DEVELOPMENT OF MATHEMATICAL CONCEPTS AND SKILLS IN PRIMARY SCHOOL CHILDREN

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I shall attempt to give an insight into how primary school children develop mathematical concepts and skills by discussing these four questions:

(1) What is a concept as opposed to a skill and how do children learn mathematical concepts?

(2) Has the age of the child any bearing on his ability to learn particular mathematical concepts?

(3) What roles do language and practice play in this development?

(4) What is the role of the teacher in fostering this development?

First, it is important to realise that concepts in general, and mathematical concepts in particular, are distinct from facts and skills, for without this realisation, it is not usually apparent why mathematical concepts cannot be taught in the same way as a collection of facts or a body of skills. As an example, if a child who has never seen an animal, either an actual one, a picture of one or the word itself, asks us what a bird is, we will not be able to give him the concept of bird, in a way that will be meaningful to him, if we merely tell him that a bird is a small flying animal covered with feathers. If the child has a good memory, he may be able to remember and retain this explanation but he will not understand its meaning because he does not know what animal means. What we can do is to point to one and say that is a bird. However, one example will not be adequate for the child to understand the concept of bird as he may think that "bird" means only the kind of bird that we have pointed to. So we will have to show him different kinds of birds, and if we cannot show him a sufficient variety, we will show him pictures of different types of birds, until we are quite certain that he has acquired the concept of birds in general, and not of any particular kind, colour or size.

In this way, though we cannot tell him the concept, we can cause it to form in his mind from a variety of examples which have the concept in common. In the same way, the child can learn the concept of dog, cat, mouse and other animals. He will then be in a position to learn the concept of animal. In this example, bird, dog and cat are concepts which have been derived from direct sensory experiences and are called primary concepts. Secondary concepts are those derived from other concepts, as was the concept of animal.

In mathematics, number is a primary concept; that of the number "five" represents the property that all collections of five objects have in common. Addition is another primary concept, derived from all actions which combine two or more collections into one. Multiplication is a secondary concept, being formed from that of addition. Similarly the algebraic statement $2x + 3x = 5x$ is a secondary concept derived from all statements of the kind $2 \times 6 + 3 \times 6 = 5 \times 6$. Understanding of this concept, the distributive property of multiplication, is dependent on a discovery of what is common to all statements of this kind and is a long way removed from knowing that $2 + 3 = 5$.

Mathematical skills, as opposed to concepts, are fundamentally techniques or methods of procedure by which a mathematical operation or problem can be worked out. The traditional way of teaching mathematics has always been to impart knowledge and to emphasise skills by telling or showing children how to obtain the correct answers and getting them to practise the skills as much as possible. We need to be aware, however, that while it is possible to drill children into learning mathematical rules and techniques, they may be doing so without an understanding of the mathematical principles which underly them. With the average child, the ability to retain, recall and apply the techniques and rules as they accumulate will quickly diminish with each passing year until the child finds that he cannot do mathematics. No doubt, mathematical skills are important, but they should be learnt with mathematical understanding.

On the question of how children learn mathematical concepts, Piaget said, "It is a great mistake to suppose that a child acquires the notion of number and other mathematical concepts just from teaching. On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults try to impose mathematical concepts on a child *prematurely,* his learning is merely verbal; true understanding of them comes only with his mental growth."

This implies that children form concepts basically through their own experiences and activities and that they take a longer time to do so than we usually think. Concept formation is the result of an accumulation of experience which in some way becomes classified and ordered. A child may have or be given the experience of looking at and handling different shapes such as squares, rectangles, triangles and circles. When he is able to classify as rectangles all rectangles which he has not come across before and also to reject all other quadrilaterals which are not rectangles, then we can say he has acquired the concept of "rectangle". It follows, then, that a necessary condition for a child to form a concept is his exposure to a variety of concrete situations in which the concept is embodied and the test of whether he has formed the concept is the ability to categorise correctly and reliably other situations as having or not having the concept. The teacher must do two things: arrange for him a variety of direct sensory experiences which have the concept in common and if it is a secondary concept, the teacher must also ensure that he has the other concepts from which it is going to be derived. However, although the teacher may arrange an environment and provide appropriate experiences to help the child, it is the child himself who must take the leap in his mind from the visual to the abstract, from the particular to the general. Until he is able to make this leap, the concept does not exist for him. We should realise though that the formation of a concept is not a singular affair; it may be formed immaturely at first, but it gradually becomes deeper as the child progresses and matures.

In his findings, Piaget has postulated that all children, no matter how they reason and think, pass through certain stages of development and he has categorised five of them, each being linked to the age of the child. The five stages are:

(1) The sensori-motor stage from birth to about 2 years;

(2) The pre-conceptual stage from 2 years to about 5 years;

(3) The intuitive stage from 5 years to about 7 years;

(4) The concrete operational stage from 7 years to about 12 years;

(5) The formal operational stage from 12 years to 16 years or more.

The teacher of mathematics in the primary school will be concerned with children who are at the intuitive and concrete operational stages. A child who is at the intuitive stage is still strongly influenced by what he sees and even though he is able to form concepts, they are immature and will break down under stress. During the concrete operational stage, a child is able to develop concepts fully. He has the ability to understand basic arithmetical and other mathematical processes and he is capable of classifying, ordering and numbering. However, in spite of having these abilities, the child has to depend on concrete situations and materials for his learning to be meaningful, and if mathematical ideas are presented entirely in symbolic form, his comprehension may be minimal. It is only during the stage of formal operations that the child develops the ability to think without reference to concrete examples. At this stage, he is able to reason in the abstract and hence to deal effectively with purely symbolic representation of mathematical ideas. Though Piaget has linked each of these stages with the child's chronological age, it should be apparent that the exact time at which each child will arrive at or be in a particular stage is dependent on other factors such as his innate capacity or intelligence, his experience and environment and his cultural background. What Piaget has clarified about children learning mathematics is that there is a great need for and value of concrete experiences in the development of abstract mathematical ideas at the primary level and that if a child is at a stage of development when he is unable to form, say, the concept of number, it will be pointless to try and teach him addition or subtraction.

Language is an essential tool of all learning and the child's proficiency in it can enhance or hinder his progress. In the learning of mathematics, in order for language to help the child rather than hinder him, he needs to develop, from the moment he enters school, familiarity with and understanding of the language found in mathematics. The language used in mathematics is different from that of normal usage in that certain words in a mathematical context have specialised meanings. The child needs a lot of exposure to and experience in hearing and using the precise use of such language for him to develop and acquire the understanding and spontaneous use of refined mathematical language. Inadequate, immature mathematical language can be a stumbling block to a child's development of mathematical concepts and skills, and it would be the responsibility of the teacher to see that he is not handicapped so.

The need for practice in mathematics is obvious, but we must be aware that practice in itself may not be a learning experience. Practice in mathematics should serve two purposes. First, it enables skills learnt to be consolidated and reinforced. Unlike a physical skill, such as swimming or cycling, which is retained even without practice for life once it has been learned, a mathematical skill needs to be consolidated and reinforced immediately after learning by practice; otherwise, it will quickly be lost. This practice, however, should be done only after the skill has been learnt with understanding of the concepts and principles that underly it, for without this understanding, the skill cannot be established firmly and confidently in the child and will soon be forgotten. If a skill has been learnt with understanding, a large amount of practice is unnecessary and what is required is merely a reasonable amount of it to fix the skill firmly in the child's mind. Furthermore, in such a case, all that is needed to recall it is a quick revision even after a considerable length of time has elapsed. In short, practice should follow comprehension and it should not be given in the hope that a large amount of it will cause understanding to dawn on the child. The other purpose of practice is to provide feedback to the child. He needs to know the results of how he is performing in order to be aware that he is progressing in the right direction or that he should correct certain mistaken ways of thinking. It is particularly important in mathematics — where various parts of the content are so interrelated that, if a child has a wrong idea about one part, he will find it difficult to make progress in others — that mistakes should be corrected and eliminated as soon as they appear; otherwise, the erroneous procedure may develop into a habit.

It should be clear that the role of the teacher in fostering the development of mathematical concepts and skills in children is fundamentally to provide the necessary and appropriate experiences which will enable each child to acquire the concepts he is required to learn. In order to do this, the teacher herself must first and foremost be aware of the relevant concepts. It is in this respect that the teacher's knowledge and understanding of mathematics play a crucial role and it is for this reason that the teacher who wishes to teach mathematics successfully should ensure that she acquires this knowledge and understanding. This does not mean that the teacher needs to undergo a course in advanced algebra or geometry, but she should be able to identify and describe in simple terms all the relevant concepts of the topics found in primary school mathematics and she should be able to explain with understanding and confidence all the mathematical techniques and procedures that are taught to primary school children. As an example, what are the concepts we want the child to learn when he is taught multiplication? The teacher has to know that multiplication can be interpreted as repeated addition, as the number of objects arranged in a rectangular array of rows and columns or as the Cartesian product of two sets. She has to decide whether the child is capable of acquiring all the three concepts or whether it would be in his interest to omit the last interpretation. Further, in her teaching, she will begin with concrete representation of the concepts and she has to judge when she can replace the concrete materials with pictorial representation and finally with pure mathematical symbols. As the child progresses in his learning of multiplication, the teacher should also ensure that the work done will lead him to form the additional basic concepts that multiplication is commutative, associative, distributive over addition and that multiplication of any number by 1 results in the same number. It is only when the child has understood the distributive law will he be able to make sense of the usual long multiplication technique that he has to learn.

Having identified the concepts the child is required to learn, the teacher will then sequence the material to be taught. In sequencing the material, the teacher ensures that there are no large gaps and that new concepts and skills to be introduced are always based on what the child has learnt previously. In this way, the learning that takes place makes use of the child's previous knowledge and builds more knowledge on to this structure. At the same time, an important feature of mathematics teaching is that the teacher should introduce appropriate mathematical language while concepts and skills are being learnt. We can think of a mathematics lesson as having two parts, one where the teacher uses concrete apparatus, pictures, diagrams and symbols to build up the child's experiences and the other in which the teacher and child talk about the situation using precise mathematical language. The teacher who realises that the child needs to acquire facility in the use of mathematical language will approach work on number and operations through problem situations which are based on the child's own experience and environment. By doing this, the teacher will not only be developing the mathematical language of the child but she will also be giving him an opportunity to develop problem solving skills.

To sum up, the teacher is primarily the provider of a mathematical environment, an organiser of learning aids and materials and a supervisor who guides the child through telling, questioning and discussion to form concepts and to learn skills. The teacher has to be aware of each child's needs and the stage of development he is at so that she will be able to help him best. She will then know whether he is ready to learn new concepts and skills or whether he needs earlier concepts and skills to be consolidated and reinforced first. In this way, the teacher will be performing a role that provides each child with the teaching and learning that suits him best.

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