Using an advanced graphing calculator in the teaching and learning of calculus

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The purpose of this study was to investigate how the use of TI-Nspire™ could enhance the teaching and learning of calculus. A conceptual framework for the use of TI-Nspire™ for learning calculus in a mathematics classroom is proposed that describes the interactions among the students, TI-Nspire™, and the learning tasks, and how they lead to the learning of calculus. A design experiment was conducted in a class of 35 students from a secondary school in Singapore. Use of TI-Nspire™ was integrated into the teaching and learning of calculus concepts in the classroom with the aid of TI-Nspire™ Navigator, a wireless classroom network system that enables instant and active interaction between students and teachers. It was found that the appropriate use of graphical, numerical and algebraic representations of calculus concepts using TI-Nspire™ enabled students to better visualize the concepts and make generalizations about relevant mathematical properties. In addition, the students were able to link multiple representations, especially algebraic and graphical representations, to improve their conceptual understanding and problem-solving skills. Six roles of TI-Nspire™ in classroom mathematical practice were identified from the findings of the experiment; TI-Nspire™ was used as an exploratory tool, graphing tool, confirmatory tool, problem-solving tool, visualization tool and calculation tool. This suggests that TI-Nspire™ is a multi-dimensional tool that supports mathematics learning. Overall, the findings of the study indicate that TI-Nspire™ is an effective tool to develop mathematical concepts and promote learning and problem solving.

Keywords: TI-Nspire™; graphing calculators; calculus; design experiments; mathematical tools

1. Introduction

The graphing calculator has been widely used as a technological tool in teaching and learning mathematics. There is a growing body of research which shows that the pedagogical affordances of the graphing calculator have a close relationship with the improved learning of mathematics [1–7]. Indeed, mathematics can be taught in a more coherent way by providing students with the opportunities to connect mathematical concepts within and between topics, and here the graphing calculator is well-positioned to offer a learning environment that enables mathematics to be experienced through multiple representations [6].

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The curriculum standards of the National Council of Teachers of Mathematics [8] suggest the use of graphing calculators to provide students with multiple representations to explore and investigate mathematical concepts. Extensive research has been conducted to answer questions such as whether graphing calculators significantly improve mathematical learning and problem solving [4,6,9]. However, many such studies have compared the use of graphing calculators to traditional chalk-and-talk learning, and little research has examined how students use graphing calculators in learning [10] and the relationship between pedagogical strategies for learning and the use of graphing calculators. Doerr and Zangor [10] noted that the details of how and why graphing calculators are used are often not reported in the research findings.

Penglase and Arnold [11] carried out an extensive review of research on graphing calculators, and found that certain teaching styles, such as interactive and inquiry-oriented, are more compatible with the use of graphing calculators than others. However, they also further point out that many studies fail to address the use of graphing calculators in an instruction context, which leads to inconsistent findings on their effectiveness in teaching and learning. This review suggests a need for studies that address the use of graphing calculators in an instruction context within a particular learning environment.

Accordingly, the purpose of this study was to investigate how the use of TI-Nspire™, an advanced graphing calculator from Texas Instruments, could enhance teaching and learning of calculus. This article provides the details and results of a design experiment, which involved the design and conduct of a TI- Nspire™ intervention programme for a class of 35 Secondary Four students (15–16 years) from a school in Singapore. Use of TI-Nspire™ was integrated into the teaching and learning of calculus concepts with the aid of TI-Nspire™ Navigator, a wireless classroom network system which connects students' TI-Nspire™ handhelds equipped with a wireless cradle to the teacher's computer and projector. It facilitates transfer of documents among the teacher and learners and allows the teacher to capture students' handheld screen images to ensure everyone is on task, and collect and analyse student work to assess student understanding.

2. Literature review and conceptual framework

In the literature on mathematics reform, learning mathematics is viewed as a social endeavour, with the mathematics classroom functioning as a community in which thinking, presenting, talking, agreeing and disagreeing are encouraged [12,13]. Mathematical tasks are given to students to develop and extend their mathematical thinking and problem-solving ability. Student interaction with mathematics tasks and tools encourages them to construct mathematical ideas and hence create meaning for their learning.

Hiebert et al. [14] observed that students should construct meaning from the tools that they use and the mathematical tasks in which they engage. They further elaborated that learners construct mathematics meaning from tools as they use them. This leads to meaningful learning, as the tools become intertwined with the mathematical tasks. Bruce [15] pointed out that the quality of mathematics tasks is important in determining whether a task engages students and whether they attain higher order learning by being given opportunities to explain and justify their reasoning.

It is widely acknowledged that calculus concepts are abstract and complex for students and that teaching and learning these concepts can be challenging and even exasperating at times [16,17]. Research has shown that many students have difficulties in learning the key concepts of calculus [18,19]. It does not help that traditional calculus courses tend to focus on algebraic drills and the practicing of calculus problems without generating an understanding of the underlying concepts [16].

Specifically in terms of calculus, many researchers have advocated that teaching and learning should focus on concepts, rather than simply techniques to improve student performance. Tall [20] emphasized the need to help students move flexibly from one representation to another in learning calculus. Using a graphing calculator extends the teaching and learning of calculus concepts from being based solely on algebraic methods to the incorporation of numeric, algebraic and graphical representations. The idea of using the graphing calculator to provide a better connection between algebraic and graphical representations of calculus has been reported in several studies [1,5,21].

It is believed that the use of technology in the teaching of calculus can be beneficial if it is accompanied with appropriately designed activities [17,22]. Hiebert et al. [14] highlighted that mathematical tools are important in helping students to make connections to build their mathematical understanding. Activities such as guided discovery and inquiry learning have been used in the design of worksheets to encourage interaction with graphing calculators to help students develop an understanding of calculus concepts.

In this study, exploratory activities using TI-Nspire™ were designed by the author, with inputs from the teacher of the participating class, hereafter the participating teacher, to provide students with opportunities to explore and generalize mathematical concepts and to help them develop confidence and ability in communicating mathematically. The preliminary framework for the use of TI-Nspire™ in the intervention programme is an adaptation of that proposed in a study by SRI International [23].

A conceptual framework for calculus learning and teaching using TI-Nspire™ was devised based on the foregoing literature review, and is illustrated in Figure 1.

This study seeks to describe how the interactions among the students, TI-Nspire™, and the learning tasks lead to the learning of calculus. It describes

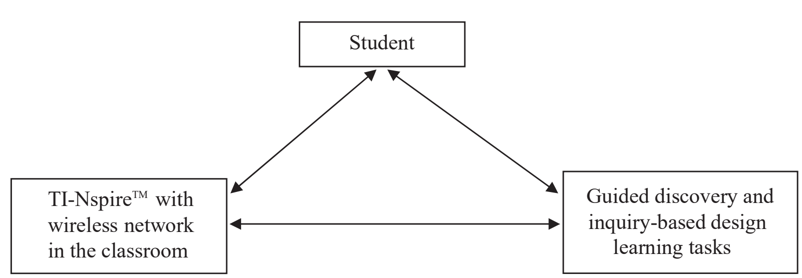

Figure 1. Conceptual framework.

what these interactions bring to the construction of calculus concepts and in what ways TI-Nspire™ is a suitable tool for such learning.

3. Methodology

3.1. *Research design*

This study employed a design experiment methodology that involved the preparation, design, implementation and retrospective analysis of a teaching experiment. A design experiment is an iterative process that features cycles of invention and revision [24]. The participating teacher worked in close partnership with the author to incorporate the use of TI-Nspire™ in lessons, examining the strengths and limitations of existing pedagogy and practice and acknowledging areas for future development with the TI-Nspire™ technology.

The subjects of the study were 35 fourth year students from a secondary school in Singapore. Each student was loaned a TI-Nspire™ handheld, two cables for transferring files and a cradle for connecting to the TI-Nspire™ Navigator. Fourteen training sessions, each of about 30 min duration, were held for the students before the start of the TI-Nspire™ intervention programme. The purpose of the training sessions was primarily to introduce the interface design and to train the participating students in the use of the algebraic, graphical and numerical functions of TI-Nspire™ while engaging the students in enriching activities involving mathematical problem solving and exploration with the aid of the TI-Nspire™.

To determine more precisely, the role of TI-Nspire™ in teaching and learning calculus in the participating class, a calculus package focusing on differentiation and integration that integrated TI-Nspire™ was designed. Students attended two to three 1 h lessons on calculus per week for a total of 15 weeks. The topics covered were Differentiation by First Principles, Equations of the Tangent and the Normal, Rate of Change, Stationary Points, Derivatives of Trigonometric Functions, Derivatives of Exponential and Logarithmic Functions, Integrations and Indefinite Integrals, Definite Integrals, Integration of Trigonometric Functions, Integration of Exponential Functions and $1/x$, and Area of a Region.

In some studies in which the use of graphing calculators was found not to improve learning, the calculator was used as an add-on to traditional teaching, rather than in an integrated way [25]. Hence, in this study, TI-Nspire™ was closely integrated with the teaching and learning process through specific activities involving the tool that served as intriguing mathematical starting points for students and encouraged them to explore, which it was hoped would lead directly to mathematical generalization. TI-Nspire™ was also employed to emphasize the complementary use of graphical, numerical and algebraic representations in teaching calculus concepts, and to encourage the use of different problem-solving approaches.

3.2. *Learning activities*

The TI-Nspire™ learning activities in the intervention programme can be broadly classified into two categories. In the first category, the activities focused on introducing mathematical concepts through exploration using TI-Nspire™. Students were encouraged to first examine the setting of a given problem or manipulate the parameters of a simulation in a TI-Nspire™ document that was

$$\frac{d}{dx}(\sin x)=\cos x$$

Notes:

a) Use TI-Nspire to verify the above result (*diffTrigoA.tns*). The gradient of the tangent line to $y = \sin(x)$ at a given point gives the value of the derivative of $y = \sin(x)$ at that point.

b) Use the tangent line to trace an approximate graph of the derivative of $y = \sin(x)$. The slope of the tangent is shown at the top right of the screen. Your goal is to move the tangent line by grabbing and dragging the point of tangency such that it is positioned directly above each of the 13 points, beginning with the leftmost point.

c) Inspect the resulting scatter plot, which represents the approximate graph of the derivative of the sine function. Write down your observations.

d) Use the Graphs and Geometry application on the next page to verify your answer.

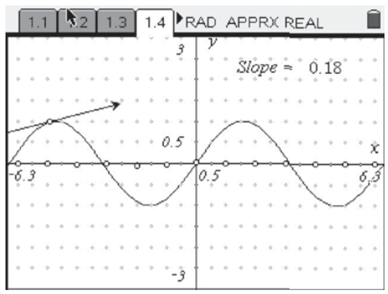

Figure 2. Learning activities with TI-Nspire^{TM}.

pre-loaded onto their handhelds. They were asked to formulate conjectures and examine and confirm those conjectures, culminating in the derivation of a formula, the generalization of the results or the summarization of the underlying concepts. A worksheet was designed to accompany these activities to facilitate guided discovery and inquiry learning. Figure 2 shows an example activity.

In the second category, the students were assigned tasks to reinforce their conceptual understanding using TI-Nspire^{TM}. In a typical activity, an exercise consisting of problems related to the concept being taught was assigned and a pre-prepared TI-Nspire^{TM} document sent to the students, who were required to complete the exercise with the aid of TI-Nspire^{TM}. Figure 3 shows an example activity.

Occasionally, a formative assessment using the ‘Quick Poll’ function of TI-Nspire^{TM} Navigator, which allows the teacher to gather instant feedback from students through the wireless network, was conducted to assess student understanding. Where time permitted, students were invited by the participating teacher to share their solutions or ideas through the use of the ‘Presentation’ function of TI-Nspire^{TM} Navigator. TI-Nspire^{TM} Navigator was also used by the participating teacher to monitor the progress of the students in the in-class tasks by periodically viewing their TI-Nspire^{TM} screens, as illustrated in Figure 4.

All of the activities and accompanying worksheets were designed after thorough discussion with the participating teacher who then co-taught the calculus package with a research assistant of the project who assisted the participating teacher with the use of TI-Nspire^{TM} and its Navigator. Activities such as guided discovery and inquiry learning were used in the design of the worksheets.

3.3. *Limitations*

This study has several methodological limitations. First, TI-Nspire^{TM} Navigator was not fully used in the intervention programme because the lessons were not

Exercise: Find $\frac{d}{dx}(\tan x)$ by hand and use TI-Nspire to verify your answer.

Notes:

- To find the derivative of tan *x* with respect to *x*, first write tan *x* in terms of sin *x* and cos *x*.
- Use the quotient rule to find the derivative of the function in terms of sin *x* and cos *x*.
- Verify your answer by graphing the derivative of tan(*x*). Enter **nDeriv(f1(x),x)** as f3(x).

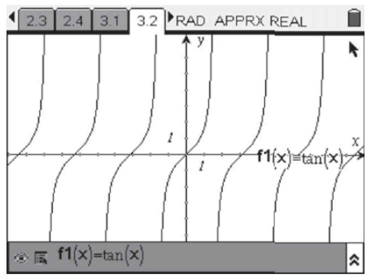

Figure 3. Class exercise using TI-Nspire^{TM}.

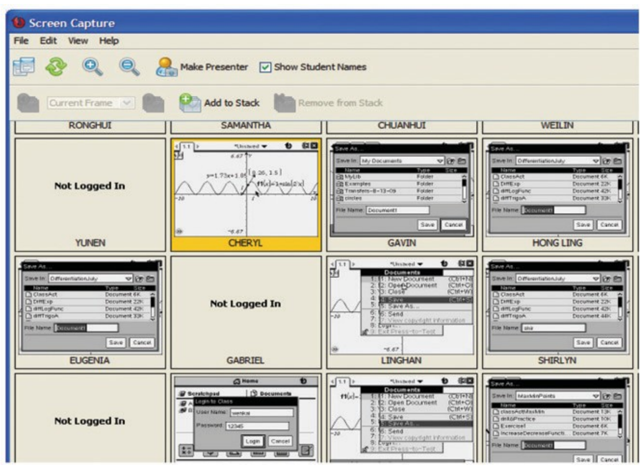

Figure 4. Use of TI-Nspire^{TM} navigator to monitor student progress.

sufficiently long. Each lesson took approximately 1 h, but it took approximately 10 min to successfully connect all TI-Nspire^{TM} handhelds in the classroom to TI-Nspire^{TM} Navigator. To ensure that the lessons were not substantially affected by time spent troubleshooting, the use of TI-Nspire^{TM} Navigator was restricted to mainly formative assessment in some of the lessons. Other uses of the system were not fully explored.

Second, the conventional teaching and learning of the mathematics lesson, which the students were more accustomed to, is more teacher-centred. Students were not used to learning through exploratory and inquiry activities, with many commenting that they were familiar with following teacher demonstrations rather than spending time on activities designed to help them generate and develop their own mathematical concepts and ideas. Although the author wanted to include more group work to encourage collaborative learning, the active learning environment was still more focused on the interaction between the teacher, students and TI-Nspire^{TM}.

Third, the study focused on topics in calculus, and the use of TI-Nspire^{TM} in the intervention programme thus focused on the graphical approach. Restricted by the scope of the topic, the study was unable to investigate the balance of the use of graphical, numerical and algebraic approaches in calculus learning. Investigating the role of TI-Nspire^{TM} in a wider range of topics would have allowed the research team to more fully examine the link between graphical, numerical and algebraic approaches in teaching and learning mathematics and in solving mathematical problems.

4. Results and discussions

Qualitative data were collected in the form of classroom observations, self-reflections and interviews. Observation, taking of field notes and video recording were carried out during each lesson. Students were also asked to write down their reflections on a reflection form designed by the author to determine which features of TI-Nspire^{TM} they had used and the difficulties that they had encountered in using TI-Nspire^{TM}. By analysing the students' reflections, the author found out what functions of TI-Nspire^{TM} the students had learned, what they would like to try out using TI-Nspire^{TM}, and what they wanted to know about TI-Nspire^{TM}.

A structured interview adapted from the study of Serhan [26] was conducted to obtain information about the students' conceptual understanding of the derivative concept. Eight randomly selected students from the participating class participated in the interview. Comments or views expressed by students during the interviews are presented as *comments* in the remaining part of this section.

In this study, TI-Nspire^{TM} was used as a pedagogical tool to complement the conventional teaching approach. During a typical lesson, the participating students used TI-Nspire^{TM} to examine the setting of a given problem, formulate conjectures or hypotheses, examine those hypotheses, recheck their solutions and finally generate a result (e.g. the chain rule) or a solution to the given problem. It was observed that the students tried to connect various representations using TI-Nspire^{TM} to understand the concept of the derivative at a point. It was also found that the students had a better understanding of the derivative concept when they were able to explore different representations of the concept as expressed by Student A (see Comment 1).

Comment 1 (*Student A*): *I can understand the concept better*, *like* $\frac{dy}{d}$ *actually means drawing the tangent line along the curve.*

On the whole, the students were excited about learning the concept and doing exercises using TI-Nspire^{TM}. They started to appreciate visualization in mathematics and the value of being able to 'see' the concept rather than simply memorizing the formula (see Comments 2 and 3).

Comment 2 (*Student B*): *We can get to see and visualize the problem better.*

Comment 3 (*Student C*): *I will if I know how to use the GC. I think so, because it simplified the thing.*

The students also solved problems using the graphical method without long computation, which again helped them to see the importance of conceptual understanding in learning mathematics rather than merely memorizing formulae. They seemed to appreciate that TI-Nspire^{TM} helped them to understand the concept of the derivative at a point through the various representations. These reactions were also reflected in the interviews, with most of the students indicating that their conceptual understanding of the derivative concept had improved.

It was also found that the students used TI-Nspire^{TM} as a tool in several different ways. For instance, they used it as a visualization tool to better understand the behaviour of graphs, the new concepts being taught, or to solve problems, and also learned how to use TI-Nspire^{TM} as a confirmatory tool to verify the correctness of their answers. Table 1 summarizes the ways in which TI-Nspire^{TM} was used during the intervention programme, both as a pedagogical tool in teaching or as a learning tool by the students.

TI-Nspire^{TM} gave the students the means not only to explore the concepts of calculus, but also to communicate those concepts mathematically. With the use of TI-Nspire^{TM} Navigator wireless network in the classroom, the students were able to present their solutions and to provide feedback to the participating teacher during the lesson. The 'Quick Poll' feature of TI-Nspire^{TM} Navigator was used to conduct

Table 1. Using TI-Nspire^{TM} as a tool in teaching and learning calculus.

Use	Description of students' actions and feedback
As an exploratory tool	TI-Nspire^{TM} was used to explore and understand the concepts of differentiation and integration. For example, students explored the differentiation of the products of two functions and derived the product rule.
As a confirmatory tool	Students used TI-Nspire^{TM} to verify their answers to the questions in the exercises. For instance, they first solved problems by hand and then confirmed their answers using the graphing functions of TI-Nspire^{TM}.
As a problem-solving tool	Students used TI-Nspire^{TM} to try different approaches to solving calculus problems. For example, in solving problems involving turning points, students used algebraic, graphical and numerical approaches.
As a visualization tool	Students used TI-Nspire^{TM} to better visualize the behaviour of functions, new concepts being taught, or problem situations. TI-Nspire^{TM} was also used by teachers to construct simulations to illustrate problems or new concepts. For example, a simulation was constructed to allow students to explore the concept of rate of change by manipulating variables and observing the dynamic changes in the graphs.
As a calculation tool	Students used TI-Nspire^{TM} to calculate values or evaluate complex expressions.
As a graphing tool	Students used TI-Nspire^{TM} to graph functions and solve problems graphically. For instance, they used the Graph and Geometry application to solve problems related to the area under a curve.

formative assessments to evaluate student understanding. In addition, TI-Nspire™ Navigator was used to monitor the progress of students in in-class tasks by periodically viewing all of their TI-Nspire™ screens on a computer. It was observed that the students participated more actively in discussions and in the learning process as a whole. The value of this teaching is best summed up by Student D (see Comment 4).

Comment 4 (*Student D*): *When GC came in*, *maths learning became more interesting and quite fun.*

The students found that they could solve some problems using the graphical method without long computation, which again emphasized the importance of conceptual understanding. These reactions were reflected in the interview results, in which most of the students indicated that their conceptual understanding of the derivative concept had improved (see Comment 5).

Comment 5 (*Student E*): [*Before*] *we learned mostly to solve by hand. If I had known the graphical method at the start*, *then I would have chosen to use it.*

The detailed analysis of students' verbal responses to the use of TI-Nspire™ in the learning of calculus concepts highlighted that learning calculus had helped them to learn TI-Nspire™. Specifically, integrating the use of TI-Nspire™ in learning calculus had helped them to:

(1) master the use of TI-Nspire™, which in turn helped them to learn calculus concepts;
(2) better visualize problems and calculus concepts;
(3) connect graphical and algebraic representations of calculus concepts;
(4) identify the application of calculus concepts from simulations designed in TI-Nspire™ and
(5) employ an alternative problem solving tool using a graphical method.

Some of the students even mentioned that calculus had become their favourite topic after the intervention programme. One student noted that she had learned more during the intervention programme through using TI-Nspire™.

The conceptual understanding of the derivative at a point was similar among the eight students from the participating class who were interviewed. The most dominant concept that they had mastered was that the derivative at a point is the gradient of the tangent line at that point (see Comment 6). Two students mentioned the instantaneous rate of change, and seven students elaborated their explanation by giving a formal definition (see Comment 7).

Comment 6 (*Student C*): *Oh! The gradient of a tangent line*.. . *Instantaneous rate of change. How fast it changes.*

Comment 7 (*Student B*): *The gradient of the tangent line at the point.* $\frac{\mathrm{d}y}{\mathrm{d}}$ ¼ nx^{n-1}. *Bring down the power number*, *the power number minus* 1.

All eight students defined the derivative of a point as finding the gradient of the tangent line at the point. All of them remembered the demonstration using TI-Nspire™ to explain the derivative of a point during the lesson, and used the visualization of the demonstration when they explained the derivative concept. This finding supports the results of other research that has indicated that students use visualization to deal with mathematics concepts, rather than formal definitions. The results of this study also indicate that the graphing calculator provides a better connection between algebraic and graphical representations.

When the students were given a problem on finding the derivative of a curve, all of them used the algebraic method to find the solution. However, they explained their answer by showing an alternative solution method. All of them were able to use graphical and numerical methods to explain their solution by showing how to find the derivative of the curve by finding the gradients of a few points on the curve.

These findings indicate that the visual and numerical representations of the derivative at a point on TI-Nspire™ helped the students to develop their conceptual understanding (see Comments 8 and 9).

Comment 8 (*Student D*): *Yes*, *draw the graph*, *draw the tangent line at the points on the graph*, *find the gradient of the tangent line*, *and then draw the graph of each point.*

Comment 9 (*Student E*): *Draw some points on the graph*, *draw the tangent lines of the points*, *and find the gradient of the lines. Use the new points to draw the graph. That is the answer. Like the lesson using GC to find the derivative of* $\sin x$ *and* $\cos x$*.*

Student E mentioned that he had learned the concept from the lesson in which they used the graphical and numerical representations from TI-Nspire™ to find the derivative of $\sin x$ and $\cos x$.

The findings from the interviews indicated that most of the students preferred to use an algebraic method to solve the problem, as they had already been taught using this approach for a long time (see Comment 10). They also needed to use algebra to show their working steps. However, they agreed that the visualization function of the graphing calculator enabled them to understand the concept better.

Comment 10 (*Student F*): *I'll choose algebra to solve it. We have been trained since* [*we were*] *young*; [*we're too*] *lazy to take out the GC.*

In summary, analysis of the interviews with the eight students revealed several findings.

(1) The dominant conceptual understanding of the derivative at a point was that it is the gradient of the tangent line at the point.
(2) Students were able to use the algebraic and graphical methods to solve derivative problems.
(3) Students were able to connect their conceptual understanding of the derivative concept with numerical and algebraic representations.
(4) Students were not able to use a symbolic definition of the derivative at a point that involved the concept of limits.
(5) The dominant approach to solving derivative problems was the algebraic method.
(6) Most of the students were unable to make a connection between their understanding of the derivative and the instantaneous rate of change.

5. Implications

The results of the study support the research hypothesis that the new mathematical experiences afforded by TI-Nspire™ offer students a rich setting in which to explore calculus concepts. The dynamic features of graphing calculators, such as multiple representations of mathematical concepts, if used appropriately, can improve students' ability to make the connection between graphical and algebraic representations. This feature was found to be particularly useful in teaching and learning calculus.

Six ways of using TI-Nspire™ as a tool in the mathematics classroom were identified in this study. In the intervention programme, TI-Nspire™ was used as an exploratory tool, graphing tool, confirmatory tool, problem-solving tool, visualization tool and calculation tool. Underlying these uses was the pedagogy of the design of the mathematical tasks and the interaction between the students and the tasks designed for use with TI-Nspire™. Interestingly, the study found that TI-Nspire™ Navigator, which creates a wireless network in the classroom, promoted communication in the learning process, and hence supported the classroom learning community. The use of TI-Nspire™ Navigator in the classroom also raised the level of participation of the students during the lessons and strengthened their commitment to the learning process.

Goos et al. [27] theorized four roles for technology in relation to teaching and learning interactions: master, servant, partner and extension of self. In the current study, uses of TI-Nspire™ as a confirmatory tool, a calculation tool or a graphing tool by the students certainly have characteristics of the *servant* role while uses of TI-Nspire™ as an exploratory tool, a problem-solving tool or a visualization tool illustrate using the technology as a *partner*. It was evident that students in this study were able to use TI-Nspire™ beyond the level of using it as a *master* as it was clear that the students were not 'subservient to the technology' (p. 312). However, there was insufficient evidence to suggest that they could use the technology as an *extension of self*, the highest level of functioning which involves 'users incorporating technological expertise as an integral part of their mathematical repertoire' (p. 312). On the other hand, Doerr and Zangor [10,28] found five patterns and modes of graphing calculator tool use: computational tool, transformational tool, data collection and analysis tool, visualizing tool and checking tool. While students in this study used TI-Nspire™ as a computational tool in evaluating numerical expressions, and as a checking tool in confirming conjectures, they did not use TI-Nspire™ as a data collection tool in gathering data or as a transformational tool in changing the nature of a given mathematical task.

Analysis of the interviews with the students from the participating class on the concept of the derivative at a point revealed that the use of TI-Nspire™, and particularly its emphasis on visual and numerical representations of the derivative, affected the students' conceptual development. Students interviewed were able to find the derivative of a given function using a visual conception of the derivative that they had learned in the intervention programme. This finding suggests that the students were able to form connections between the different representations of the derivative in calculus using TI-Nspire™, which helped them to develop a conceptual understanding of the derivative. The conventional teaching and learning of mathematics over-emphasizes algebraic methods, and the students had been trained to use such methods to solve most mathematics problems. However, the use of a graphing calculator can change the teaching and learning of calculus concepts from relying solely on algebraic method to encompassing numeric, algebraic and graphical methods, as indicated by Tall [20]. Several studies have shown that the connection of different representations leads to the development of conceptual understanding and the improvement of students' problem-solving skills [7,9,21,29].

It was found that some of the students claimed to have been taught mainly using algebraic methods since they were young. Thus, the predominant approach that they used in problem solving remained algebraic even after using TI-Nspire^{TM}, although they saw the usefulness of the graphical approach in solving problems and understanding concepts better. Some even claimed that they only realized the accuracy and efficiency of the graphical method in problem solving after the intervention programme. These findings suggest that the use of a graphing calculator should be integrated into mathematics teaching and learning at an early stage, such as Secondary 1.

Teachers play an important role in conventional mathematics teaching and learning in the classroom. Most of the time, they are in full control of the teaching and learning situation and act as the knowledge provider. However, the teacher's role changes when TI-Nspire^{TM} is used in the classroom from instructor to learning facilitator. Students play an active role in the learning process, and construct their learning through guided discovery and inquiry-based learning tasks that they can then share using TI-Nspire^{TM} Navigator.

The visualization aspect of TI-Nspire^{TM} engendered a better conceptual understanding through demonstration graphs and simulations. In addition, the mathematics tasks were designed in a way that facilitated student learning. The conventional mode of learning that focuses on the regurgitation of what teachers have taught does not necessarily allow students to gain a full understanding of concepts, which are of utmost important in calculus.

6. Conclusion

The findings from this research demonstrate the potential of TI-Nspire^{TM} as a tool to promote learning and conceptual understanding in mathematics. The encouraging results recommend the use of the graphing calculator to provide multiple representations for mathematics learning and problem solving. A conceptual framework (Figure 1) is also proposed that shows the importance of the interactions among the students, TI-Nspire^{TM}, and the learning tasks in the learning of calculus. In the future, TI-Nspire^{TM} could be used to stimulate students to think mathematically so that they will engage strongly with mathematical structures and concepts in ways that are not possible with traditional paper and pencil approaches. However, before it is used for this purpose, training should be provided so that teachers and students can become familiar with the features and functions of TI-Nspire^{TM} as a pedagogical tool. The transferability of the research approach of this study to other school settings should also be explored.

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