A BLEND OF MATHEMATICS AND MORAL EDUCATION

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A problem on set theory for secondary school students involves finding the greatest and least possible values of the intersection of two sets. To exemplify,

n (ℰ) = 100 where n (ℰ) is the number of elements in the universal set ℰ. Similarly for n (A) and n (B).
n (A) = 75
n (B) = 65

Find the greatest and least possible values of n ($A \cap B$).
(Source: "O" Level Mathematics, Alternative C, Dec 1975, Paper 1, Q. 4)

In order to demonstrate the solution to the above problem, a similar problem is stated using smaller numbers. Let us take, for example,

n (ℰ) = 6
n (A) = 4
n (B) = 3

and, as in the above problem, the greatest and least possible values of n ($A \cap B$) are to be found.

An Activity

The following activity is suggested: 7 pupils are picked at random and are asked to divide themselves into two groups, namely, Group A comprising 4 members and Group B, the remaining 3. The pupils are told to place 6 chairs in front of the classroom.

The attention of the whole class is now drawn to the fact that there are more pupils than chairs. Obviously, for all 7 pupils to be seated, some sharing of chairs must take place. In sharing chairs, however, participating pupils are told that any one chair can be shared by only 2 pupils and that these 2 pupils must be made up of one member from Group A and the other member from Group B. The activity is divided into two parts:

(i) The teacher first states that the two groups, A and B, are to share the 7 chairs such that the greatest amount of sharing takes place; in other words, as many chairs as possible are to be shared. To guide pupils along, the pupils of Group A are asked to be seated first and those in Group B thereafter. The "onlookers" are asked these questions and to note down their answers:

- How many "shared" chairs are there?
- How many empty chairs are there?

(ii) As in the above, the pupils in Group A are to be seated first. However, this time the teacher tells the pupils looking on that the members of Group A are most reluctant to share their chairs but they will share if asked. Now the members of Group B are to be seated as well. Thus we have a situation in which the least amount of sharing takes place. As in (i) above, "onlookers" are asked to note the number of shared chairs and empty chairs. In addition, they are also asked how many members in Group B have each a chair to themselves.

A Try-out

This activity (taking about 30 minutes) was in fact tried out by the writer in three secondary three classes. For part (i), the answers given by "onlookers" were: there were 3 "shared" chairs, and two empty chairs. The teacher, then, drew the corresponding Venn diagram.

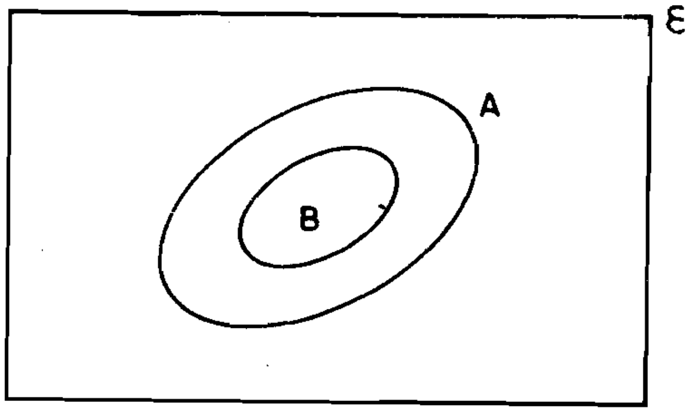

Figure 1

Pupils were told that in activity (i) in which they were just onlookers, n ($A \cap B$) in fact was the number of "shared" chairs. Here the teacher pointed out that where the greatest amount of sharing took place the whole of the smaller set was within the larger set; in this case, set B was entirely inside set A. The 2 empty chairs constituted a need to use a rectangle to encompass the universal set.

For part (ii), the answers given by "onlookers" were as follows: there was only one "shared" chair and no empty chairs. Also two members in Group B each had a chair to themselves. When asked whether there was a need to draw a rectangle for the universal set, the pupils unanimously replied no. The teacher then drew the appropriate Venn diagram for finding the least possible value of $n(A \cap B)$.

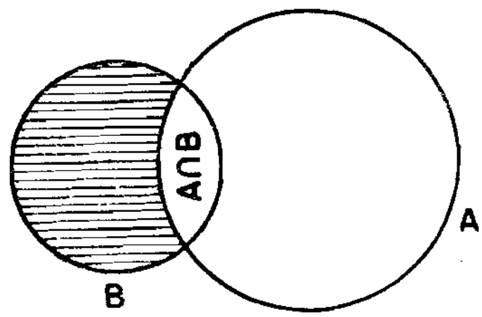

Figure 2

The following explanation is given. The fact that 2 members of group B each had a chair to themselves is equivalent to n(B only) [Refer to the shaded part.]. Now n(B only) = n(ℰ) - n(A)
= 6 — 4
= 2

From Figure (2) ∴ n ($A \cap B$) = n(B) — n(B only)
= 3 — 2
= 1

This explains mathematically why one "shared" chair resulted when the least amount of sharing took place.

Findings and Advantages of the Activity

More than 90% of pupils in the three secondary three classes stated that the activity was certainly meaningful in that they understood not only how to approach a problem requiring them to find the greatest and lowest values of the intersection of two sets but also the fact that where people were willing to share, it did everyone good. The teacher also added that when pupils were willing to share, it certainly strengthened unity within the school. It was rewarding to note that after the activity all pupils gave correct answers to the original problem.

Among the advantages of the activity, as seen by the writer, are the following:

- In rephrasing the problem using smaller numbers, it not only provides for demonstration but also simplifies explanation.

- The activity is easily conducted as chairs are conveniently available in the classroom. Moreover, an activity involving pupil participation helps to break the monotony of a lesson.