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A Semester-Long Flipped Calculus Course for Pre-Service Teachers in Singapore

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This paper reports on a study on a semester-long flipped university mathematics course (Calculus II) taught to a cohort of pre-service teachers enrolled in the Bachelor of Science (Education) programme at the National Institute of Education, which is the autonomous teacher training institute of Nanyang Technological University, Singapore. The current study is the second phase of a three-phase project which developed a comprehensive framework to guide the design of three stages of flipped learning activities: pre-class tasks; in-class interactions; and post-class consolidation. A mixed-methods research design was used to collect quantitative and qualitative data over many occasions, through methods such as weekly surveys, to investigate students' perceptions of flipped learning activities. Results of the study suggest that the students generally found the flipped learning activities helpful and enjoyable.

Keywords: flipped learning, calculus, pre-service teachers, Singapore

Introduction

University mathematics is a compulsory course in numerous professional programmes, particularly teacher education in science, technology, engineering and mathematics (STEM) disciplines, business studies, psychology and mathematics. Yet many undergraduate students find it challenging to master. To alleviate this problem, university mathematics lecturers have experimented with various pedagogies to make university mathematics more interactive, enjoyable, and student-centred. Computer-mediated learning (CML) is one of these pedagogies made possible by the availability of powerful computers and online resources. An increasingly popular form of CML is flipped learning, also known as flipped classrooms, inverted classrooms, or inverted instruction (Lage et al., 2000).

The core feature of flipped learning is that students learn new basic content on their own before they come to class (pre-class tasks). Class time can then be used more productively to deal with more difficult work through group discussion and the guidance of the lecturer. The traditional approach of delivering lectures or whole-class teaching in class sessions is 'flipped', becoming a pre-class task undertaken by the students themselves. In addition to learning academic content, students can engage in self-directed learning by completing pre-class tasks, and develop collaborative skills through in-class discussion and problem solving.

Rationale

The National Institute of Education (NIE) is responsible for training most of the teachers in

Singapore's government schools. The participants in this three-phase project were pre-service mathematics student teachers enrolled in the four-year Bachelor of Science (Education) programme. Pre-service training in flipped learning has two key stages. First, the student teachers must develop the competence to self-learn mathematics through the flipped model when they take academic mathematics courses. This active and metacognitive way of learning supports the lifelong learning of mathematics. However, as Dewey noted, 'We do not learn from experience... we learn from reflecting on experience'.[1] Hence, in this project, the student teachers not only experienced flipped learning but also reflected on their experience by responding to written surveys and interviews with the researchers. Second, student teachers need to understand the theories, research and practices associated with different types of flipped learning, and this set of competences is acquired through the mathematics curriculum. The goal of the three-phase project was to work with student teachers in the first stage of their flipped learning training, in terms of lived experiences and reflections. This would prepare them to implement flipped learning in future lessons, should they need to do so. As these cohorts of student teachers were likely to be digital natives, engaging them to use CML in learning mathematics was not likely to be an onerous task.

International interest in flipped learning in K-12 and university education has evolved rapidly for different disciplines. The following observations made in international studies are noteworthy.

- There is a core conceptualisation of flipped learning, but its implementation has varied considerably by discipline and grade level (K-12 and university).
- Advocates of flipped learning claim that this new learning model enables students to develop 21st century competencies, such as self-directed learning, critical thinking, and collaboration skills (e.g. Hamdan et al., 2013; Herreid & Schiller, 2013). However, it also inculcates the self-learning of mathematics at the tertiary level, a metacognitive competence that is important to lifelong learning. For university mathematics, flipped learning can be used to supplement the traditional lecture and tutorial model.
- There have been many more studies of the impacts of flipped learning on students' perceptions of it than studies of students' academic performance after flipped learning. Further, the findings for perceptions and performance have been inconsistent, depending on various factors, such as discipline, grade level, duration of the course, and quality of implementation.
- Few international studies have involved student teachers. This project provides new insights into how student teachers learn mathematics under this pedagogy.
- Studies of flipped learning for university mathematics have tended to be short-term or small-scale. For example, Novak et al. (2017) focused on only one flipped lecture because they did not wish their "flipped" attempt to be a "flop" (p.647). Ogden et al. (2014) studied two implementations of flipped courses in college algebra. Talbert (2014) discussed three flipped classroom designs for linear algebra: "as a one-time class design[ed] to teach a single topic; as a way to design a recurring series of workshops; and as a way of designing an entire linear algebra course" (p. 361). This project grew out of Talbert's third design. It involved two semester-long flipped courses, which, it

[1] https://www.goodreads.com/quotes/664197-we-do-not-learn-from-experience-we-learn-from-reflecting

was hoped, would produce a deeper understanding of the long-term influence of flipped learning and how implementation can change over a longer period of time.

As flipped learning is novel among universities in this region, this project makes a modest contribution to the international literature on the flipped learning of university mathematics, especially among ASEAN countries. It can lead to more effective implementations of this exciting pedagogy to better prepare professionals for careers that require mathematics. Publishing the findings of this project will also enhance communication among university lecturers who plan to experiment or are experimenting with this pedagogy.

Purpose of Study

This project had two principal goals. One was related to praxis and the other was related to research. The main praxis goal was to develop a comprehensive framework to guide the design of a variety of flipped learning activities based on strong theories of both mathematics education and general education. This framework covered three stages of flipped learning: pre-class tasks; in-class interactions; and post-class consolidation. It includes the roles of traditional lectures and tutorials. A comprehensive framework is likely to be more effective than a single flipped activity, such as watching video clips before coming to class.

The main research goal was to investigate the learning experiences of the student teachers who studied academic mathematics through flipped learning. Their learning experiences include both perceptions and academic performance. This report details the findings related to the following research questions:

(1) What is the nature of the online component of the flipped learning experienced by the students?
(2) What is the nature of the face-to-face interactions during in-class sessions?

This project was exploratory; it did not set out to test specific hypotheses about flipped learning. Nevertheless, certain features of flipped learning in mathematics were investigated. The findings could facilitate future practice and research on the use of flipped learning for university mathematics.

All necessary ethics approvals were obtained prior to the start of the project from the institutional review board of Nanyang Technological University. The approval reference number is IRB-2018-07-037.

Participants

This project was conducted in three phases (Phases 1, 2 and 3) for two cohorts (Cohorts A and B) of student teachers (referred to as ‘students’ for the rest of this article). They were treated as ‘students’, not as ‘research participants’, and they were given all of the assistance necessary to master the course content. The main results of Phase 1 which involved only Cohort A have been reported in Ng et al. (2020). This article reports the results and findings in Phase 2 which involved Cohort B.

Calculus II was a second year mathematics courses in the four-year Bachelor of Science (Education) programme. It was an academic course, without explicit links to mathematics pedagogy. Each course included 12 weeks of in-class instruction, with a one-week mid-semester break after week seven, when the class did not meet. Every week, the class met for a two-hour 'lecture' and a one-hour 'tutorial'. These were the institution's labels. The flipped activities for these sessions were different from the usual long lectures and review of assignment problems.

The students in each cohort of this project were in the second year of their Bachelor of Science (Education) programme, which integrates the study of Science and Education over a four-year period. They were awarded teaching scholarships by the Singapore Ministry of Education to enrol in this pre-service teacher programme, and as awardees, they had strong mathematics backgrounds from their K-12 education. In the first year of their programme, they passed Calculus I, Finite Mathematics, Number Theory, Linear Algebra I, and Computational Mathematics, all of which were taught in the traditional way. Many of them earned good grades in these courses.

Cohort B comprised only four female students, with no male students, and they will be referred to as Students A, B, C and D in this article. All four students gave informed consent to participate in this study by means of completing and returning duly signed consent forms approved for use by the institutional review board.

The four students were very competent in mathematics, although one student (Student D) was slightly better than the other three and more inquisitive while Student B was slightly weaker than the rest. They all came prepared for class, paid attention during class, and participated actively in class. Student C was the most hardworking and motivated one among the four students. Calculus II was the four students' first exposure to flipped learning.

Methodology

This project was an exploratory study of flipped learning in two second-year university mathematics courses. It was 'exploratory' because we did not set out to test specific hypotheses about flipped learning based on any particular learning-teaching theory or specific types of implementation. Based on the findings of this project, we hope to generate hypotheses for future research and recommendations for the praxis of flipped learning.

The project comprised three phases involving two cohorts of students. Each phase was considered a 'case study'. A mixed-methods research design was used to collect quantitative and qualitative data for each 'case' over many occasions, through methods such as weekly surveys (akin to the equivalent time series design for a single group) (Cohen et al., 2018). This intensive data collection differed from the typical one-off surveys of student perceptions or attitudes, as reported in the education literature. This approach also allowed us to make iterated changes to the flipped activities (after considering the weekly surveys and online quizzes) and the research instruments between phases, while retaining the major items. With this approach, some features of a design experiment were included (Sandoval, 2013).

There was no comparison group due to the small enrolment in the two cohorts (19 and 4 students respectively). Thus, we were unable to compare the effects of flipped learning versus the traditional lecture-cum-tutorial model, which had been used in many other mathematics courses taken by the students in this project.

Evaluation of the project followed the equivalent time series design for a single group (see e.g. Creswell, 2014). This within-group design was used because it was not feasible to split the small classes into two groups (control versus experiment) under the constraint that all of the students were expected to experience the flipped condition as a curriculum goal of the courses. As undergraduate mathematics classes at NIE are very small (usually no more than 20 students), it was not possible to create larger classes for the study. To address the lack of generalisability resulting from the small sample size, and to strengthen the measures of the degree of implementation, this study examined in depth the flipped learning experiences of each participating student throughout the duration of each course. The differences in such experiences, if any, were then used to suggest plausible explanations or hypotheses that could potentially contribute valuable knowledge to applications of the flipped learning model. The present study was a case study in which 'case' referred to the class taking Calculus II. It investigated how members of this 'case' responded to a new pedagogy (flipped lessons) using multiple sources of evidence. The entire project examined this 'case' under two iterations (cohorts). This was intended to strengthen the validity of the findings compared to one-off case studies. Within this case study, a mixed methods design was used, which is a methodology that advances the systematic integration of quantitative and qualitative data within a single investigation or sustained programme of inquiry. The basic premise of this methodology is that such integration permits a more complete and synergistic utilisation of data than separate quantitative and qualitative data collection and analysis alone. In the present project, the qualitative and quantitative data we collected validated and corroborated each other, creating a solid foundation for drawing well-substantiated conclusions.

Flipped Calculus

The flipped Calculus II course comprised a total of 12 weeks of in-class sessions and 11 weeks of pre-class activities, with a one-week mid-semester break after Week 7. There were a total of 12 two-hour and 12 one-hour in-class sessions (1 one-hour session and 1 two-hour session per week over 12 weeks). The learning activities were classified under three categories: pre-class tasks, in-class interactions, and post-class consolidation. One week prior to the first class, the students were notified through emails to complete the Week 1 pre-class tasks. At the first class, they were briefed on the structure of the flipped model, and their written consent to participate in this study was sought. For subsequent weeks, the videos and the supplementary materials for the upcoming week were uploaded to Blackboard, the online learning management system that all the students had to use for this course. They had one week to complete the pre-class tasks before coming to class. These learning activities are described in the following subsections.

Pre-class tasks

Pre-class tasks gave the students their first encounter with the new materials to be covered in the upcoming in-class discussion. There were five types of pre-class tasks for each course.

(1) *Synopsis*. This was a short paragraph that provided an overview of the topic and the videos for the upcoming week. Figure 1 shows a sample of the synopses.

In this lecture, we first define the partial derivatives of real-valued function of two variables $f(x, y)$ with respect to x and with respect to y at a point (a, b) using limits. We shall state the rules for finding partial derivatives of $f(x, y)$ without using the limit definition. We also illustrate with an example how to find the partial derivatives of functions defined implicitly. We then introduce higher order partial derivatives (like the second, third, ... derivatives of a function of one variable), and state Clairaut's Theorem, which gives a sufficient condition on when two second order partial derivatives are equal. We shall introduce the notion of differentiability of $f(x, y)$ at a point, and state a sufficient condition for $f(x, y)$ to be differentiable in terms of the first order partial derivatives. Lastly, we shall prove that if $f(x, y)$ is differentiable at a point, then it is continuous at that point.

Figure 1. Week 8 synopsis on partial derivative

(2) *Video lectures.* These were mini-lectures narrated by the lecturer. The authors as the researchers perceived video lectures delivered by the course lecturer to be more effective that those delivered by third-party narrators as they could be better aligned with the course content, the students would be familiar with the lecturer's delivery style, and there was a sense of belonging to a learning-teaching community composed of the lecturer and the students. The weekly content was covered in three to five videos, each about 3 to 13 minutes long.
Breaking long lectures into shorter videos encouraged fuller attention and made it easier for the students to watch and re-watch any of the videos under their own control, which encouraged self-learning. The researchers had deemed the requirements for watching these videos to be quite manageable. In general, these videos covered only standard definitions, theorems, simple examples, and routine procedures, so they did not demand too much prior knowledge. Other researchers have found that knowledge requirements can lead to frustration. In this study, the students could pause the video any time and write their own notes. To help them self-check their understanding of the content, each video was linked to an embedded pop quiz, a summary sheet, and a worksheet. This design was based on the critical role of self-assessment through feedback, as advocated in CML. The videos were captured using Camtasia screen-recording and video-editing software, and were then uploaded to Blackboard. Camtasia allows the PowerPoint slides shown on the laptop screen to be recorded, including the things written on the screen using a stylus pen and the lecturer's voice-over. The students had to watch these videos online, instead of downloading them onto their own device for offline viewing. The topics for the videos are given in Table 1 below.

Table 1.
Topics in Calculus II covered in the videos

Week	Topics	No. of videos
1	Infinite sequences and their convergence, limit laws	6
2	Bounded monotonic sequences, definition of infinite series and its convergence, geometric series	5
3	Tests for convergence of series	4

4	Tests for convergence of series, absolute and conditional convergence of series	4
5	Power series, differentiation and integration of power series	4
6	Taylor and Maclaurin series, functions of two variables	5
7	Limit and continuity	6
	Mid-semester break	
8	Partial derivatives, differentiability	4
9	The chain rule (1-hour midterm test followed by 1-hour lecture in class; no videos)	0
10	Directional derivatives, tangent plane, extreme values	4
11	Extreme values, double integrals	5
12	Double integrals	2

(3) *Summary sheet* for every video. This was a printed sheet consisting of one or two questions that captured the main points of the video. The students wrote down their answers manually (Mueller & Oppenheimer, 2014) and checked them in their own ways, for example, by re-visiting the video or referring to their own notes. Figure 2 shows part of the summary sheet for the topic on partial derivative while Figure 3 shows a snapshot of the video to which the summary sheet is related. Working through the summary sheet after watching each video allowed the students to at least acquire its essential content, and this helped them link the content across the videos, resulting in more meaningful learning.

1. Video Clip 8.1: Fill in the blanks.

Let $f(x, y)$ be a real-valued function such that the partial derivatives exist.

(a) To find $f_x(x, y)$, we treat ______ as a constant and differentiate $f(x, y)$ with respect to ________.

(b) To find $f_y(x, y)$, we treat ______ as a constant and differentiate $f(x, y)$ with respect to ________.

Figure 2. A summary question for a video on partial derivative (Week 8).

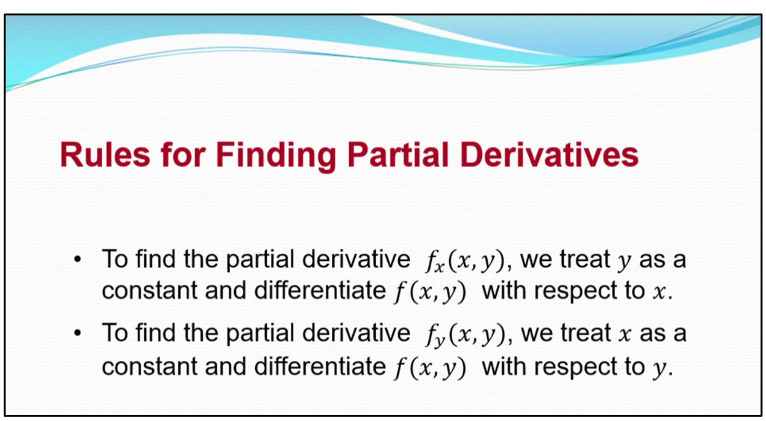

Figure 3. A snapshot of the video corresponding to summary sheet in Figure 2.

(4) *Worksheet problems and activities*. Each worksheet had several procedural problems, similar to the examples explained in the videos. Some videos did not have accompanying worksheets. Solving these problems and completing the activities were another way for the students to check their understanding of the content and to note any difficulties they wanted to raise in the class sessions. Figure 4 shows a sample of the worksheet problems and activities.

Let

$$f(x,y) = \begin{cases} \dfrac{yx^2\cos y}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Show that $f(x,y)$ is not continuous at $(0,0)$.

(b) Show that $f_x(0,0)$ and $f_y(0,0)$ exist.

(c) Is $f(x,y)$ differentiable at $(0,0)$? Explain.

Figure 4. A sample of worksheet problems

(5) *Weekly online quizzes*. Each weekly online quiz assessed the students' understanding of the content covered in all of the videos shown that week. Each quiz had 7 to 10 items, with multiple-choice (some items had more than one correct answer) and true/false questions. The students' answers were 'marked' immediately as either right or wrong, and their responses were captured on Blackboard. Their 'marks' contributed to an 'effort' grade (for participation, irrespective of right or wrong answers), which accounted for 10% of the course grade. This formative cum summative assessment mode was one small way to motivate the students to take the pre-class learning seriously. Figure 5 shows a sample of the weekly online quizzes.

QUESTION 1

If $\sum_{n=1}^{\infty} a_n$ is a series with positive terms such that $\dfrac{a_{n+1}}{a_n} = \dfrac{2}{\left(1+\frac{1}{n}\right)^n}$ for all $n \geq 1$, then the series $\sum_{n=1}^{\infty} a_n$

○ A. converges.

○ B. diverges.

○ C. may converge or may diverge.

Figure 5. A sample of a question in a weekly online quiz.

Discussion questions were downloaded weekly from Blackboard, but the students did not have to solve them before coming to class. They were conceptual and more difficult than the worksheet problems.

In addition to these five types of assigned pre-class tasks, the students were encouraged to consult the textbook, because each course followed the sequence and content in the textbook quite closely. This close alignment helped students who needed different or detailed explanations or solutions to examples that were not given in the videos. Relevant online resources were also given in the synopsis or summary sheets.

Completing these assigned pre-class tasks was expected to take about 60 minutes per week, which the students considered to be manageable. The video lectures and supplementary materials enabled the students to learn mathematics on their own by listening, writing, practising and reviewing their understanding after gaining immediate feedback on their attempts to solve basic problems. This created an enriched flipped learning experience for those who worked conscientiously to complete these tasks.

As mentioned previously, one week prior to the first class in each course, the students were notified by email to complete the Week 1 pre-class tasks. In subsequent weeks, the videos and the supplementary materials for the upcoming week were uploaded to Blackboard, and the students were given one week to complete the pre-class tasks before the next class session.

In-class interaction

The weekly in-class interactions served several learning-teaching functions, as planned for the two-hour lecture session and the one-hour tutorial session, discussed below. The two-hour 'lecture' typically proceeded as follows.

(1) The lecturer explained the corrections to the students' mistakes in the online quizzes and re-explained specific points in the videos related to these mistakes. The students were encouraged to ask questions based on the notes they took when they worked on the pre-class tasks. Most learning-teaching theories emphasise that mistakes are inevitable when learning new content for the first time, and that providing specific feedback that students can act on will deepen learning.

(2) The lecturer delivered mini-lectures on the more difficult content, such as alternative solutions, complex concepts and complicated proofs. These mini-lectures expanded on the content covered in the weekly videos.

(3) The students compared their solutions of the worksheet problems and activities with each other, and the lecturer circulated throughout the class to provide at-seat help and check on the students' solutions.

(4) Students were called on to present their solutions on the whiteboard to the whole class. The classmates and the lecturer commented on the solutions, and the presenter was encouraged to 'defend' the solutions. This helped the students develop competence to talk about mathematics and to listen carefully to others' mathematical ideas. Such competence is especially important for future teachers to acquire through their mathematics courses.

(5) To leverage the benefits of cooperative learning, the class was given about one third of the session to tackle the discussion questions in pairs. Subsequently, the lecturer led whole-class interactions on their solutions to these discussion questions.

(6) The session typically ended with a brief summary given by the lecturer.

The one-hour tutorial was used to discuss 'challenging' problems in the problem sets, which were provided for post-class consolidation. Typically, the students were called on to present their solutions to these challenging problems on the whiteboard and to answer questions from their peers and the lecturer. If necessary, the lecturer guided the class to arrive at the correct solutions to problems that they were unable to solve on their own.

Post-class consolidation

The following activities were planned to help the students consolidate their learning after the in-class sessions.

(1) *Problem sets and posted online solutions*. For the flipped calculus course, there were several problem sets. Each problem set consisted of several 'procedural' problems and 'challenging' problems, covering the class content for one to two weeks. These problem sets were delivered through Blackboard approximately two weeks before their respective tutorial sessions. The students were directed to post their solutions to the assigned 'procedural' problems on Blackboard. It was expected that most of them could solve such problems. Students could either scan and upload their handwritten solutions or type their solutions; an equation editor was available for them to type mathematical symbols. Posting solutions to the procedural problems only freed up more time to discuss the challenging problems during the tutorials. This was in line with a key feature of flipped learning: focusing class time on higher cognitive activities. Nevertheless, the students were encouraged to try to solve the challenging problems, because they would be called on to present their solutions during the tutorials. On average, each student posted about five solutions to the procedural problems. The students could read and comment on the posted online solutions before the lecturer commented on those solutions and suggested alternative methods. This approach engaged the students in peer learning and allowed them to obtain almost immediate feedback from their lecturer and peers, if they logged on regularly. This gave the flexible assessment role of CML leverage over face-to-face discussions.

(2) Those who did not do well in the first attempt could complete the online quiz again, but the scores for their subsequent attempts were not included in the computation of the 'effort' grade. This form of retrieval practice, where the answers have been discussed in class, is another example of using 're-visit' feedback to consolidate learning. However, this was optional. The students had to take responsibility for their own learning.

(3) The students were encouraged to seek help from the lecturer in his office outside the class schedule. However, they typically clarified their doubts during class sessions and only requested individual consultations just before the final examination.

Data Collection and Analyses

Several research instruments were designed to collect quantitative and qualitative data. They are delineated as follows.

Weekly surveys of pre-class tasks

The weekly surveys were used to collect two types of data: student perceptions of the five types of assigned pre-class tasks and the amount of time they spent watching the videos and completing the worksheet/activities (time on tasks).

The students rated each task on a 6-point scale (1 to 6), for two dimensions: 'help me learn', and 'enjoyable'. All four students in Cohort B consented to participate in the research submitted the hardcopy of these surveys (including their names or matriculation numbers) to the lecturer at the first in-class session of the following week.

For time on tasks, the researchers used the times reported by the students. Although Blackboard, the learning management system used in the university, provides log-on times when the students watch the videos and answer online quizzes, the researchers also wanted to collect time spent on the supplementary materials, which could not be captured online. The Blackboard system could record the time that the students were logged on, but there were issues about interpreting this. They might have been logged on but were not working on the task. Hence, the study relied on self-reported times.

Mid-semester survey and end-of-course survey

The mid-semester survey was conducted at the end of Week 7 and the end of course survey was administered on the last day of the course in Week 12. In these two surveys, the students rated the workload for each activity (1 = not enough; 2 = just right; 3 = too much). They rated their perceptions of the pre-class tasks, in-class interactions and post-class consolidation on two dimensions ('helps me learn' and 'enjoyable'). They also responded to three open-ended questions: re-visited the videos; used additional learning resources; suggestions for the second half of the semester (mid-semester survey) or flipped learning of university mathematics.

It is common practice to 'validate' instruments that measure attitudes or perceptions using statements about psychological constructs. However, for this project, the three types of surveys assessed perceptions by asking students to rate each activity on only two dimensions: *helpful* and *enjoyable*. Such basic ratings did not require 'validation'. It was more important to ensure that the data were 'trustworthy'. The data from this project were considered trustworthy because the students were serious about giving their honest opinions, and there were very few out of range values and/or missing data.

End-of-course interview

This interview was conducted at the end of the course, but before the final examination. It was designed to collect qualitative data to provide further insights into flipped learning, supplementing the quantitative data from the surveys. The interviews were audio-taped and transcribed.

Given the small number of students in this project, basic descriptive statistics are reported. Comments from the interviews and observations from the class sessions are cited to provide insight into the quantitative results.

Findings and Results

This section describes results and findings of phase 2 of this project which is for Cohort B.

Time spent on pre-class tasks

Time spent on assigned learning tasks is a critical factor in effective learning. It demonstrates whether the student are conscientious about learning and may predict their final performance.

The weekly surveys asked the students to indicate how much time they spent on watching each video and the time they spent on completing the weekly worksheets/activities. The average amount of time the 4 students spent on watching the videos and completing the weekly worksheets/activities ranged from approximately 103 minutes (Week 4) to 178 minutes (Week 7). In their mid-semester and end-of-course surveys, all four students indicated that this workload was 'just right'. Hence, flipped learning did not add drastic demands to their workload. Indeed, the amount of time spent on the pre-class work for the flipped calculus course was comparable to the amount of time which students were expected to spend on their out-of-class work in the traditional lecture-tutorial model; in the non-flipped version, the instructor typically expected students to spend at least one hour after a one-hour lecture to go through and make sense of what was covered in the lecture.

Figure 6 shows the amount of time the 4 students spent on pre-class activities weekly. Student B, who was relatively weaker than the other 3 students, had spent more time on the activities than the others from Week 7 onwards when the demand of the Calculus course got higher.

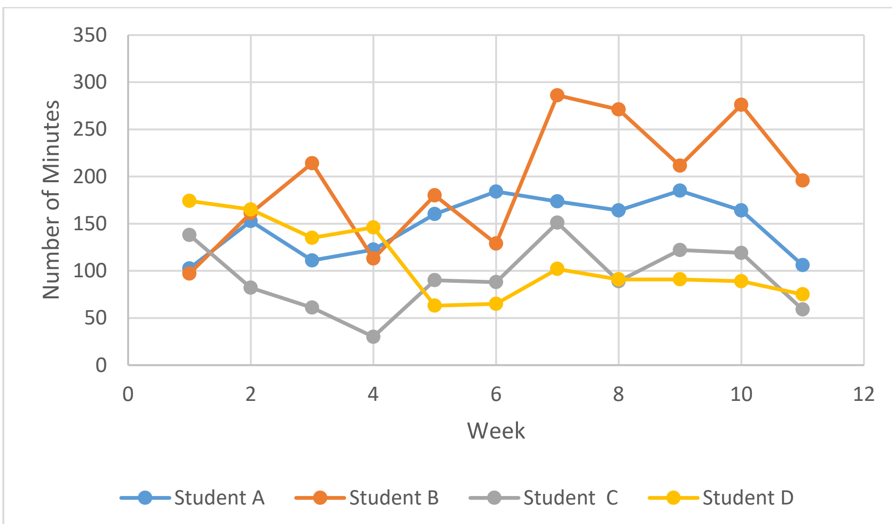

Figure 6. Time spent on pre-class activities.

The weekly data were aggregated to give the overall time spent on the videos and the worksheets, for the entire semester. Table 2 provides a summary of these times (to the nearest minute). On average, the students reported spending more time completing the worksheets or activities than watching the videos. This was not surprising because the videos were watched to gain understanding, whereas solving the worksheet problems required the students to apply their understanding, which was more demanding.

Table 2.
Time (minutes) spent on videos, worksheets/activities, and both activities over 11 weeks

	Time (in minutes)
Videos	
Mean	383
Shortest	328
Longest	431
Worksheets/Activities	

Mean	1496
Shortest	1029
Longest	2134
Total time	
Mean	1879
Shortest	1357
Longest	2520

Perceptions of flipped activities

Student perceptions were measured by the weekly surveys, the mid-semester survey, the end-of-course survey and the end-of-course interview.

The students responded in fairly similar ways to the questions in the weekly surveys, mid-semester survey, and end of course surveys. They rated the amount of work for almost every flipped learning activity in each course as 'just right' (value 2). In all three surveys, they also gave most of these activities scores of 5 or 6 (scale 1 to 6) in terms of 'help me learn', and 'enjoyable'. Hence, the students quite uniformly expressed positive experiences in these flipped courses.

The mean ratings for the students' perceptions of the amount of workload (scale 1 to 3) and all of the flipped activities, in terms of being helpful and enjoyable (scale 1 to 6), were based on the mid-semester and end-of-course surveys. They are given in Table 3. Overall, the students found that the amount of workload for each activity was 'just right'. Some commented that the workload became heavier in the later weeks of the course, but this could have been an issue of time management. The workload for in-class explanations, and in-class discussions with classmates were slightly less than 'just right'. These areas can be considered for future practice and research.

Table 3.
Student ratings of workload and flipped activities from mid-semester and end-of-course surveys

	Workload		**Helpful**		**Enjoyable**	
	Mid	End	Mid	End	Mid	End
Pre-class tasks						
Synopsis	2.0	2.0	4.5	5.0	4.5	4.5
Videos	2.25	2.5	4.75	4.5	5.0	4.5
Summary Questions	2.25	2.0	4.75	5.0	5.0	5.0
Worksheets/Activities	2.0	2.0	5.25	5.5	4.75	5.25
Online Quizzes	2.0	2.0	5.25	5.75	4.75	5.25
In-class Interactions						
In-class Discussion Questions	2.0	2.0	5.0	5.5	4.0	5.0
In-Class Explanations	1.75	1.75	5.0	5.25	4.5	5.25
In-Class Discussions with Classmates	2.0	1.75	5.25	5.5	5.0	5.5
Post-class Consolidation						
Problem Sets	2.0	2.0	5.5	5.5	4.75	5.25
Posted Online Solutions	1.75	2.25	5.0	5.25	5.0	5.0

Perceptions of pre-class tasks
In both surveys, the students consistently gave high ratings to the tasks that helped them learn and they enjoyed completing most of them. In terms of helpfulness, the tasks can be arranged in descending order as follows: Online quizzes > Worksheets/Activities > Summary questions > Videos/Synopsis.

Tackling the worksheet problems and activities was deemed very helpful and enjoyable. This supports the principle that active learning facilitates learning. Watching the videos was also very helpful and enjoyable. All four students commented on the positive features of the videos. They said the videos provided clear explanations, were bite-size and easy to follow and emphasised the main concepts. The videos explained more than the textbook, used animation and did not repeat the content from class. Furthermore, the videos could be paused or replayed, and had a nice pace.

Online quizzes were perceived to be the most helpful, but were deemed less enjoyable in the mid-semester survey for various reasons. The students said they were tricky and stressful, and for Student A, there was a last minute rush to complete them. Three students commented that the questions were quite easy and straightforward, just like definitions. Because these online quizzes contributed to the students' effort grade, all of them submitted most of the online quizzes. Student B said the online quizzes were not so enjoyable, but they were helpful in consolidating his learning. Almost all students found completing the summary questions rather helpful and fairly enjoyable while they found reading the brief synopsis somewhat helpful and enjoyable.

The perceptions of almost all of the tasks (except videos) became more positive after the mid-semester survey and before the end-of-course survey. In particular, the students appreciated how helpful the synopses were (mean: 4.5 to 5.0), and enjoyed completing the worksheets/activities (mean: 4.75 to 5.25) and online quizzes (mean: 4.75 to 5.25).

A plausible hypothesis to explore is that as the students gained more experience with these well-planned pre-class tasks, they appreciated how completing the tasks helped them master the content and some of them enjoyed the positive experiences more than they did in the earlier phase of the course.

During the first session of each week, the lecturer collected the printed copies of the weekly survey and took notes of any unusual responses that needed his attention. For almost all of the weekly surveys, only a few minor points were found and they were easily dispensed with. This suggests that there was consistency in the students' perceptions across the weeks.

Perceptions of in-class interactions
The three types of in-class interactions were ranked in descending order as follows, for both helpful and enjoyable: In-class discussions with classmates > In-class discussion questions > In-class explanations. The means were generally higher than they were for all of the pre-class tasks, with the exception of online quizzes. The students preferred face-to-face explanations and discussions over self-learning, even though the differences were small. Student A, for example, explained that the disadvantage of self-learning via videos was that 'when we are not sure, we can't directly ask questions on the spot when we watch the videos'.

In-class discussions with classmates had the highest mean for being helpful and enjoyable in both surveys. Some students emphasised the need to prepare for class by completing the pre-class tasks. Student C explained that 80% of the assigned pre-class tasks were 'within our capacity … So if we use that class time to go through that, 80% of it is kind of wasted … there [could be] more time to actually do more thinking and more discussions'.

In-class explanations were also much appreciated. The lecturer used a variety of explanations: mini-lectures on more difficult content, comments on students' mistakes in online quizzes, corrections and on-the-spot feedback for the solutions presented by the students.

The discussion questions required higher order thinking and were found to be helpful. Student B commented on them, saying, 'like what are the worst questions that can come out in exams, that kind of thing. At least I'm more mentally prepared'. However, the students did not enjoy answering the questions compared to the explanations and discussions.

In another in-class interaction students were encouraged to ask questions about things they still did not understand. This aspect of the interaction was not assessed. Student D recognised this, saying, 'when you have already learned all the content then you go into class it's really more about asking questions rather than receiving new information'. Perhaps due to the small class size, the students felt comfortable asking questions in front of the class.

Perceptions of post-class consolidation

The problem sets and posted online solutions were perceived to be very helpful, and not any less enjoyable than some of the other activities. Three of the students uploaded scanned copies of their handwritten solutions, while Student C typed her solutions online, using an equation editor. This task could have been made more enjoyable through the more efficient use of technologies to prepare mathematical solutions including symbols and diagrams and submit them online.

Student D would have preferred the problem set to contain more questions. However, Student B was worried about the workload entailed in posting solutions online and related tasks. Nonetheless, she said that the questions were 'definitely helpful, because I believe that when you are triggered to think, it definitely will help your learning'.

The students talked about the many benefits of posting online solutions and reading the solutions posted by their group members. The questions tested their ability to apply content. They received helpful feedback from group members and the lecturer. They then used the solutions (after feedback) to prepare for tests and examinations and build confidence in their own work. As each student had to post only five solutions to the online questions throughout the course, this was considered an acceptable workload.

<u>Weekly online quizzes</u>

The online quizzes assessed the students' understanding of all of the video content for a particular week. Their responses were automatically marked in Blackboard, but only participation (not the number of correct answers) was taken into account for the 'effort' mark (10%). This low-stakes assessment provided a certain amount of extrinsic motivation for the students to complete these quizzes. The lecturer examined their responses to determine how to deepen their understanding in the upcoming class sessions.

For each week, the mean score obtained from the individual student scores was divided by the maximum possible score to obtain the percentage scores. These percentage scores for the online quizzes over the 11 weeks are plotted in Figure 7. Overall, the scores suggest that the students learned the basic content covered in the videos throughout the course. The fluctuations over the weeks may reflect the levels of difficulty of the quiz items or the effort the students made.

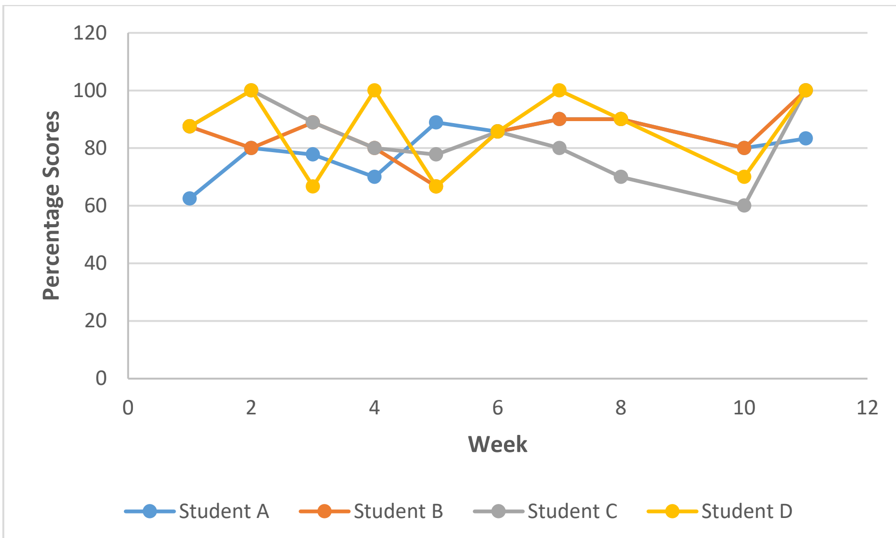

Figure 7. A plot of the scores for the online quizzes.

Open-ended questions

Some insights can be gained from the students' written comments to the open-ended questions in the surveys and the weekly interviews, at the mid-semester break, and the end of the course. Several new points are discussed below.

<u>Re-watch videos</u>

All of the students wrote in the weekly surveys that they did not re-watch the videos after the in-class sessions or before the upcoming examinations. Some of the reasons given were as follows: no time; sufficient information provided in notes; and 'the notes are more accessible because I do not have to find a particular concept in a specific part of the video' (Student A). However, during her interview, Student A said that she would re-watch a video if she was distracted, and Student C said that 'if I don't understand and I need to go through the example again, I will re-watch it'. Apparently, re-watching took place within a given session but not after the in-class discussion or when the students prepared for exams.

<u>Refer to resources other than the textbook</u>

Almost all of the students wrote that they did not refer to any other resources. Student C wrote that she 'did not know we are supposed to' refer to an online animation of a directional derivative. Student A found that the animation helped her visualise the concept in 3D.

<u>Prepare for final examination</u>

The students' preparation for examinations changed after using the flipped learning materials, which were unavailable in the traditional mode. Student A listed five types of materials she used to revise in preparation for Calculus II: tutorial questions (discussion questions), lecture notes, summary questions, worksheet activities, and online quizzes. These were more than the three things she had relied on in the past: tutorial questions, lecture notes, and past year papers. Student B explained that when she revised her lecture notes, she could 'remember what he said in the videos', and she wrote this down beside her notes. She had been unable to do this in the past.

Other students used the flipped learning materials in slightly different ways. For example, Student D used the textbook, reviewed the discussion questions, re-did some of the questions in the problem sets, and looked at papers from the past year, but she did not re-watch the videos.

Class presentations

There was a new open-ended question in the end-of-course survey. All of the students said that giving presentations was a good experience for several reasons, as follows.

- They could identify their misconceptions, as commented on by their peers and the lecturer. Student B noted that she realised her mistakes when she wrote on the whiteboard.
- They learned how to present their solutions better and to use proper notations.
- Discussion and thinking were promoted.
- Presenting motivated them and helped them build confidence. Student C said that she tried to seize opportunities to present her solutions.

Preparing to conduct flipped lessons in school mathematics

One of the questions in the end-of-course survey asked the students to comment on whether their own experience of learning mathematics through flipped learning had prepared them to use flipped lessons in school mathematics in the future. One student wrote 'maybe' but did not elaborate. The other three students gave positive responses. Student D wrote: 'Yes, as I can put myself in the shoes of the students and I can adjust the tasks accordingly to suit their needs'.

This was a small indication of the success of the programme and a reason to prepare student teachers to implement flipped learning in schools. It would be helpful to collect more responses to answer this question in greater depth.

Discussion and Conclusion

The researchers conceptualise flipped learning as a powerful integration of sound teaching/learning theories, strong disciplinary contents (mathematics in this study), and accessible internet tools. Sound teaching/learning theories cover advance organisers, clarity of objectives, learning progressions, and self-monitoring (metacognition) of learning with embedded questions; strong mathematics disciplinarity includes the use of precise mathematics language, examples and counter-examples, and different modes of mathematical representation; accessible internet tools (via Blackboard) which enable immediate continual quizzes and immediate feedback, peer discussions, and record keeping.

These concepts and the associated practices are applied to provide comprehensive pre-class materials and activities to motivate and to engage students to complete them under self-directed learning, to inculcate deep processing of mathematics contents through direct instruction of more sophisticated mini-lectures, discussions, and group problem-solving at in-class sessions, and to consolidate learning through group posting and discussion of solutions to new problems after class sessions.

By dividing flipped learning into these three stages with their distinctive learning tasks and processes, this "integrated flipped learning" model is the researchers' contribution to the theoretical foundations for flipped learning so that it is richer and more impactful than short flipped learning sessions. In this study, the researchers tested this "integrated flipped learning" through Calculus II for a cohort of NIE student teachers, lasting one whole semester.

The findings from this study may not be generalisable because they were based on a small sample size. Further, the study involved only students with good mathematical backgrounds; no comparison groups were included. In addition, the study conceptualized and operationalized the flipped learning experience to one which is characterized by a spectrum of pre-class, in-class, and post-class activities in flipped learning, rather than a general perception. While this approach is useful for understanding the nuances of flipped learning, it might also make it more difficult for larger findings and implications to be drawn out. Nevertheless, the following points may provide ideas for further practice and directions for future research on flipped learning.

In this study the undergraduate mathematics courses (Calculus II) used flipped learning for an entire semester lasting 12 weeks. Engaging students in flipped learning over the whole semester, instead of just one lecture or a small section of a course, provides ample opportunities for them to meet the challenges of this new learning model and to develop the "culture" of "preparation required before attending class" (Novak et al., 2017, p. 656). Although students may express favourable perceptions of a new learning model because of its novel effect, these perceptions may change under extended exposure. Nonetheless, throughout the semester-long course, the students in this study consistently rated their flipped experiences very highly in terms of enjoyment and helpfulness.

Further research could explore the long-term perceptions and impacts of flipped learning, including the inculcation of fruitful learning strategies, such as being prepared for upcoming classes, writing notes, engaging in active in-class discussions, and using resources to prepare for the final examination.

A plausible explanation for the students' consistently positive perceptions in this project relates to the well-designed structure of the flipped mode. The pre-class tasks catered to the different ways students from diverse mathematics backgrounds could learn mathematics, such as providing clear explanations in the videos, offering immediate feedback on online quizzes and highlighting key content through synopses and summary sheets. Flipped learning must involve more than simply asking students to watch pre-recorded video lectures. Even though the instructor in this project spent 120 hours recording and editing the video lectures, this demand was a one-off that will not need to be repeated for subsequent runs of the course. The time he spent designing and implementing the three types of learning activities was also not burdensome, partly because he could convert previously used materials into the flipped mode, and partly because of his past experience of teaching flipped courses. Another advantage of flipped learning is that the materials can be used again in subsequent offerings, with revisions based on evidence collected from the students.

The students in this study reported spending, on average, around two hours per week completing the pre-class tasks. Most of them found this additional out of class study load 'just right', and said that the tasks embedded in this 'acceptable' load were very helpful and enjoyable. Thus, they achieved a fine balance between the benefits of self-directed learning and the demands of time and effort. In the literature, one factor that might prevent tertiary instructors from experimenting with flipped learning is the extra workload on instructors and students. This study demonstrated that this additional demand is both manageable and beneficial, especially after repeated trials. Instructors from different institutions teaching students from diverse disciplines might encounter challenges that are different from the researchers'. Sharing these experiences will help spread flipped learning in tertiary institutions.

Education research has consistently found that prior cognitive achievement is a stronger predictor of academic performance than current perceptions, attitudes or feelings toward a course. This should not imply, however, that perceptions are not relevant to successful flipped learning. Indeed, as noted earlier, students must be convinced of the need to learn mathematics in non-traditional ways, such as flipped learning, and this factor must be carefully managed and assessed, for example, through surveys and interviews.

In this study, there were individual differences in the time spent on pre-class tasks. However, the students still consistently reported that these tasks were helpful and enjoyable. One implication is that students benefit from working in their own way on different types of learning activities, and that this variety supports the self-learning of basic mathematics in preparation for class discussion. A slight improvement in the students' already favourable perceptions was found after the mid-semester and before the end of the course. This is a strong indication that long-term engagement with carefully planned learning activities can bring about more positive perceptions.

Having students complete a full course in flipped learning was especially relevant for the cohort of students in this study because they were pre-service teachers who could be called upon to implement flipped learning in their future mathematics lessons. In Calculus II the students were required to provide feedback and reflect on their flipped learning experiences on a weekly basis. The combination of direct experiences and reflection will hopefully prepare them to empathise more with the needs and feelings of their future students under this learning model. For example, they may be able to transfer to their school situations how they overcame the challenges of mastering new mathematics using flipped learning.

As noted above, the students involved in this study entered the course with strong mathematical backgrounds. Future research could investigate flipped activities that work well for students from diverse mathematics backgrounds and from different disciplines, such as teacher education, STEM, economics, and the humanities. The literature has not reported the impacts of flipped learning on such diverse groups of students or their perceptions of flipped mathematics courses.

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