PROBLEM SOLVING IN MATHEMATICS

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In **Journey Into Maths,** Bell, Wigley and Rooke suggest that "the most characteristic self-justifying activity in mathematics is *problem solving* in its various aspects – first, searching for patterns and secondly applying to practical problems and situations. Problem solving is the end to which the learning of a particular content is the means."

Before presenting some practical notes which teachers could consider in trying to inculcate an awareness, facility and encouragement of problem solving in school mathematics we might profitably review Dienes' **The Power of Mathematics** and enquire as he does "How is mathematics learned?" His reply: "It is very daring to ask ourselves to explain how mathematics is learned. In fact it is somewhat daring to ask how anything at all is learned. We may well ask ourselves the more fundamental question of what does learning really consist?"

In connection with our efforts as teachers to encourage the learning of mathematics as something to be enjoyed as well as endured we might look at some of the following components of Mathematics Learning.

"Mathematics" as Bell suggests is a characteristic way of organising our experience of the world which is employed wherever there is "pattern" or "structure". As a complex human activity, mathematics comprises a number of components of dissimilar nature.

These include the need for pupils to experience among others the following aspects:

1. Recognition – especially visual and tactile, of objects which can be used in practical mathematical work.
2. Language – building up suitable vocabulary and expressions which are reflections of real situations.

As Lucienne Felix states, "we do not have to begin with definitions. Quite the contrary. Nothing can be defined from nothing. What we must do is to begin describing a *mathematical situation."* This induces learning the correct "label" words.

3. Notation – Using relevant notation symbols and expressions to describe meaningful mathematical things.

4. Concept development – enjoyment of the relevant real experience to become aware of and assimilate the fundamental idea of the notion which is being considered, eg. that at primary level an introduction to the concept of *area* may be achieved by studying "an amount of surface". That it may be measured with another area and this can be explored through shapes which will *tessellate.*

5. Relationships – Concepts are the "objects" or "things" of mathematics, e.g. "triangle", "three", "volume", the mathematical development of individuals requires a study of the *relationships* which connect these.

6. Facts – Learning of relevant facts which depend for their retention on repetition and practice.

7. Skills – These may be regarded as well defined sequences of operations such as arithmetical algorithms, methods of geometrical construction and methods for solving algebraic equations.

8. Strategies – These combine the use of the facts, concepts and skills which have been learned in the recognition of circumstances where new mathematical knowledge is required and new investigations are carried out. These

are expressed in relevant language with appropriate symbols and notation.

An introduction to problem solving in mathematics

Ever since mathematics has been an important school subject, *problem solving* has been difficult to teach. It has been classified by many educators as a skill or a group of skills. Are these skills teachable?

What is a Problem?

There is not any clear-cut agreement about this. One suggestion is that it is a situation which confronts us and which requires a solution.

As pupils develop mathematical ability, what were problems initially become eventually routine exercises.

Textbook Problems

While most mathematics textbooks contain sections called "word problems", not all are real problems. In many cases a *model solution* has already been presented by the teacher. The student or pupil is merely applying an *algorithm,* that is, a *routine technique* as taught. If the pupil applies it correctly, avoids calculation errors, he gets the correct *solution.* This kind of work is really an *exercise.* It is valuable. It provides practice in *technique and reinforces* ideas.

At first it may of course be a problem to the pupil but then it becomes an exercise.

2 pencils and 3 more pencils altogether
make how many pencils?

This is a problem with an *algorithm*

$$2 + 3 = 5.$$

Then after practice it becomes routine. Some educators therefore distinguish between a *problem* and a well known process of calculation, that is, a *mechanical exercise.* Further exposure is also necessary to other aspects of *problem solving.*

A Dictionary Definition of a Problem

"A doubtful or difficult question." For teachers it is necessary to consider the following points:

- Should children be set difficult problems?
- Failure may produce frustrations and dislike.
- How do we relate the level of *problem difficulty* to the pupil's ability?
- Enjoying problem solving, achieving success are important ingredients for *education*.

What is Problem Solving?

Problem solving is a *process.* It is the means by which a student uses previously acquired

- *skills*
- *concepts*
- *principles*

to satisfy an unfamiliar task. So we have:

- *information and facts*
- *the ability to use information and facts.*

Teach through problems, not for them.

A pupil comment is good for thought. He said

> "Teacher Mr. T. was the best we had, he always seemed to go from the problem to the mathematics. All the others went the other way round."

Many pupils react to a new learning situation which is illustrated by a problem, as a challenge. It releases potential and positive thinking especially when the pupil can say "I've got it." "I can see it." "I understand."

What Makes a Person Good at Solving Problems?

A good *problem solver* has:

- a *desire* to solve problems
- a *perseverance* to go on

An approach to be encouraged

A. Read

1. Note key words.
2. Study problem setting.
3. What is being asked for?
4. Restate the problem, if possible, in your words.

B. Explore

1. Draw a diagram (or make a small model).
2. Make a chart (record data).
3. Look for patterns.

C. Investigate

1. Experiment.
2. Look for a simpler solution.
3. Make a guess (conjecture).
4. Form a hypothesis.
5. Assume a solution.

D. Teacher to Pupil

1. Create an atmosphere of success.
2. Encourage pupils to solve problems.
3. Teach pupils how to read problems.
4. Involve pupils in the problem.
5. Encourage pupils to write problems.
6. Encourage pupils to make diagrams.

E. Use Creative Questions

1. Do you see a pattern?
2. What is another way of doing this?
3. What kind of problem does this remind you about?
4. What would happen if we change this?

An example of a problem

How many squares are there on a chessboard?

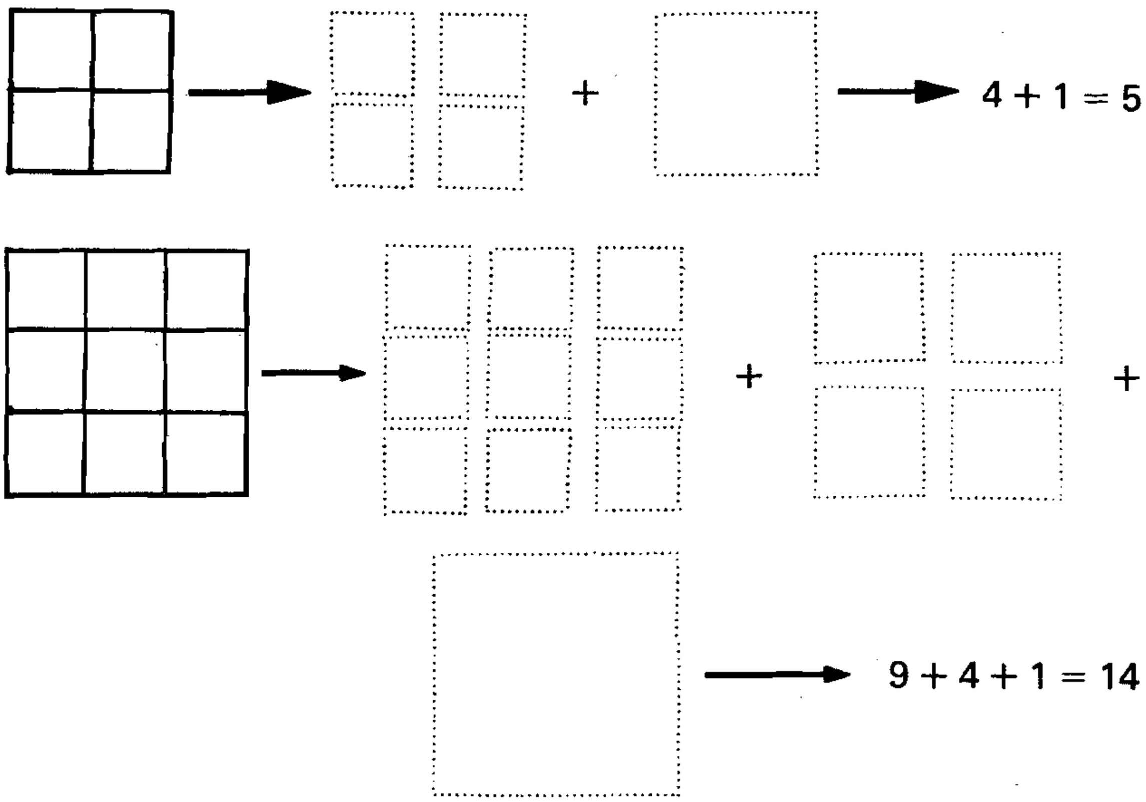
A first reply might be 64.

The problem may be extended and an investigation may be encouraged to explore how many different size squares there are on the chessboard.

A Plan

This may begin with an examination of simpler cases.

An example:



What kind of numbers are being added?

$$3^2 + 2^2 + 1^2 = 14$$

So we have a pattern.

Does it apply to the other cases?

Can we find a solution for any squares which are covered with square lattices as is the chessboard?

References

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