METACOGNITIVE BEHAVIOURS OF PRIMARY 6 STUDENTS IN MATHEMATICAL PROBLEM SOLVING IN A PROBLEM-BASED LEARNING SETTING

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ABSTRACT

Mathematics classroom instruction is often characterized by approaches towards helping children gain mastery of skills and the development of concepts. Where metacognitive behaviours are concerned, little is known by practitioners how they are surfaced and what characterize these behaviours. This paper provides a scheme for checking the cognitive and metacognitive behaviours of a group of Primary 6 students. It examined excerpts of student-student and student-teacher interactions and discussed how the metacognitive behaviours were facilitated through a problem-based learning platform. The excerpts show evidence that children under such a problem-solving context do exercise different types of cognitive and metacognitive behaviours towards problem resolution.

INTRODUCTION

The Singapore Mathematics Curriculum recognizes metacognition to be one of the essential components to successful problem solving (MOE, 2007). This paper focuses on the metacognitive aspects of a group of Primary 6 students solving a complex authentic problem which required them to work in small collaborative groups alongside the scaffolding provided by the teacher-facilitator to stretch their thinking. Solving problems in this context is commonly known as problem-based learning (PBL). This preliminary paper is part of a larger study in determining children's mathematical problem-solving behaviours in a PBL setting.

THEORETICAL BASIS FOR MATHEMATICAL PROBLEM SOLVING IN A PBL SETTING

Many researchers and educators believe that learning should be an active process. It is when learning is active do students construct or reconstruct their knowledge networks and create meaning and build interpersonal interpretations of the world through their experiences and interactions. This is something the traditional teaching of mathematics is unable to fulfill due to its limitations. Its prescriptive mode of instruction serves to help students imitate procedures without acquiring a deep conceptual understanding of mathematics (Schoenfeld, 1988).

The constructivist philosophy advocates that learning takes place when constructive processes operate and learners form, elaborate, and challenge their own mental structures until a satisfactory one emerges (Perkins, 1991). In mathematical problem solving reform movements, researchers have argued that the classical four stages to problem solving (Polya, 1973) is likened to a linear approach characterized by highly structured sets of signals and cues within the well-defined knowledge domains of the solver that would end with an identifiable correct answer (Anderson, 1990). Such did not truly reflect the problem solving process. The later problem-solving frameworks place more emphasis on metacognition (Schoenfeld, 1985; Garofalo and Lester, 1985) and depict problem solving as an iterative or cyclic process (Artzt and Armour-Thomas, 1992; Carlson & Bloom, 2005). These newer frameworks emphasize more about strategic and monitoring behaviours that students should manifest towards successful goal resolution.

Also, these newer frameworks adopt the constructivist philosophy that reflects the problem solving perspective that encourages mathematization, that is, the use of mathematical language for expressing, communicating, reasoning, computing, abstracting, generalizing, and formalizing (Romberg & Kaput, 1999). It is believed that students through the exploration of problems will learn to verify, interpret and generalize their findings, and in doing so, learn to reason through the problem situations and thus develop good habits in making and evaluating conjectures, and of constructing and validating sound arguments. Mathematics education reforms have called for such learning environments where problem solving becomes the vehicle for learning (NCTM, 2000).

The PBL setting could be seen as an appropriate platform to foster critical thinking and mathematical reasoning. Its three main tenets of PBL, the complex authentic problem task, teacher-scaffolding, and small-group collaboration, are seen as powerful means to enhance children's thinking and to develop problem solving skills. The main aim of PBL is learning rather than the completion of the project (Uden & Beaumont, 2006). It uses problems as stimulus for learning and to assist in the development of the skills themselves. The teacher-facilitator has the role of helping students to explore the problem situation, develop their critical thinking skills, help them to reflect on their experiences, challenge their thinking, and monitor their progress (Woods, 1996). Finally, collaborative learning presents the students with opportunities to exchange diverse views, share and defend ideas, as well as critique one another's ideas. It is believed that when students are collaborating actively, they go beyond their current developmental levels as reflected in Vygotsky's (1978) concept of “Zone of Proximal Development” (ZPD).

The information-processing theory also seems to underlie PBL (Schmidt, 1983). To be able to use current knowledge to understand new information, to transfer knowledge to new situations, and to be able to communicate and reason while solving problems all point to a meaningful learning environment that promotes mathematical thinking.

METACOGNITION AND MATHEMATICAL PROBLEM SOLVING

Metacognition, generally accepted as the ability to monitor one’s thinking since its conception by Flavell (1979), is fundamental to children’s reasoning process. It has been largely recognized that metacognition plays the executive role in overseeing and regulating cognitive processes to achieve cognitive goals. The metacognitive ability of children allows them to understand when, why, where, and how to apply their knowledge towards successful solving of mathematics problems (Carr & Jessup, 1995) and a study into how experts solve problems suggested that they exercise metacognitive thinking by not only possessing more well-connected knowledge and rich schemata, they also regularly monitor and regulate their problem solving efforts (Goos, 2002). On the other hand, students who perform poorly in mathematics are those who are unable to recognize their mistakes, monitor their work, select appropriate strategies, or express their thought (Lucangeli & Cabrele, 2006; Carlson & Bloom, 2005). Recent research has focused on metacognitive activity in group learning whereby students monitor one another's understanding. For example, the works of Goos (ibid) involving collaborative groups distinguished routine monitoring from controlled monitoring, with the latter showing characteristics of raising an alert whenever errors are detected, or there is a lack of progress, or there are anomalous results. She claims that metacognitive blindness would result if students fail to notice something amiss. Such "higher" metacognition is likened to thinking critically.

Because the beneficial aspects of metacognition and its relevance to problem solving are widely acknowledged, and not enough is known about the metacognitive thinking of students when they work in small collaborative groups in problem-solving situations (Goos, ibid), there is therefore a need to understand more about the types of metacognitive monitoring and control behaviours that students use and contribute to the metacognitive development of oneself and other students.

ANALYZING COGNITIVE-METACOGNITIVE BEHAVIOURS

Studies in metacognition over the past twenty years have evolved through different disciplines and different adaptations to its meaning have emerged. In this sense, researchers have found it difficult to truly differentiate between the cognitive and metacognitive processes as each seems to be intertwined and that the interrelationship has been complex (Veenman, Van Hout-Wolters, Afflerbach, 2006). However, researchers have tried to distinguish between cognitive and metacognitive behaviours.

Studies in cognitive-metacognitive behaviours that stemmed from Schoenfeld’s (1985), and Artzt and Armour-Thomas’ (ibid) frameworks made use of the method of protocol analysis where protocols of problem-solving sessions were video-taped and problem-solving behaviours coded according to a scheme. Thus, their frameworks depict the cognitive-metacognitive behaviours that students would ideally manifest during problem solving. For example, Artzt and Armour-Thomas' (ibid) cognitive-metacognitive episodic framework of small group problem solving can be summarized as follows:

Reading (cognitive): The student is observed to be reading aloud or silently the problem or listening to someone else reading the problem.

Understanding (metacognitive): The student uses domain-specific knowledge that is relevant by way of restating the problem, asking for clarification, writing key facts or making a diagram of the problem, deliberating on the problem requirements, recalling similar problems or noting the presence or absence of important information.

Analysis (metacognitive): The student decomposes the problem into smaller elements and reformulates the problem by seeking relationships between the givens and goals.

Planning (metacognitive): The student selects solution steps or strategies, or combines them.

Exploration (cognitive and metacognitive): If the student engages in a variety of calculations without any apparent structure to the work, it is deemed as a cognitive behaviour, whereas if he or she monitors the progress of his or her or other's actions and decides whether to continue or terminate the process is metacognitive.

Implementation (cognitive and metacognitive): If the student executes based on what he or she already knows, then it is considered cognitive. However, it would be metacognitive if he or she makes metacognitive decisions that are characterized by checking or revising previously considered decisions.

Verification (cognitive and metacognitive): Mere checking of calculations is said to be a cognitive behaviour while evaluating the solution outcomes with respect to understanding, analysis, planning and/or implementing if they make sense constitutes to metacognitive behaviour.

Another episode known as "watching and listening" was not included in the present study as the intent was to capture the active behaviours. The above cognitive-metacognitive framework recognized the central role that metacognition plays in problem solving.

METHOD

Subjects and Groupings

146 Primary 6 students from two primary schools took part in this preliminary study. 42 students from Class A and 40 students from Class B of a school in the North Zone and 32

students each from Class A and Class B from a school in the East Zone participated in the study. The students in Class A of both schools were considered the higher-ability pupils while the students in Class B of both schools were of mixed-abilities. The students in each class were grouped according interest (friendship) and mathematics academic results (P6 Preliminary Assessment). As this study has a qualitative design, only the problem-solving behaviours of one group of five students from an A-class were analyzed.

The PBL Task

As the problem solving was to take place in a PBL setting, of importance is the quality of problem task designed to stimulate thinking. The task should be complex or ill-structured, where information is not all provided, and should have a real-life context. The problem task should also require them to tap on their learned mathematical concepts to be applied in this new situation. The element of mathematical modeling has also been factored into the design of the task as presented in Figure 1. This problem task was printed in a task sheet for each group.

> Your team is participating in a math project competition where you will need to present your findings in two days' time. In the competition, each team has been given only two square sheets made of vanguard. The team can decide if they want to use one vanguard sheet for trial. The team is supposed to make the biggest box (volume) using only ONE vanguard sheet. How would your team plan to solve the problem of making the biggest box? Show in detail how you reach a solution to convince the judge.
> (*Note that your box must have a base but need not have a cover*)

Figure 1. The problem task

Data Collection and Analyses Procedures

Video and audio-recording of the problem-solving process were captured from one group of students in each class. The video and audio recordings were then transcribed to identify and analyze mathematical and metacognitive thinking. The students' written notes were to be used to match their thinking processes.

Two important events took place a week before the actual PBL sessions. The researcher conducted a short PBL familiarization session with the teachers covering conceptions of PBL, the intended task, the task solutions, scaffolding questions, and student preparation aspects. A few days prior to the PBL session, the teachers demonstrated and practiced thinking-aloud with the students. Students were also briefed about the study and the importance of helping behaviours when working in groups.

On the day of the actual problem solving, the problem task was given to each group. The students in each group negotiated their own roles (scribe, time-keeper, resource manager, and presenter). A modified KWL[*]-template (Appendix 1) adapted from King (1991) was also provided for them to record their discussion and working. The modified KWL-template can be said to function as a simple form of metacognitive instruction. During the problem solving, the teacher would spend some time in each group to listen to their discussion, ask questions to get students to explain their thinking and trigger further thinking. At no point was the teacher to supply any answers.

It was deemed that the cognitive and metacognitive behaviours documented in the works of Artzt and Armour-Thomas (ibid) and Goos, Galbraith and Renshaw's (2000) could

[*] KWL is the acronym for "What I **know**", "What I **want** to know" and "What I **learned**". It is an instructional technique often presented as a graphic organizer to help learners chart their thinking and strategies.

represent a more current thinking in how the metacognitive behaviours are framed. By adapting the frameworks with modification as well as through a grounded approach when analyzing the protocols, a cognitive-metacognitive problem-solving behaviour taxonomy was developed to code the verbalizations that represented the students' problem solving behaviours that would befit the context of a PBL setting. Independent coding were done by the authors and discussion were held later to further unpack the meanings of the problem-solving behavioural domains and descriptors. The taxonomy was improved upon and intercoder-agreement returned a value of 76% using Cohen’s (1960) kappa. The taxonomy presented in Figure 2 differentiates between different domains of cognitive and metacognitive behaviours and their respective indicators.

The five students are labeled as S1, S2, S3, S4 and S5. The teacher is labeled T. In the event that it could not be made out who said something, it is labeled C. Where the conversation is not audible, it is indicated as “(…)”, and where the speaker did not end his statement, it is indicated as "…" For the types of monitoring behaviours, they are labeled as UPT, DLN, CPS, EI and CT according to the five categories in the scheme above. A statement or clause that does not contribute to the metacognitive monitoring scheme is labeled as O.

Cognitive-Metacognitive Problem-Solving Behaviours	**Examples of Prompts**
Metacognitive - Understanding Problem Task (M-UPT) * Clarifying problem requirement, objective or goal * Explaining problem requirement, objective or goal * Paraphrasing problem requirement, objective or goal * Questioning / probing requirement, objective or goal **Cognitive (C-RDG)** * Reading from text	"The problem means…" "What does the problem mean?" "Can you explain this part of the problem?" "The problem says there are 5 cars."
Metacognitive - Discerning Learning Needs (M-DLN) * Analyzing knowledge / concepts / skills / resources needed to manage problem * Identifying that which is needed (gaps) to solve problem * Acknowledging one's inadequacies **Cognitive (C-RAP)** * Recalling and applying facts or procedures	"What can we use?" "What mathematics is involved?" "Do we need …?" "I don't know how or what…" "We need to know…" "Use length times width"
Metacognitive - Checking Planning Situation (M-CPS) * Providing progress status with reason / understanding * Showing concern about progress * Checking for understanding **Cognitive (C-RPT)** * Mere checking of calculations * Mere reporting of calculations	"How can we…?" "If we don't proceed, we'll…?" "So far, what we found helps in…" "The answer to this operation is…" "How to do the calculation?"
Metacognitive - Exploring / Analyzing Ideas (M-EAI) * Articulating one's ideas for scrutiny * Promoting a strategy or heuristic * Looking for alternatives * Rationalizing solution or choice of solution * Analyzing ideas / solutions **Cognitive (C-EAI)** * Engages in a variety of random calculations that does not converge towards solving the problem.	"What if we…" "We should be doing this…" "Could you do…" "My idea is to …" "Let's try …" (without monitoring) "Do this…" (without monitoring)
Metacognitive - Critiquing Thinking (M-CT) * Questioning if procedures or strategies are correctly applied * Disputing fallacious ideas * Challenging one's assumptions or decisions * Calling for checks to be made in view of inconsistency * Checking if solution satisfies problem requirement, objective or goal * Casting doubts about efficacy of idea	"It didn't work out because…" "Can I/you check…?" "It should be …rather than…" "It works because…" "There's an error in…" "How can it be…?" "Let's not be too hasty…"

Figure 2. A taxonomy of cognitive-metacognitive problem solving behaviours in a PBL setting

ANALYSES

Excerpt 1 – Focus on Understanding of Problem Task

1	S5	So hard.	M-UPT Showing task concern
2	S3	How to?	M-UPT Showing strategy concern
3	S4	What is our goal?	M-UPT Questioning problem objective
4	S3	We are supposed to make the biggest box with only one vanguard sheet.	M-UPT Explaining problem requirement
5	S1	You mean make the biggest box with only one vanguard sheet?	M-UPT Clarifying problem requirement
6	S5	No, can use two also.	M-UPT Clarifying problem requirement
7	S1	My first thought is “How can we make this kind of a thing?” *(Gesticulating with open palms)*	M-DLN Acknowledging inadequacy
		(S1 and S2 began folding the vanguard sheet to try and shape it to a box. The others still looking intently at the problem)	
9	T	What's the goal?	
10	S4	To win the competition.	C-RDG Reading
11	S3	To convince the judge.	C-RDG Reading
12	S1	First must measure the length and breadth of the vanguard sheet.	M-EAI Articulating idea
13	S5	What do we know about solving the problem?	M-DLN Analyzing needs
14	S1	We need to make the biggest box using both vanguard sheets. *(Brandishing her pencil as she talked)*	M-UPT Clarifying problem requirement

The initial problem-solving process started with the students trying to grapple with what the problem was about. S4 was holding the task sheet while the rest of the students gathered beside him to read silently. The initial response of a student, S5, to the problem task was the acknowledgement that it was difficult (1). This is likened to what Flavell (1979) would term as metacognitive knowledge of task, a reflection based on the demands of the task. S3 followed up by requesting how to proceed (2), an indication of an strategy concern due to the demands of the task. The subsequent interactions among S4, S3, S1 and S5 were centered on understanding the problem task (3, 4, 5 and 6) through discussing important pieces of information to clarify the goals. S1 then interjected to question about their learning needs as interpreted through her gestures (7) which in a sense was trying to identify what was needed to proceed.

When the teacher came about to monitor the group's understanding of the task (8), S4 and S3 were able to relate the requirements from the task sheet about their goals (9 and 10). These are coded as cognitive behaviours as they were exact wordings lifted from the task sheet after their reading through. The modified KWL-template seems to serve its purpose in capturing the group consensus of the group goal. S5 who was the scribe wrote "Make the biggest box using one vanguard sheet" and "Convince the judge" as their ultimate purposes (see Appendix 1). What followed was the introduction of ideas or strategies towards for consideration. S1 articulated the need to take measurements (12), and the delineation of learning needs (13) and unpacking the meaning of the problem task continued (14). This beginning excerpt shows that the students did not merely proceeded after knowing what the problem required. There was constant revisiting of the task to make better sense of it.

Excerpt 2 – Focus on Discerning Learning Needs

1	T	Have you written down what you know about solving the problem?	
2	S2	Yes.	O
3	T	XXX, what do you know about solving the problem.	
4	S2	Must find out the length and breadth of the vanguard sheet and what shape the box should be, whether it is rectangle or square.	M-DLN Identifying knowledge

			gap
		(S4 started to write on paper)	
5	T	So have you all come up with any strategy?	
		(S4 started punching the calculator and then showing it to S2)	
6	S2	We are finding the area. The area is 2601. Actually rounded off already.	M-CPS Providing progress status
7	T	What do you not know about solving the problem?	
8	S4	We do not know the length and breadth of the vanguard sheet and we don't know what the shape of the box should be.	M-DLN Identifying knowledge gap

This excerpt captures the students metacognitive processing in terms of reflecting on their learning needs. As the teacher checked with the group on their learning needs (1), the group was able to relate what they knew about solving the problem by articulating the specific mathematical processes needed (4). At that point in time, the group had already analyzed their learning needs and they knew that to make progression, they had to bridge the gap of finding the length and breadth of the vanguard sheet as well as determine if the box should take the shape of a square or rectangle (8). This inference is also based on the connection made to the modified KWL-template with respect to their responses on "What we know about solving the problem" and "What we do not know about solving the problem" (see Appendix 1).

Excerpt 3 – Focus on Checking the Planning Situation

		(All students were standing. S2 and S5 toyed with folding the vanguard sheet)	
1	S2	Now measure the volume.	C-EAI Recommending what's known
2	S1	How can we measure the volume without measuring the height? *(Turning to S2)*	M-CT Questioning procedures
3	S2	We've just measured the height. The height is 3.6. So we take 50.8 minus 3.6. *(Took the calculator and started punching numbers)*	M-EAI Analyzing solution

4	S5	So what did we find out? *(Getting ready to write)*	M-CPS Showing concern about progress
5	S3	This side is 50.8, right? So you have to minus 3.6, minus 3.6, 2 times. *(Facing S5)*	M-EAI Rationalizing solution
6	S1	43.8 x 43.8 x 3.6 *(S2 showing the calculator to S1)*	C-EAI Applying procedure
7	T	What is the volume? You can use the calculator to work out and draw your conclusion.	
8	S3	6906.384	C-RPT Reporting answer
9	S4	We need the exact volume? *(Facing S1 and S2)*	M-UPT Questioning problem requirement
10	S5	Yeah. Exact volume, 43.6. That's exact. *(Writing)*	C-RPT Reporting answer
11	C	6843.456	C-RPT Reporting answer
12	S4	Well done. Underline it.	O Aesthetic or emphasis concern
13	S5	But how do you know this is the biggest? *(Bending forward with scissors)*	M-CT Challenging assumption
14	S2	We're just trying it out.	C-EAI General exploration

This excerpt starts at a point where the group had identified what the net or blueprint of a box should be like. It does seem that the monitoring of the planning situation occurs more intensely when mathematics calculation was needed. The suggestion by S2 to measure the volume as a general recommendation (1) at this stage cannot be considered a "new" or "unknown" strategy. But this recommendation sparked some interesting interactions that gave rise to a situation where the discussion was focused on checking on the progress status particularly with respect to getting the appropriate the dimensions of the box. In line (4), S5 asked for a status check, and what followed was a flurry of reports based on the calculations the members had obtained in (8), (10) and (11). The reporting of computed results are coded as cognitive behaviours as they were responses to a request that was metacognitive in behaviour. Interspersed among the protocols of checking for planning, it is also noted that students were engaged in some form of critical thinking to question and challenge assumptions (2 and 13).

Excerpt 4 – Focus on Exploring Ideas

1	S1	So the bigger the box, the bigger the volume. That means we make the bigger the box, the height is higher, the base may be…*(Gesticulating with one hand held up high)*	M-EAI Analyzing solution
2	S5	For example if you cut 20cm, right? Then this one becomes very small you know? If you use 20 cm, right? Then you will minus 40 cm, right? Then you will get 10.8. So 10.8 x 10.8 x 20, hey you get lesser. *(Punching numbers in the calculator)*	M-EAI Analyzing solution
3	S3	8652 (reading from calculator)	C-RPT Reporting answer
4	S2	Wait let's try again.	M-CT Checking for consistency
		(S1, S2 and S3 were taking turns to use the calculator)	
5	S5	Put at 10 cm first.	C-EAI Restatement from (5)
6	S3	We must try 11, 12, 13, 14.	M-EAI Promoting alternatives
7	S4	Now let's do guess and check method. So everyone, now must do guess and check method.	M-EAI Promoting a strategy
8	S2	Why don't we make a table, like 10cm, 11 cm, 12 cm, 13 cm, 14 cm? *(Gesturing how a table is drawn with his hand)*	M-EAI Promoting a strategy
9	S1	Now let's try 11.	M-EAI Analyzing solution (comparing)

The group was beginning to realize that the modeling task required them to hypothesize and work towards an optimal solution although they had yet to find the range for the lengths of the squares to be cut out. In line (1), it could be seen that the group's hypothesis-reasoning became more evident. S1 was hypothesizing that by cutting out bigger squares, one would get bigger volumes for the box based on what she had observed thus far. S5 was also exploring and analyzing the situation and to her delight as well as surprise, she found that by making the cut-out squares bigger, she got a smaller volume (2). They had to try again to confirm this phenomenon (4). This apparently led to the other members to try and adopt a more systematic way to approach the solution. S4 suggested a Guess-and-Check strategy (7), and this was followed by S2 in proposing to use a table to systematically try the cut-out squares for lengths of 11 cm, 12 cm, 13 cm and 14 cm (8).

Metacognitive Critical Thinking

Analysis of the full transcript (not enclosed) revealed that students mediated socially towards problem resolution. Metacognitive thinking was evident and helpful in enabling the checking and revision of plans, strategies, as well as in the overall course of action throughout the problem-solving process. However, metacognitive critical thinking raised the level of metacognitive thinking such that members were not blind to what could have passed off as illusions of understanding. For example, in *Excerpt 3*, line (2), S1 raised issue with finding the volume without first measuring the height which led S2 to justify that he had indeed measured the height (3). In the same excerpt, line (13), S5 challenged if the volume they found was to be accepted as the biggest. In *Excerpt 4*, line (4), we see an instance where S2 called for checks to be made again in view of a new discovery. Such metacognitive critical thinking exhibited by the students is important towards making correct judgments and decisions and as Goos (ibid) would put it, it prevents the students from having metacognitive mirages and failures.

GENERAL DISCUSSION

The present study investigated how metacognitive monitoring behaviours are facilitated during problem solving in a PBL setting. Based on the defined scheme of cognitive -

metacognitive behaviours, it is evident that students do exhibit metacognitive behaviours to understand the problem task, determine their learning needs, check their planning situation, promote ideas and strategies, and evaluate ideas critically. The elicitation of such monitoring behaviours can be said to have been made possible via the interaction of the key tenets in the architecture of the PBL setting, specifically, the task complexity, peer-collaboration, and teacher-scaffolding.

Task Complexity

Making the biggest box is a complex and novel problem task for the students. The complexity of the task facilitated student-student interaction, student-task interaction and student-teacher interaction, which in a way provided ample opportunities for students to make manifest their metacognitive thinking. It is evident from the excerpts that the students' understanding of the problem situation was not initially consistent, and through dialog, they brought the problem situation into closer correspondence with each other's knowledge and understanding. In a sense the task complexity had brought about a series of dialog that forced awareness of the students' cognitive processes.

Teacher-Scaffolding

The role of the teacher-facilitator is to empower students cognitively and meaningfully through scaffolding. Ho (2004) pointed out that teachers act as coaches of the students' cognitive processes which involve goal setting, modeling, guiding, facilitating, monitoring and providing feedback to the students. In coaching, these cognitive processes are not meant to be taught but instead the emphasis and practice are on fostering "thoughtfulness". As evident from the excerpts, the teacher enhanced the students' metacognitive thinking through the questions asked to deliberately make their thinking visible by getting them to explain their understanding of the problem or hinting for the need to explore alternative solutions.

Small-Group Collaboration

In a learning environment where there is group collaboration, it is deemed that prior knowledge mobilized by one learner could activate what might have been inaccessible knowledge to another learner. Once collective knowledge is accessed, learners also begin to elaborate on what they knew to close certain knowledge gaps (Schmidt & Moust, 2000). This has been evident in the excerpts where students proposed ideas for exploration, commented on their peers' ideas, and sought clarification from peers about their own ideas and their peers' ideas.

LIMITATIONS

The use of the transcripts from only one class suggests that it is limited in its presentation towards a broader view of the mathematical reasoning and metacognitive behaviours manifested by the students. Also, metacognitive problem-solving behaviours are examined based on the use of only one problem task. Finally, thinking-aloud has not been easy for the students especially when they are not used to such a communicative approach towards problem solving. A better extension to this study would be able to examine the transcripts from the different classes with different problem tasks over time so as to develop and establish a more analytic problem-solving taxonomy that could include metacognitive behaviours, mathematical reasoning behaviours and affective behaviours as well.

IMPLICATIONS

From the discussion above, there are several implications and indications for future research directions. First, a PBL setting can be an appropriate platform for problem solving. Its key features of task complexity, group collaboration and teacher-scaffolding can be powerful means to enhance social interaction, mathematical reasoning and metacognitive thinking. In other words, these behaviours can be influenced through interacting with an environment where the task (physical), the teacher and peers (social) serve as mediators.

Cognitive conflicts can arise from engaging the task or the peers. When this happens, the conflict resolution rather than the conflict is deemed to promote rich mathematical discourse. Although disagreements may arise but if the students remain resolute towards solving the problem, their collaboration and collective will would enhance both their cognitive and metacognitive processes.

This study has shown that students exercise metacognitive thinking when they solve problems in a PBL setting. Not only that, their metacognitive thinking has been very much related to the application of mathematical reasoning and academic skills as well. This study could also be extended to find out how facilitating metacognitive thinking in a PBL context could impact mathematical reasoning thus addressing a concern in enacting the intended mathematics curriculum which local studies often claim that mathematics classroom practices focus on the development of mastery of skills and concepts, and paying little attention to the other components like processes, metacognition and attitude (Ho, 2006; Foong, 2004; Kaur, 2003). At present, Singapore primary schools are exploring pedagogies that promote active learning. This study could provide a means for educators to consider how metacognitive behaviours have been facilitated when children solve complex authentic problems in a PBL setting.

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Appendix 1

Modified KWL Template

Our Goal(s)

→ Make the biggest box using one vanguard sheet
→ Convince the judge

What we know about solving the problem

→ We can use one vanguard sheet
→ Make the biggest box
→ Calculate the area of the vanguard sheet first.

What we do not know about solving the problem

→ The length ~~of~~ and breadth of the vanguard sheet
→ What the shape of the box should be
→ What the measurements of the box ~~wil~~ should be.

What we found out about solving the problem

→ The length and breadth of vanguard sheet
→ However ~~wa~~ shape the box is, the volume would be the same ✗
→ We can try to make a bigger box by shortening the height. ✗
→ We were wrong from the start.
→ The longer the height of the box, the larger the volume of the b
→ But if the box is too tall, the volume is less.
→ The height at 8.5 cm makes the box with the biggest volume.

How the problem is solved

→ Found out the length and breadth of vanguard sheet.
→ Tried the height at 5cm.
→ Found out we were wrong.
→ Used guess and check method.
→ Added 1 decimal place to the number to calculate.
→ Figured out the right height.
→ Cut the remaining piece of vanguard sheet.

Solution:

8.5cm × 33.8cm × 33.8cm
= 9710.74 cm³ (volume of box)