences of Section II "find angle" type of tasks left her susceptible to doubts about her ability to complete such tasks. With respect to verbal persuasion, there were traces of positive influences in the encouragement culture engendered by Teacher Meng Tin. On vicarious experiences, the positive influence came from her friends who motivated her to engage with the task productively (cf., Item 14b); but she was also afraid of embarrassment when her weakness relative to her classmates were exposed in the public domain (cf., Item 11b). There were indications of affective states as shown by her facial expressions and articulated during the interview but they were not inserted into the analysis in this paper as I cannot ascertain directly the affect-efficacy link. Suffice to say, these affective impulses played a part too in the complex interplay of factors in the efficacy switches.

But this study also uncovered another "source" of positive self-efficacy: the notes. As such, I would like to surmise here a sharpened form of its potential role in buttressing positive efficacy. For Amalina, I think the set of notes provides a kind of intermediary that leads to personal mastery experiences as a more reliable source of efficacy. This is so because the set of notes contains her own written working of the tasks — and so it performs the function of a build-up of her personal mastery of the tasks. Nevertheless, the difference in her sense of efficacy with and without the availability of the notes shows that the notes are not equated to full personal mastery but a lead-up to it. But the notes also contain in-built mathematical content that were deliberately structured by Teacher Meng Tin. [It will take us too far a detour to establish this point here on intentional structuring of instructional materials by Singapore teachers. On this, the readers may refer to Leong et al. (2019); and Leong et al. (2021).] In this sense, the notes can also be seen as a record of and a link to the teacher's instructional work in class — and with it, all the positive helps and encouragements of the teacher. As such, the notes become a kind of remote conduit for ongoing verbal persuasion by the teacher. Moreover, by reading the notes after class, it provides a link to how she actually experienced learning during the lesson. According to Bandura (2003), this (re-)looking at oneself with a view of self-modeling is also a form of vicarious experience: "Seeing oneself perform successfully [in this case, as recorded in the notes] … strengthens beliefs in one's capabilities" (p. 94).

Thus, it seems the set of notes is not so much *another source* of efficacy than a concrete integration of multiple sources of efficacy. However, unlike the traditional sources of efficacy which are less controllable by individuals for direct help in the improvement of their personal efficacy (especially those who has low efficacy in the domain-in-focus to start with), the notes — as illustrated by Amalina — are readily accessible and provide a near-instant effect in propping up efficacy, at least to a degree that would bring a student to engage (rather than avoid) the task. This insight potentially opens up a whole new area of consideration for teachers. Instructional materials designed by teachers for students have traditionally been viewed mainly from the perspective of helping students in their content development. That these instructional materials can also potentially be useful for supporting students' efficacy may have implications in how teachers design these materials also for this purpose. There is a need for further research in this erstwhile unexplored area.

This study also highlights the contours of efficacy judgments across mathematical domains of different grain sizes. The efficacy links and subtleties at the task, type, and lesson levels as experienced by the student can provide corresponding sensitivities in teachers to exploit opportunities to improve efficacy at each of these levels.

At the task level, I think these considerations by way of educational implications are appropriate as arising from the findings: (a) A student's signs of negative efficacy towards a particular task should not be readily dismissed as insignificant in the "bigger scheme of things" — resisting simplistic conclusions such as, "so long as he can do most questions, it doesn't matter if he can't do THIS one." As shown in this study, this negative efficacy can contribute negatively to a broader class of questions. (b) A student's mistakes in a task should be pinpointed and ring-fenced so that the actual mathematical error be corrected before he wrongly generalizes his error to and weakens his efficacy of a broader class of questions (cf., the prototypical error in Item 11 was ascribed by Amalina as "I can't do find angles"). Concomitantly, the part of the student's workings in the same task that are correct should be complimented so as to help him protect the resilience of efficacy for that reserve of domain. (c) Verbal encouragements for a student's success even in a single task matters — it can help confirm within him positive mathematics efficacy beyond that task.

For type of tasks, (a) in the design of a type of tasks, there ought to be careful sensitivities to the entry and gradation of the tasks so that it is easier for most students to build positive efficacy through personal mastery of successful completion of the series of tasks. Abrupt "jumps" in the cognitive demand for the majority of students between tasks may accentuate the risk of lowered efficacy in this process. (b) The development of efficacy for a type of task is not necessarily tied to the chronological period when that particular type (e.g., Section I) is taught; other sections of the module (e.g., Section III) can contain elements that overlap content-wise with the type and hence serve to reinforce or improve efficacy for it. The implication in design of materials is that inter-section contents should contain deliberate and substantial overlaps to provide multiple opportunities for building efficacy for types of tasks. (c) Negative efficacy towards a type of task can be addressed not just at the type level but also incrementally at the task level (since positive efficacy for a task can have positive influence to the perceived type it belongs to). Such a task should be one where the first part of the task contains elements in which most students are first positive towards, with the intent that this positiveness can "carry over" to the later part of the task that belongs to the target type.

At lesson-level, (a) during lesson planning, teachers can consider a conscious inclusion of a substantial number of "reservoirs" of positive efficacy experiences for students. This does not mean that types of tasks (e.g., challenging tasks of high cognitive demand) that can potentially trigger negative efficacy cannot be included in a lesson, but their number and their level of challenge ought to be judiciously considered so that it does not "tip over" to engender an overall negative efficacy towards the mathematical contents of the lesson. (b) There are non-content aspects of the lesson that can contribute to the building of positive efficacy experiences for the lesson. In the case of Amalina, the helpful "friends" she could turn to readily for guidance and the encouraging teacher that set up a supportive classroom climate were important factors.

Though beyond the scope of this study, it seems natural as a thought experiment to extend beyond even the lesson level to broader domains, such as the levels of topic (e.g., Trigonometry at Year 9), module (e.g., Trigonometry at secondary levels), and the subject of mathematics itself. One can imagine that the Venn Diagram as shown in Fig. 7 may be enlarged to include additions of these respective supersets. This means that vicious cycles spinning among these states can be easily formed in students: "I can't do this type of tasks," "I can't do Trigonometry," "I can't do mathematics." The encouraging findings arising from this study is that teachers who detect such degenerative descents in students need not throw up their hands in resignation — as if nothing can be done once students formed low views of their mathematical capabilities. Rather, teachers can begin the intervention right at the task level and turn the tide into a virtuous cycle involving interaction among these states instead: "I can do this type of tasks," "I can do Trigonometry," "I can do mathematics." But it does mean that there is unlikely to be shortcuts in this process apart from deliberate cultivation of positive efficacy from lesson to lesson — so that these positive reservoirs of lesson experiences can in turn build up towards a higher-level efficacy, and so on.

Conclusion

This study uncovers some of the contours of mathematics self-efficacy that are under-reported in extant literature. That this is so is attributable to a methodological shift of focus — on switches of efficacy states instead of viewing them as largely "stable" — and a change of analytical method — of using suitable frame sizes across different data sources to analyze these switches instead of relying on standard interview-based units of analyses. As a result, we can view up close the zoomed-in details of the efficacy challenges against the backdrop of other broader contextual factors. To the student, efficacy-related experiences are not isolated but integrated: sources of self-efficacy does not merely influence one at a time; a task is not perceived merely as a task by itself; and (even embarrassing) encounters with teachers are set against the wider frame of past experiences with them.

Apart from methodological contributions, I suggest that this nuanced view of self-efficacy development also provides a useful lens for educators to approach the improvement of students' mathematics self-efficacy. We can frame the enterprise as one involving multiple entry points — through instructional materials, student–teacher interactions, student–student interactions — and at different levels of grain sizes — at task level, type level, and lesson level. Teachers can take on board this more holistic framing when designing curriculum that targets self-efficacy improvement as an ostensible goal of mathematics instruction.

Further research can explore extended regions of the contour beyond the scope of this study. One area is the development of self-efficacy across the boundary that separates school life and home life. Would, for example, the teacher-given notes continue to play a useful role as a buttress to self-efficacy even in a home that is not supportive of mathematics learning? Another area of study is in the development of mathematics personal efficacy across even longer periods of time and at even broader levels of module, topic, and mathematics as a subject. The ebb and flow of efficacy across these zoomed-out frames can provide further insights into the confluence of factors affecting the shifts and nuances in self-efficacy.

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Declarations

Ethical approval The research reported in this manuscript was cleared by the Ethics Committee of the Nanyang Technological University.

Informed consent We obtained informed consent from all the subjects in the study reported in the manuscript. In the case of minors, the consent was also obtained from their guardians.

Conflict of interest The author declares no competing interests.

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