ALGEBRA IS FUN

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Games can be used in mathematics classrooms, at both primary and secondary levels, as a means to motivate pupils to learn the subject. Many of the games are tricks using numbers and playing cards. Pupils enjoy playing such games. However, it is more meaningful and interesting to pupils if they are led to discover how such tricks work. In fact, pupils require only simple algebra to enable them to see how such games are invented. Here are a few illustrations, adapted from Frohlichstein (1962) and Stern (1973).

Number Games

The first game is a number trick. It involves only two players, A and B. Player A asks Player B to do the following in sequence:

(a) Think of any three digits.

(b) Multiply the first digit by 2.

(c) Add 5 to the product.

(d) Multiply the sum by 5.

(e) Add the second digit to the product.

(f) Multiply the sum by 10.

(g) Add the third digit to the result.

(h) Tell me your final answer.

From the final answer that Player B has given, Player A is able to tell exactly the three digits that Player B is thinking of. Player A does not make a guess but he can tell the numbers in Player B's mind with the help of simple algebra.

Let x, y and z be the digits that Player B is thinking of. Then following the steps that Player B has taken, we arrive at the algebraic expression, $100x + 10y + z + 250$, which corresponds to the final answer that Player B has given. Hence, to tell the three digits that Player B is thinking of, what Player A needs to do is to subtract 250 from the final answer, which Player B has given to him, and then read the digits in the result.

Another number game to be illustrated is again for two players. One player asks the other player to do the following in sequence:

(a) Take the month of your birthday, using 1 to represent January, 2 to represent February, etc.

(b) Multiply by 2.

(c) Add 5 to the product.

(d) Multiply the sum by 50.

(e) Add your age.

(f) Subtract 365 from the sum.

(g) Add 115 to the result.

(h) Tell me your final answer.

Here, the first player is supposed to tell the month of the second player's birth and his age. How can he do it? Well, let us see what algebraic expression we can obtain following the steps that the second player has taken.

Suppose that m is the number representing the month of the second player's birth and that he is n years old (assuming $n < 100$). Then we arrive at another algebraic expression, $100m + n$, which corresponds to the second player's final answer. This algebraic expression indicates that the tens and ones digits of the final answer will give the age of the second player while the thousands and hundreds digits will give the number representing the month of his birth. It should be noted that if he is less than 10 years old, then there will be a zero in the tens place of the final answer given by him. Also, if he was not born in December or November or October, then the final answer will be a three-digit number. Hence, the first player is able to tell the month of the second player's birth and his age from the digits of the final answer given by the latter.

Card Tricks

Card tricks seem to be some kind of magic to pupils. Certain card tricks use simple algebra. Let us see how a magician plays one such card trick. The magician needs to have an ordinary deck of 52 playing cards. This is how he performs in front of his audience:

(a) Places all the 52 playing cards face down on the table.

(b) Takes the top card and turns it face up.

(c) Counts from the face value of this card and turns cards face up from the deck until he counts to thirteen. (Ace has a face value of '1', Jack '11', Queen '12' and King '13'.)

(d) Forms other piles in the same manner until he uses all the cards or does not have enough cards to reach the magic number '13'.

(e) Turns all piles of cards face down.

(f) Places any left-over cards in a discard pile.

(g) Asks any observer to take any three piles of cards and put the other piles of cards together with the left-over cards in the discard pile.

(h) Asks the observer to choose any two of the three piles taken by him and turn the top card of each of the chosen piles.

(i) Starts to count the cards in the discard pile and tells the observer the face value of the top card of the third pile of cards without turning it up.

How can the magician know the face value of the top card of the third pile of cards without turning it up? The mystery can be solved simply by algebra.

There are three piles of cards taken by the observer. Suppose 'a' is the face value of the top card of the first pile, 'b' is the face value of the one from the second pile and 'c' is the face value of the one from the third pile. Since the magician has started with 'a' and counted to 13, there are 14-a cards in the first pile. Likewise, the second pile has 14-b cards, and the third pile has 14-c cards. Suppose 'd' is the number of the removed and left-over cards in the discard pile. Then we have

$$d = 52 - (14\text{-}a + 14\text{-}b + 14\text{-}c)$$

$$d = 10 + a + b + c$$

From this equation, we get

$$a = d - (b + c + 10)$$

or $$b = d - (a + c + 10)$$

or $$c = d - (a + b + 10)$$

Hence, to find the face value of the top card of the third pile, the magician adds 10 to the sum of the face values of the cards that are turned up, removes that many cards from the discard pile, and counts the number of cards left in the discard pile.

Concluding Remarks

The games illustrated show that pupils need to know only simple algebra to find out how the tricks work. If teachers feel that their pupils can understand the algebra involved in the tricks, they should guide their pupils to analyse the tricks algebraically. This will make it more enjoyable and useful for the pupils, who can then be motivated to invent their own games. They can find that learning algebra is fun.

References

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