Title Tools and approaches for integrating computational thinking and mathematics: A scoping review of current empirical studies Author(s) Shiau-Wei Chan, Chee-Kit Looi, Weng Kin Ho and Mi Song Kim

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Tools and Approaches for Integrating Computational Thinking and Mathematics: A Scoping Review of Current Empirical Studies

Abstract

review of learning tools for integrating CT and from 2015 to 2021. The review shower eMath-connected integration (CTIMCI).
Fed assessments, only about half of the assess c evidence of their validity and reliability. Filigi The importance of computational thinking (CT) as a 21st-century skill for future generations has been a key consideration in the reforms of many national and regional educational systems. Much attention has been paid to integrating CT into the traditional subject classrooms. This paper describes a scoping review of learning tools for integrating CT and mathematics in current empirical studies published from 2015 to 2021. The review showed that most of the studies implemented CT-intensive Math-connected integration (CTIMCI). While the majority of the studies employed integrated assessments, only about half of the assessments have been validated by providing psychometric evidence of their validity and reliability. Five major types of CT tools had been identified, i.e., digital tangibles, apps and games, programming languages, formative or summative assessments, and other technological tools. In many instances, the tools also provide functions of assessment of CT skills. The most assessed CT competencies were including algorithms and algorithmic thinking, abstraction, testing and debugging, loops, and sequences. Geometry and Measurement was the most assessed mathematics topic. Our scoping review is beneficial in the investigation of the literature on CT and mathematics education, as well as guides those who are interested in developing curriculum, programs, or assessments that involve the integration of CT and mathematics.

Keywords: computational thinking; mathematics; tools; approaches; scoping review; empirical studies; assessment

The idea of computational thinking (CT) originated from Papert (1980) in his book "Mindstorms: Children, computers and powerful ideas." As "learning to communicate with a computer may change the way other learning takes place" (Papert, 1996, p. 6), CT has been advocated by many researchers to "forge ideas" (p.13). For example, Wing (2006) contended that "to reading, writing, and arithmetic, we should add computational thinking to every child's analytical ability" (p.33). CT is viewed as an essential ability that ought to be possessed by everyone, not just beneficial for computer scientists. Further, Wing (2010) described CT as "the thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent." (p. 1). The information processing agent can be a human being, a computer, or a machine (Wing, 2006).

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and scholars Other researchers and scholars came up with their definitions relative to their specific research fields, but to date, there is no consensus on an operational definition of CT yet (Hsu et al., 2018). Zhong et al. (2016) argued that the definitions of CT can be categorized into three different perspectives, namely CT is a problem-solving process, CT is a crucial form of expression, and CT is the three-dimensional CT framework by Brennan and Resnick (2012). Hoyles and Noss (2015) viewed CT as involving decomposition (solving a problem includes solving a set of smaller problems), abstraction (seeing a problem at different detail levels), pattern recognition (seeing a new problem as associated with previous problems), and algorithmic thinking (seeing tasks as to smaller linked discrete steps). These four elements are regarded as a four-stage problem-solving process in accordance with the study of Tabesh (2017). Moreover, Buitrago Flórez et al. (2017) proclaimed CT as a means of reasoning that accumulates some high-level practices and skills in the core of computing, but was pertinent to various areas far beyond computer science.

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technique to encourage students to be invo
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c C careers as well as to assist them to become science, technology, engineering, and mathematics (STEM)-savvy citizens (Weintrop et al., 2016). English (2018) claimed that CT and mathematics are natural partners. Weintrop et al. (2016) and Ho et al. (2017) also declared that CT and mathematics have a mutual relationship, which is about utilizing CT to enrich mathematics and employing mathematics context to enrich CT. Hardin and Horton (2017) stated that immersing CT in the mathematics syllabus not only provides computational skills to the students but also enables the students to better comprehend mathematical concepts and skills. CT practices also can serve as an effective technique to encourage students to be involved in powerful ideas of mathematics and stimulate their mathematical habits of mind (Pei et al., 2018). Durak and Saritepeci (2018) and Durak et al. (2019) suggested that students' achievement in mathematics positively influences their CT skill levels. Hence, the instructors ought to design the appropriate curricula and learning tools related to CT practices and infuse them into the mathematics Nevertheless, it could be a challenge to merge the content areas of CT and mathematics

while at the same time conserving the integrity of the two domains. It is hard to find a good match between CT concepts and certain mathematical content (Israel & Lash, 2019). The integrated approach to instruction may affect the design of appropriate assessments. The assessments used can be problematic as the interactions between these two disciplines may disprove the outcomes of the assessments and result in measurement interference. This is due to the outcomes in one discipline that sometimes might be easier to be measured when compared to

another discipline (Bortz et al., 2020). Hence, this study intends to uncover how CT has been incorporated into the mathematics classroom from recent empirical studies.

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the use o Several reviews of CT in mathematics instruction have been published in previous years such as studies from Barcelos et al. (2018) and Hickmott et al. (2018). In these earlier reviews, there is no detailed information about the types and forms of CT tools, and the assessed CT competencies and mathematics topics in the integrated instruction. This current scoping review seeks to fill in the missing literature. It is important to explore these research gaps as earlier studies such as Berkaliev et al. (2014) emphasized the power of computational tools in solving problems in mathematics that cannot be done manually. The usage of computational tools could improve the students' performance in mathematics. diSessa (2018) also stressed that the application of computational tools affected the students' experience in learning mathematics. Thus, this paper aims to examine the integration of CT into mathematics instruction, provide a more organized view of the use of CT tools and approaches in mathematics education, and determine the CT competencies and mathematics topics being assessed in the integrated instruction. Three research questions stated below drive this study:

(1) How has CT been integrated into mathematics instruction in recent empirical studies?

(2) What are the types and forms of CT tools being applied in the integrated instruction?

(3) What are the CT competencies and mathematics topics being assessed in the recent empirical studies?

2. Literature Review

Several scholars and researchers discussed the inclusion of CT in tandem with mathematics, but their ideas on the integration of CT into the mathematics classroom varied from

thematical thinking were algebra, calculus,
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atting between different semiotic represe
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p each other. For instance, Barr and Stephenson (2011) enumerated nine main CT concepts and capabilities that could be incorporated into mathematics instruction, namely data collection, data analysis, data representation, abstraction, problem decomposition, algorithms and procedures, simulation, parallelization, and automation. According to Sneider et al.'s (2014) Venn diagram model, the common factors of CT and mathematics included analyzing and interpreting data, modeling, problem-solving, and statistics and probability. Meanwhile, the non-overlapping factors of CT were robotics, programming, networking, and simulation whereas the nonoverlapping factors of mathematical thinking were algebra, calculus, arithmetic, and set theory. Barcelos and Silveira (2014) identified three transferrable high-order skills between CT and mathematics: (a) alternating between different semiotic representations, (b) establishing relationships and identifying patterns, and (c) building descriptive and representative models. Weintrop et al. (2016) developed a model that integrates mathematics and CT, resulting in four major categories of data practices, modeling and simulation practices, computational problemsolving practices, and systems thinking practices.

To incorporate CT and mathematics, Waterman et al. (2018) proposed three levels of CT integration: exist, enhance, and extend. CT skills, practices, and concepts that already exist in the lessons can be explained with instances of how they can be linked to technology. For example, the existing activity of using physical models to comprehend certain phenomena is connected to CT concepts. Besides, extra lessons or tasks are added to enhance the disciplinary concept and come up with a clear link to computing concepts. For instance, the students are required to collect data on their own, construct a visual representation manually, and analyze the data. New lessons or series of lessons extend the disciplinary concepts as a foundation for computer science exploration, usually involving programming tasks. For example, the students utilize a computer

simulation and alter underlying code or variables to explore how dynamic systems change over time.

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TT, then the Israel and Lash (2019) also put forth three levels of integration for CT and mathematics, namely no integration, partial integration, and full integration. No integration refers to the lessons of CT or mathematics being taught separately in the classroom. In the lesson, the teachers were either focused on CT or mathematics, for example, students were introduced to the Scratch blocks "Ask a question and wait" and "Answer" in the Sensing tab in the unplugged CT lesson. Meanwhile, partial integration means CT is employed to strengthen mathematics, or mathematics is employed to strengthen CT in the lessons. By adapting Kiray's (2012) model, if CT outcomes are more dominant than the mathematics outcomes, then the instruction is regarded as CT-intensive Math-connected Integration (CTIMCI); if mathematics outcomes are more dominant compared to CT, then the instruction is viewed as Math-intensive CT-connected Integration (MICTCI). For instance, students were allowed to demonstrate their comprehension of fractional parts through a CT activity in which students design their own fractional part stories with animations in Scratch. The mathematics content was greatly focused in the lesson and CT activity was built into it. Full integration is the content of both CT and mathematics are taught together or the affordances of CT or mathematics are employed to teach the other domain. For example, students would learn about different polygons by animating them in Scratch. In the lesson, the students would walk the shapes of the polygons and then code those shapes. In this case, the teachers taught mathematics concepts directly through CT activities rather than teaching mathematics lessons, and then had students demonstrate understanding through CT (Israel & Lash, 2019).

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lies put under the category of full integration.
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e There were several recent reviews conducted on the integration of CT and mathematics. A recent review study implemented by Nordby et al. (2022) analyzed ten empirical studies to present an overview of the existing literature from 2000 to 2021 on CT activities in primary mathematics education and to explain how they are integrated into primary mathematics teaching and learning. They found two types of CT activities in the primary mathematics research, i.e., activities focusing on skills (for instance abstraction, decomposition, looping, debugging, conditionals, and sequencing) and process-oriented activities (creativity, communication, engagement, and engagement). In the reviewed studies, there were five studies put under the category of partial integration, one study put under the category of a mix of partial and full integration, and three studies put under the category of full integration. Kallia et al (2021) conducted a systematic literature review to determine what characterizes CT in mathematics education and which CT characteristics can be addressed within

the subject. They conducted a Delphi study that gathered the opinions of 25 math and computer science experts on the possibilities for addressing CT in mathematics education. The Delphi study's findings, which support the literature review's conclusions, indicate three key parts of CT that need to be addressed in mathematics education, namely, problem solving, cognitive processes, and transposition.

Another review study was performed by Hickmott et al. (2018) by recognizing peerreviewed articles on CT in K-12 educational settings that were published between 2006 and 2016 and figuring out how these research-related CT to mathematics learning. The results indicated that the majority of the studies: (1) originate from computer science academics rather than education experts, (2) involve mathematics but primarily focus on teaching programming skills, (3) present small-scale research designs on self-reported attitudes or beliefs, and (4) seldom deal

with concepts in mathematical domain areas such as statistics, probability, functions, and measurement. Barcelos et al. (2018) conducted a review of the articles published between 2008 and 2017, to identify studies that looked at how the relationship between mathematics and CT has been proven through didactic activities at all levels of education. They discovered that the didactical activities used in the studies were related to a wide range of mathematical concepts and that the activities were carried out utilizing a variety of computer tools.

3. Method

s study was intended "to summarize and diss
gaps in the existing literature" (Arksey & O'
ucted using the five framework stages of Arl
earch questions; (b) identifying relevant stud
o collating, summarizing and reporting Scoping review in this study was intended "to summarize and disseminate research findings" and "to identify research gaps in the existing literature" (Arksey & O'Malley, 2005, p. 6-7). This scoping review was conducted using the five framework stages of Arksey and O'Malley (2005), i.e. (a) identifying the research questions; (b) identifying relevant studies; (c) study selection; (d) charting the data; and (e) collating, summarizing and reporting the results. Six databases were utilized as the search engines to identify the relevant studies, namely *Scopus*, *Web of Science*, *Science Direct*, *SpringerLink*, *Taylor & Francis Online*, and *ERIC (Education Resources Information Center)*. The keywords performed for the document search in any part of the article including title, abstract, keywords, and full text, were: "computational thinking" AND ("mathematics" OR "math"). As Table 1 shows, the total search results were 2233 documents including Scopus (491), Web of Science (363), Science Direct (341), SpringerLink (584), Taylor & Francis Online (297), and ERIC (157).

-Insert Table 1 here-

Subsequently, three-round of screening was conducted for study selection to filter out the inappropriate articles as demonstrated in Figure 1. For the first round of screening, the duplicates

rs, and book series; (c) Must be empirical respresentations of framework or literature reversentations; (e) Must be empirical studies that
ublished in the English language. Based of ally, 73 articles are recognized to be a of the articles ($n = 336$) and the articles without full text ($n = 23$) were excluded. Hence, a total of 1874 articles were filtered for the screening for relevance in the second round. The irrelevant articles $(n = 743)$ were removed in the second round of screening. Then, a total of 1131 articles were filtered for exclusion in the third round. In the third round of screening, 1058 articles that did not meet the requirements of the inclusion criteria were eliminated. The inclusion and exclusion criteria for the scoping review comprise (a) Must be published between the year 2015 and 2021 (the last seven years); (b) Must be peer-reviewed journal articles, not conference proceedings, book chapters, and book series; (c) Must be empirical research, either qualitative or quantitative, and not just presentations of framework or literature review; (d) Must involve the integration of CT and mathematics; (e) Must be empirical studies that employed some form of CT tools; (f) Must be published in the English language. Based on all these inclusion and exclusion criteria, eventually, 73 articles are recognized to be applicable and chosen for the next stage of analysis.

Next, 64 unique CT tools were determined from the 73 articles. Each unique CT tool was coded by the CT competencies based on three CT dimensions for Brennan and Resnick's (2012) framework, i.e., computational concepts, computational practices, and computational perspectives. Some of the sub-categories of CT competencies were adapted according to the frameworks of Zhong et al. (2016), Grover and Pea (2018), and the 3D Hybrid CT Framework (Adams et al., 2019). During the review process, two researchers compared and classified all the 73 articles as well as tabulated them into them according to the predefined criteria, namely year, authors, level of integration, separated/integrated assessments, validated/not validated, types of CT tools, forms of CT tools, CT competencies, and mathematics topics. Then, they reviewed the articles based on the aforementioned predefined criteria and worked collaboratively on the

coding process. Lastly, the intercoder agreement was sought to confirm the reliability of the coding scheme (MacPhail et al., 2016) and it was 90%. The parts that had disagreements, were resolved through further review and discussion until reaching a mutual agreement.

-Insert Figure 1 here-

4. Findings and Discussions

4.1 How has CT been integrated into mathematics instruction in recent empirical studies?

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integrated into mathematics instruction in the ated/integrated assessments, and being valida-
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one of the partial integrations, i.e., CT-
or ins In this study, 73 peer-reviewed journal articles were chosen for review. Appendix A shows how has CT been integrated into mathematics instruction in these articles, including the level of integration, separated/integrated assessments, and being validated/not validated. Figure 2 indicates the levels of integration for CT and mathematics. Most of the studies ($n = 41, 56.2\%$) were categorized under one of the partial integrations, i.e., CT-intensive Math-connected Integration (CTIMCI). For instance, Panskyi et al. (2019) determined the impact of informal computer science education on a visual creative programming course using Dr. Scratch for students aged 9-14. Across all the sessions, their Dr. Scratch projects were mainly assessed using seven CT and algorithmic thinking dimensions including logical thinking, parallelism, abstraction, problem decomposition, user interactivity, synchronization, data representation, and algorithmic notions of flow control. Mathematics was only partially integrated as one of the 12 sessions of teaching where the students learned the basics of mathematics and geometry.

We found 31 studies (42.5%) of fully integrated CT and mathematics in the classrooms. For example, Nam et al. (2019) determined the impact of a card-coded robotics syllabus and integrated the activities of sequencing and mathematical problem solving into the kindergarten curriculum. The treatment group attended the eight-week intervention that emphasized CT components of algorithmic thinking, patterns, efficient thinking, and procedural thinking. The

students were given a pretest and posttest of sequencing and mathematical problem solving before and after treatment. Therefore, the study was categorized as full integration. There was only one study (1.4%) of Math-intensive CT-connected Integration (MICTCI) conducted by Messer et al. (2018). The authors examined the influence of an educational programming intervention on mathematics skills, spatial awareness, and working memory among children aged 5 to 6 years old. A total of 41 students from a UK primary school were randomly assigned to one of three groups: programming with iPad technology, programming with paper and pencils, or a comparison condition containing pencil and paper mathematics addition and subtraction tasks. In this case, only one group was given the intervention of programming using iPad technology to reinforce mathematics, indicating partial integration (MICTCI).

-Insert Figure 2 here-

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ematics assessments in the reviewed studies
13: Separated, Integrated, The CT and mathematics assessments in the reviewed studies were utilized in three ways as demonstrated in Figure 3: Separated, Integrated, and Separated & Integrated. The majority of the studies ($n = 37, 50.7\%$) employed an integrated assessment method to evaluate both concepts of CT and mathematics. For instance, one of the studies performed by Miller (2019) with primary students used pretest and posttest with a focus on patterning and coding along with video-recorded instructions. There were 10 items of patterning in the pretest and posttest which included forming patterns, making predictions about patterns, determining repeating patterns, and generalizing patterns. Meanwhile, there were also 10 coding items where all patterning concepts were mapped onto coding contexts.

Furthermore, 34 studies (46.6%) utilized separated assessments to assess students' concepts of CT and mathematics. A study by Rodriguez-Martinez et al. (2019) employed Computational Thinking Test (CTt) to measure four computational concepts such as handling of

ent in STEM-related tasks; goals, beliefs, and
a parents about STEM. These two ins
ted assessment that was utilized was semi-
ssess the constructs related to concepts of co
-Insert Figure 3 here-
portion of validated or no events, loops or iterations, conditional sentences, and sequences in a Grade 6 classroom. The Mathematical Knowledge Test (MKT) was used as another instrument to assess the chosen standard of mathematics learning. Both tests were administered as pretest and posttest. Additionally, two studies (2.7%) involved the use of separated and integrated assessments. For example, Tran (2018) applied a pretest and posttest on CT with 10 items to identify five concepts of computer science, namely, algorithm, debugging, sequence, conditionals, and looping. Another instrument that was used was 44 items of pretest and posttest surveys with five Likert scales to evaluate enjoyment in STEM-related tasks; goals, beliefs, and aspirations in STEM; and attitudes from peers and parents about STEM. These two instruments were separated assessments. The integrated assessment that was utilized was semi-structured interviews with open-ended questions to assess the constructs related to concepts of computer science and STEM.

-Insert Figure 3 here-

Figure 4 exhibits the portion of validated or not validated assessments in the integrated instruction. It is crucial to identify whether the assessment was validated or not as the validated assessments provided the psychometric evidence including validity and reliability to make sure the appropriate reporting of accomplishments among the test-takers (McMillan et al., 2011) and to confirm that the assessments being used were measuring what it should be measuring (Lai, 2013). There were 36 studies (49.3%) that validated their assessments. For instance, in the study of Magana et al. (2016), the reliabilities for control appraisals and value appraisals of the pretest and posttest self-belief measures were 0.86, 0.90, 0.80, and 0.91 respectively. The construct validity was obtained via factor analysis. Learning measures, another assessment was validated through face validity which was carried out by three experts. On the other hand, the assessments employed in the 37 studies (50.7%) were not validated. These studies applied their assessments

or modified existing assessments for their research without reporting validity or reliability indicators. Some studies asserted that the assessment they used had been validated elsewhere, but they did not validate it themselves when applied in different contexts, such as in other countries. In fact, every assessment needs to be reevaluated for its validity and reliability prior to its use (Lai, 2013).

-Insert Figure 4 here-

4.2 What are the types and forms of CT tools being applied in the integrated instruction?

4.2.1 Types of CT tools

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identified. There were five major types of

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Three former types were adap Table 2 displays the types of CT tools used in the 73 reviewed studies. From the 73 articles, 64 unique CT tools were identified. There were five major types of CT tools, namely digital tangibles, apps and games, programming languages, formative or summative assessments, and other technological tools. Three former types were adapted from the study of Namukasa et al. (2017). Two new categories were added based on the reviewed studies, which were formative or summative assessments, and other technological tools.

-Insert Table 2 here-

As indicated in Figure 5, the majority of the types of CT tools in the selected literature were programming languages ($n = 21, 32.8\%$). Digital tangibles and formative or summative assessments were the second major type of CT tools with a frequency of 16 (25.0%). Meanwhile, the third major type of CT tools was apps and Games (*n* = 7, 10.9%) and the fourth major type of CT tools was other technological tools ($n = 4, 6.25\%$).

-Insert Figure 5 here-

4.2.1.1 Digital tangibles

mamically involved in problem-solving an 2019). 16 out of 64 unique CT tools were cl
bbotics/Lego Mindstorms EV3/NXT robots.
/programmable robots and circuits, Ardu
Mindstorms robots, TurtleBot, KIBO robo
no, Codey Rocky, Digital tangibles were defined as "physical artefacts designed to trigger various digital events, potentially provide innovative ways for children to play and learn" (Price & Pontual Falcao, 2011, p. 500). Code was produced to control programmable robots or circuits (Gadanidis et al., 2018). Educational robotics is generally deemed as a relevant medium for CT skill development (Durak et al., 2019). These digital tangibles allow young students to make abstract programming ideas and concepts more visible and accessible as well as more flexible for social learning. Not only that, children are dynamically involved in problem-solving and learn powerful concepts from robotics (Bers et al., 2019). 16 out of 64 unique CT tools were classified as digital tangibles including LEGO®EV3 robotics/Lego Mindstorms EV3/NXT robots, Lego® Education WeDo 2.0 robotics kit, robots/programmable robots and circuits, Arduino, mBot robot, SRAprogramming Lego NXT Mindstorms robots, TurtleBot, KIBO robotics kit, Sphero, micro:bit, Zowi, Dash & Dot, Anprino, Codey Rocky, Bee-Bot robot, and Robotics Dream ER kits.

4.2.1.2 Apps and Games

On the other hand, the category of apps and games includes games and web-based simulations. Altanis et al. (2018) claimed that the use of games can enhance CT and mathematical understanding. The application of apps also can foster CT skills as asserted by Ehsan et al. (2017). In this study, seven CT tools grouped under apps and games were Scalable Game, Kinect games, Bee-bot iPad app, SmartMeasure App, Hour of Code, Lightbot, and Digital Educational Material (DEM) "Evolution".

4.2.1.3 Programming Languages

Previous research showed that the integration of programming had improved performance in mathematics (Saez-Lopez et al., 2019). In this study, the 21 CT tools categorized as programming languages included Scratch, Scratch Jr., Dr. Scratch, MATLAB, Mathematica, Thermo-Calc, COMSOL, Statistical programming language R, Python, Google's Blockly/Code.org coding problems, Sketchpad, Logo, BlockyTalky, LaPlaya, ViMAP, Visual Studio IDE, Bitbloq, mBlock, Ardublockly, Processing, and Bootstrap Algebra. In Figure 6, there were seven major kinds of programming languages in the reviewed research, i.e., blockbased programming, text-based programming, spatial programming, functional programming, agent-based programming, visual basic.net programming, and object-oriented programming.

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1, visual basic net programming, and object-of

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Slockly or Code.org, LaPlaya, BlockyTalk

1

the most prevalent CT tool. The us Most of the reviewed studies used block-based programming including Scratch, Scratch Jr., Dr. Scratch, Google's Blockly or Code.org, LaPlaya, BlockyTalky, Bitbloq, mBlock, and Ardublockly. Scratch was the most prevalent CT tool. The usage of Scratch enabled the students to engage with tough mathematical concepts in new, meaningful, and generalizable ways (Benton et al., 2018). By using Scratch, students were able to create computer programs without worrying about syntax or spelling in block-based coding environments (Gadanidis et al., 2018). MATLAB, Mathematica, Thermo-Calc, COMSOL, Python, and statistical programming R were text-based programming. Two functional languages employed in this study were Logo and Bootstrap Algebra. There was only one study that utilized spatial programming (sketchpad), agent-based programming (ViMAP), Visual Basic.net (vb.net) programming (Visual Studio IDE), and object-oriented programming (Processing).

-Insert Figure 6 here-

4.2.1.4 Formative or Summative Assessments

Formative assessment is used to enhance teaching and learning, identify the problems faced by the students ongoing, before and during instruction, and assess what is working and what needs to be improved. Meanwhile, a summative assessment is utilized to assess learning outcomes after the instruction, and evaluate students' understanding of the material and their readiness for the next level of activity (Dixson & Worrel, 2016). There are 16 CT tools: computational cognition test, Visual Blocks Creative Computing Test (VBCCT), CT Test (CTt), CT survey, pretest and posttest on computational thinking, CT Skills Scale (CTSS), Robotics Activities Attitudes Scale (RAAS), Bebras tasks, Solve-Its assessments, VELA activities, CT Lesson Screener, CT Lesson Enhancer, CT concept test, CT practice test, CT test, and Math + C worksheets. 4.2.1.5 Other Technological Tools

EXECUTE: (RAAS), Bebras tasks, Solve-Its assessmes

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d that enables th Other technological tools are the Easy Java Simulator (Ejs) tool, Lattice Land, the Science Behind Pixar (Pixar) exhibition, and spreadsheet. For instance, Lattice Land is deemed as a mathematical microworld that enables the students to investigate the geometrical idea by manipulating the polygon drawn with a discrete point on a plane. This microworld serves as a powerful tool that facilitates the students' CT skills and mathematical habits of mind (Pei et al., 2018).

4.2.2 Forms of CT Tools

The forms of CT tools used in the integrated instruction are reviewed in Appendix A including assessment methods, format, cognitive/non-cognitive, and automatic/non-automatic. There were several assessment methods used in this study: in-class tasks (assignments, open-

at tools which unable to give scoring or 1

it was found that most of the studies ($n = 3$

er, followed by paper ($n = 20, 27.4\%$), and

of the studies assessed cognitive skills ($n = 3$

and non-cognitive skills were 26 (ended tasks, modules, classroom activities, etc.), knowledge tests (pretest and posttest, performance, etc.), artefact assessment (projects), survey (questionnaire), interview, and observation. Three formats of CT tools were computer (e.g., iPad, robots, Python), paper (e.g., test, survey), or both (computer and paper). The example of cognitive skills were CT skills, mathematics skills, spatial awareness, and working memory, while non-cognitive skills were self-efficacy, attitudes, and perceptions. The assessment tools that provide scoring or feedback in an automatic way to the learners were considered as automatic tools. Non-automatic tools referred to the assessment tools which unable to give scoring or feedback automatically. In Appendix A and Figure 7, it was found that most of the studies $(n = 34, 46.6\%)$ involved the use of both computer and paper, followed by paper $(n = 20, 27.4\%)$, and computer $(n = 19, 26.0\%)$. Furthermore, nearly half of the studies assessed cognitive skills ($n = 38, 52.0\%$). The studies that assessed both cognitive and non-cognitive skills were 26 (35.6%), while the least studies assessed non-cognitive skills with the number of nine (12.3%). The CT automatic tools were Dr. Scratch (2 studies), Kinect games (1 study), and Digital Educational Material (DEM) "Evolution" (1 study). Other CT tools from the 64 studies (93.8%) were viewed as nonautomatic tools, such as Scratch, Python, Arduino, and Computational Thinking Test (CTt).

-Insert Figure 7 here-

4.3 What are the CT competencies and mathematics topics being assessed in the recent empirical studies?

In this study, 64 unique CT tools have been mapped under the sub-categories of CT competencies which were organized based on the three CT dimensions of Brennan and Resnick (2012)'s framework, i.e., computational concepts, computational practices, and computational

perspectives. The framework's three primary dimensions are computational concepts (the concepts designers engage with as they program, such as iteration, parallelism, etc.), computational practices (the practices designers develop as they engage with the concepts, such as debugging projects or remixing others' work), and computational perspectives (the perspectives designers form about the world around them and about themselves)" (p. 1). As shown in Table 3, some of the computational concepts and practices from Brennan and Resnick's (2012) framework have been utilized including sequences, loops, events, parallelism, conditionals, operators, data, testing and debugging, and reusing and remixing.

ta, testing and debugging, and reusing and reviously at the state of the state of the ams et al. (2019) were logic and logical ern recognition, abstraction, generalization, com both frameworks of Grover and Pea (20 omputa Furthermore, computational concepts and practices adapted from both frameworks of Grover and Pea (2018) and Adams et al. (2019) were logic and logical thinking, algorithms and algorithmic thinking, pattern recognition, abstraction, generalization, automation, and modeling. Problem decomposition from both frameworks of Grover and Pea (2018) and Adams et al. (2019) were subsumed under computational concepts. Creativity in the Grover and Pea's (2018) framework and problem solving in the Adams et al.'s (2019) framework were employed. User interactivity and creation from the framework of Adams et al. (2019) were reorganized into computational concepts and computational practices respectively. Collaborating/collaboration in the frameworks of Zhong et al. (2016), Pinkard et al. (2019), and Adams et al. (2019) was used. Expressing, connecting and questioning in Brennan and Resnick's (2012) framework were revised to expressing and questioning about technology world, and connecting to the technical world. Some new sub-categories of CT competencies have been added based on recent empirical studies such as functions, subroutines, variables, procedural thinking, efficient thinking, agency, access, experimentation, problem formulation, confidence, appraisals, sense of identity, and so forth.

-Insert Table 3 here-

From Table 3, it was noticed that most of the CT tools were used to assess the algorithms and algorithmic thinking $(n = 32)$. The second and most assessed CT competency was abstraction ($n = 26$), and the third most assessed CT competency was testing and debugging ($n =$ 23). Loops $(n = 22)$ was the fourth most assessed CT competency, while the sequences $(n = 21)$ was the fifth most assessed CT competency. This was followed by decomposition $(n = 20)$, patterns/pattern recognition ($n = 17$), parallelism ($n = 15$), logic and logical thinking ($n = 14$), variables/Naming $(n = 13)$, and iteration/repetition $(n = 13)$. The most assessed CT competencies are demonstrated in Figure 8.

-Insert Figure 8 here-

9, and iteration/repetition ($n = 13$). The most

e 8.

-Insert Figure 8 here-

d studies, we found that there were some

instruction. The five mathematics topics we

8), i.e., Numbers and Operations, Algebra,

cs and Prob From the reviewed studies, we found that there were some mathematics topics being assessed in the integrated instruction. The five mathematics topics were modified from the study of Hickmott et al. (2018), i.e., Numbers and Operations, Algebra, Calculus, Geometry and Measurement, and Statistics and Probability. However, not all the reviewed studies revealed their mathematics topics. The results for the mathematics topics and sub-topics being assessed were displayed in Table 4. The most assessed mathematics topic was Geometry and Measurement with a frequency of 23 studies. This followed by Number and Operations ($n = 15$), Algebra ($n =$ 11), Statistics and Probability ($n = 5$), and Calculus ($n = 1$).

-Insert Table 4 here-

5. Discussion and Conclusion

Our scoping review reports on the integration of CT into the mathematics classroom, the types and forms of CT tools in the integrated instructions, as well as the CT competencies being

assessed in the integrated instruction. Regarding research question one, the results indicated that the majority of the reviewed studies employed CT-intensive Math-connected integration (CTIMCI) instead of full integration and Math-intensive CT-connected Integration (MICTCI). It means that the instructors or researchers tended to focus on the CT outcomes rather than the mathematics outcomes in the integrated classrooms. This result differs from those reported in the earlier studies including the studies of Israel and Lash (2019) and Nordby et al. (2022). Israel and Lash (2019) claimed that most of the lessons did not incorporate any integration between CT and mathematics, then followed by partial integration and full integration. Meanwhile, in the review of Nordby et al. (2022), they found that the majority of the studies which employed partial integration focused on math content with CT built-in as a technique to exhibit their understanding of mathematical concepts.

ed by partial integration and full integration.
they found that the majority of the studie
math content with CT built-in as a te
atical concepts.
d assessments were utilized in most of the s
were likely to integrate the as Furthermore, integrated assessments were utilized in most of the studies. In other words, the instructors or researchers were likely to integrate the assessments of CT and mathematics instead of separating both assessments. During the integrated instruction, the integrated assessments with specified identification of domains ought to be employed to yield a more complete picture of students' strengths and weaknesses in both domains (Bortz et al., 2020). At the same time, the students will gain benefit from the interplay of integrated domains towards achieving a deeper understanding of conceptual knowledge. Meanwhile, nearly half of the assessments in the reviewed studies were not validated which corroborates the statement of Tang et al. (2020) where many assessments lacked evidence of validity and reliability. It is crucial to develop a valid and reliable assessments to ensure the quality of the research findings (Chan et al., 2021) and allow the instructors and researchers to confidently use the assessments during the class and enhance the wide distribution of the assessments (Tang et al., 2020).

I forms of assessment methods were in-cla
ys, interviews, and observation. These findin
chers on how to design the assessments of
matics (e.g., partial integration or full integral
f CT and mathematics in the instruction.
 For research question two, there were five main types of CT tools being utilized in the reviewed studies including digital tangibles, apps and games, programming languages, formative or summative assessments, and other technological tools. Additionally, most of the studies used CT tools that were non-automatic, assessed cognitive skills, as well as involved both computer and paper in the integrated instruction. They were consistent with the findings from the study of Cutumisu et al. (2019), except for the format of CT tools. This study mostly used both computer and paper in the integrated instruction, but Cutumisu et al. (2019) mostly involved computer only in the instruction. Several forms of assessment methods were in-class tasks, knowledge tests, artefact assessment, surveys, interviews, and observation. These findings can be instrumental for the instructors and researchers on how to design the assessments of integrated lessons in the contexts of CT and mathematics (e.g., partial integration or full integration) as well as effectively conveying the concepts of CT and mathematics in the instruction.

With regard to research question three, it was also found that algorithms and algorithmic thinking were the most assessed CT competencies, followed by abstraction, testing and debugging, loops, and sequences. Two of the CT competencies (algorithmic thinking and abstraction) were also put in the leading positions from the review of Kallia et al. (2021). Zhang and Nouri (2019) declared that loops and sequences were essential skills no matter what programming language was chosen. However, testing and debugging was not frequently highlighted in the empirical papers reviewed by Kallia et al. (2021). The findings of this study allow the teachers, scholars, and researchers to have a better understanding of the CT tools and CT competencies that can be gained in the mathematics classroom. Geometry and Measurement was the most assessed mathematics topics. This is followed by Numbers and Operations, and Algebra. This result differs from the findings of the review study from Hickmott et al. (2018)

where most of the studies involved Numbers and Operations and followed by Algebra and Geometry. This scoping review has contributed to the areas of integration of CT in mathematics education. The findings of this scoping review have provided more information about CT and mathematics education on the existing literature and research themes. It can serve as a benchmark for many parties, including government, policymakers, school administrators, teachers, researchers, and scholars.

here were some limitations. First of all, this and seven years, i.e., between the years 201 integrated instruction for CT and mather ars. Secondly, only empirical studies were use of the reported practices of integrated no Although this scoping review involved a comprehensive review of current CT tools in the mathematics classroom, there were some limitations. First of all, this review only focused on the articles published in the last seven years, i.e., between the years 2015 and 2021. We made this decision as the area of integrated instruction for CT and mathematics has not been fully developed until recent years. Secondly, only empirical studies were used for the analysis as they had "verifiable" evidence of the reported practices of integrated instruction of CT and mathematics. Thirdly, although our search scope extends to multiple databases, we excluded the studies that did not state the CT tools and CT competencies measured. Therefore, it is suggested that future studies on CT in mathematics education should elucidate the CT tools and CT competencies being assessed, preferably with detailed descriptions including definitions.

After reviewing the articles, there are some recommendations for future studies. Despite the studies done on CT and mathematics education, validated CT tools are still lacking in the mathematics classroom at different stages. Most of the CT tools did not undergo a comprehensive psychometric validation process to warrant their integration in the syllabus including the validity and reliability of the instruments. Such a situation will cause problems related to the infusion CT in the educational systems as argued by Roman-Gonzalez et al. (2019). Thus, future studies should provide more evidence of validity and reliability as it is crucial to

construct universal CT tools which are appropriate to be administered across different contexts such as gender, ages and grade levels. Moreover, most of the studies tended to focus more on CT outcomes rather than mathematics outcomes. It is recommended to have a balance or full integration for both CT and mathematics in future studies. Bortz et al (2020) also proposed that if the assessments of CT and mathematics are integrated, the scoring systems should be differentiated to uncover learning in each domain including detailing the ways in which the integration alters epistemology.

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Table 1. Summary of the Number of Articles from the Initial Search and Final Search

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Table 2. Types of CT Tools

Types of CT Tools	CT Tools	Studies
Digital Tangibles	LEGO®EV3 robotics/ Lego Mindstorms EV3/NXT robots	(Leonard et al., 2016; Francis & Davis, 2018; Piedade et al., 2020; Suters, & Suters, 2020)
	Lego® Education WeDo 2.0 robotics kit	(Jaipal-Jamani & Angeli, 2017; Chiazzese et al., 2019)
	Robots/Programmable robots	(Gadanidis et al., 2017; Kopcha et al., 2017; Ching et
	and circuits Arduino	al., 2019) (Martín-Ramos et al., 2018; Cui & Ng, 2020; Ng & Cui,
		2021)
	mBot robot	(Saez-Lopez et al., 2019; Piedade et al., 2020)
	SRA-programming Lego NXT Mindstorms robots	(Fanchamps et al., 2019)
	TurtleBot	(Nam et al., 2019)
	KIBO robotics kit	(Bers et al., 2019; Jurado et al., 2020)
	Sphero	(Cendros Araujo et al., 2019)
	micro:bit	(Cendros Araujo et al., 2019)
	Zowi	(Piedade et al., 2020)
	Dash ¨	(Piedade et al., 2020)
	Anprino	(Piedade et al., 2020)
	Codey Rocky	(Piedade et al., 2020)
	Bee-Bot robot	(Munoz et al., 2020)
	Robotics Dream ER kits	(Sisman et al., 2020)
Apps and Games	Scalable Game	(Leonard et al., 2016)
	Kinect games Bee-bot iPad app	(Altanis et al., 2018) (Messer et al., 2018)
	SmartMeasure app	(Valovicova et al., 2020)
	Hour of Code	(Stigberg, & Stigberg, 2020)
	Lightbot	(Stigberg, & Stigberg, 2020)
	Digital Educational Material	(Rico Lugo et al., 2018)
	(DEM) "Evolution"	
Programming	Scratch	(Grover et al. 2015; Mouza et al., 2016; Benton et al.,
Languages		2017; Gadanidis, 2017; Gadanidis et al., 2017;
		Gadanidis et al., 2018; Cendros Araujo et al., 2019;
		Grover et al., 2019; Israel & Lash, 2019; Rodriguez-
		Martinez et al., 2019; Marcelino et al., 2018; Miller, 2019; Gleasman, & Kim, 2020; Olteanu, 2020)
	Scratch Jr.	(Falloon, 2016; Sung et al., 2017; Stigberg, & Stigberg,
		2020)
	Dr. Scratch	(Altanis et al., 2018; Papadakis & Kalogiannakis, 2019; Panskyi et al., 2019)
	MATLAB	(Magana et al., 2016; Vieira et al., 2019)
	Mathematica	(Magana et al., 2016)
	Thermo-Calc	(Magana et al., 2016)
	COMSOL	(Magana et al., 2016)
	Statistical programming	(Benakli et al., 2017)
	language R	
	Python	(Gadanidis, 2017; Gadanidis et al., 2017; Cendros
		Araujo et al., 2019; Lin et al., 2019; Lockwood & De
		Chenne, 2019; Stigberg, & Stigberg, 2020)
	Google's Blockly/Code.org	(Gadanidis, 2017; Gadanidis et al., 2018; Roman-
	coding problems	Gonzalez et al., 2018; Arfe et al., 2019; Israel & Lash,
	Sketchpad	2019; Piedade et al., 2020) (Sinclair, & Patterson, 2018)

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Table 4. Mathematics topics assessed

 $R_{\rm H}$ Figure 1. Flow diagram for study selection procedure

Figure 3. Assessment Methods

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Appendix A. Tools and approaches of integrated instruction in the reviewed empirical studies

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