The Role of Metacognition in the Learning of Mathematics among Low-Achieving Students

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Metacognition is often regarded as a higher order thinking skill that benefits mainly higher achieving students. Weaker students, on the other hand, often encounter mathematics as a form of drill and practice exercise. It is no wonder that many weaker mathematics students become school dropouts and live to believe that mathematics is beyond the common folk. As we enter into the new knowledge-based economy, we need to raise the educational level of the masses — we need to seek breakthroughs. This paper takes a look at some strategies to promote the use and learning of metacognitive skills for weaker students.

INTRODUCTION

The international scene on thinking is proliferated with experts in the area of thinking offering theories, models and research findings to facilitate the infusion of thinking in school curriculum. Howard Gardner's theory of multiple intelligence, Daniel Goleman's introduction of emotional intelligence, Robert Sylwester's use of brain research, Robert Marzano's teaching with dimensions of learning, Richard Paul's wheel of reasoning and Robert Swartz's infusion of the teaching of thinking into content instruction are just samples of ideas derived from the pool of international expertise in thinking that have generated much interest locally.

In order to teach thinking more effectively and systematically in schools, much interest has been centred around models to examine human thinking processes. One such model which offers a schematic representation of the thinking process is the map of the thinking domain drawn up by Swartz & Perkins (1990) (Figure 1).

Swartz classified the thinking skills into three broad categories, namely, creative thinking, retention and use of information, and critical thinking. Creative thinking skills refer to skills at generating ideas; retention and use of information skills refer to skills that foster learning for understanding and the active use of knowledge; while critical thinking skills refer to skills at assessing the reasonableness of ideas. Swartz makes a distinction between these thinking skills and the goal oriented thinking processes, which are decision making and problem solving. However, Swartz and Parks (1994) felt that "Teaching the thinking skills ... without helping students learn how to use them in

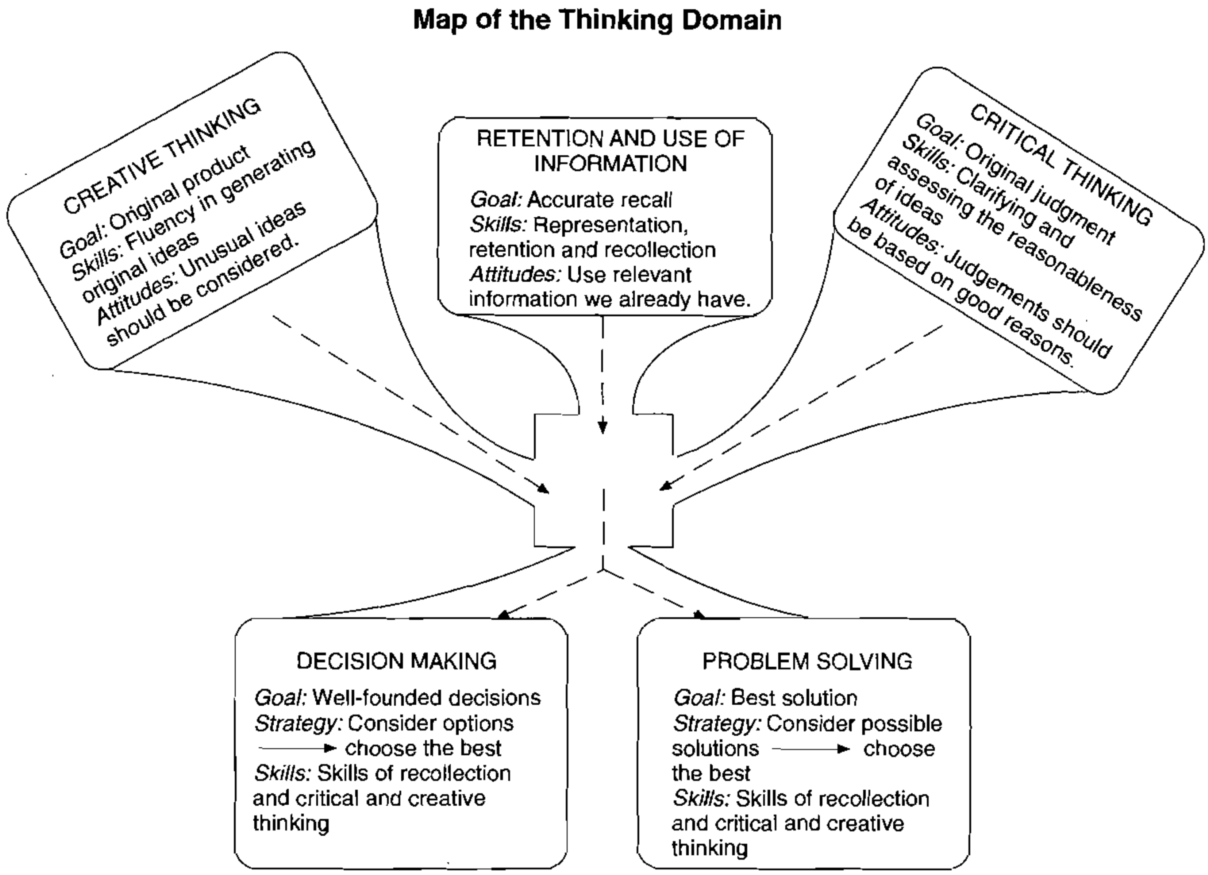

Figure 1. Map of the Thinking Domain (Swartz & Perkins, 1990)

decision making and problem solving accomplishes only part of the task of teaching thinking." Teaching strategies for problem solving and decision making, without teaching students the skills needed to use these strategies effectively, is similarly limited. If we teach lessons on individual thinking skills and lessons on decision making and problem solving, we can show how these thinking skills are connected with good decision making and problem solving. Students will then have the thinking tools they need to face their most challenging tasks in using information and ideas.

WHAT IS METACOGNITION?

Further examination of Swartz's map of the thinking domain reveals yet another component of thinking — the part which links the thinking skills to the thinking processes. Swartz and Perkins (1990) call this part of the thinking, metacognition, "a crosscutting superordinate kind of thinking relevant to all the others", which refers to "one's knowledge about, awareness of, and control over one's own mind and thinking".

THE SINGAPORE MATHEMATICS CURRICULUM

The importance of metacognition has been reflected in the Singapore Mathematics curriculum since 1992. The framework for the Singapore Mathematics Curriculum (Ministry of Education, 2000) shown below (Figure 2) features problem solving as central to the aim of the teaching of mathematics, and metacognition is regarded as one of the five key inter-related components in the attainment of problem solving ability.

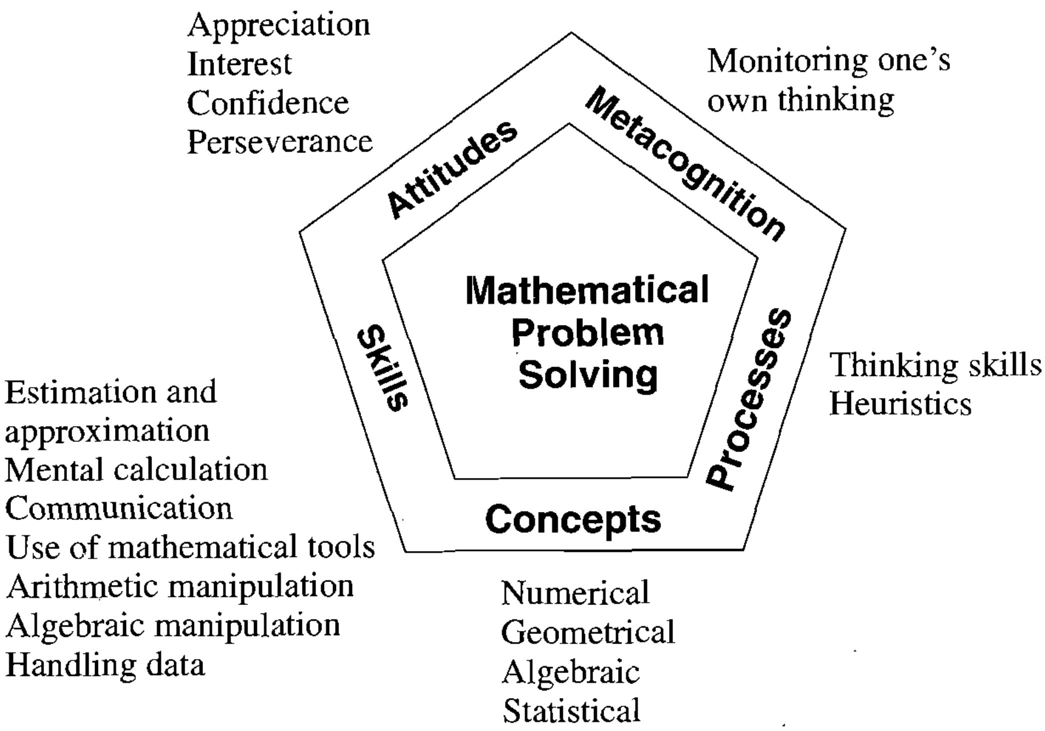

Figure 2. Framework of the Singapore Mathematics Curriculum

In view of the importance of metacognition in the learning of mathematics, it is thus of urgent need to examine the effect of a metacognitive classroom environment in the mathematical learning and achievement of students. Furthermore, by considering the effect on lower secondary students, which provides the link between the primary level and upper secondary and post-secondary levels, it allows for better generalization of the effects on the other levels.

METACOGNITION AND MATHEMATICAL PROBLEM SOLVING

With reference to the earlier discussion of Swartz's map of the thinking domain, mathematics lends itself well to the teaching of thinking. With the current renewed interest in the constructivistic approach towards the learning of mathematics, mathematics tasks are no longer restricted to the ones that are convergent and routine; mathematics is viewed as a human activity (Freudenthal, 1973) that involves problem solving. Just as depicted by Swartz's map of the thinking domain, mathematics educators also see the important role of metacognition in mathematical problem solving. It is no wonder that Schoenfeld (1992) noted that "(mathematical) problem solving and metacognition are perhaps the two most overworked ... buzzwords of the 1980's", and "problem solving has been used with multiple meanings ...; metacognition has multiple and almost disjointed meanings". Therefore, in order to establish a common platform for discussion, it is important to operationalize the two "buzzwords of the 1980's" here. The following understanding will be adopted:

> *Problem solving, as against the drill-and-practice mode of solving routine problems, refers here to the process involving the use of higher order thinking skills to solve novel problems, and whereby the solver, though possessing the necessary resources, does not have a direct or immediate path to a solution.*
>
> *Metacognition refers to the individual's declarative knowledge and procedural knowledge about his or her cognitive processes as well as self-regulatory procedures, including monitoring and "on-line" decision making.*

The interest in both mathematical problem solving and metacognition in the 1980's culminated in the detailed and extensive study by Lester, Garofalo and Kroll (1989) on the role of metacognition in seventh-graders' mathematical problem-solving, which has since then been quoted in many other subsequent related research studies. Interest, though not as rife, continued into the 1990s. Fortunato et al. in 1991 suggested one way of implementing metacognitive strategy instruction to help students to solve non-traditional problems, while Kjos and Long (1994) included the development of metacognitive abilities as one of the characteristics in their instructional approach to improve critical thinking and problem solving in Fifth Grade Mathematics. There is also a recent study by Mevarech and Kramarski (1997) investigating the effect of metacognitive activities, among other components, on students' mathematics achievement in instructional method for the teaching of mathematics in heterogeneous classrooms.

In fact, a metacognitive approach towards the teaching of mathematical problem solving has been tried and implemented with varying degree of success for a variety of students, ranging from primary school students (Kjos et al., 1994), to secondary students (Lester et al., 1989), and even to undergraduate students (Narode, 1985). Generally, it was found that the metacognitive approach towards problem-solving instruction is likely to be most effective when it is provided in a systematically organized manner under the direction of the teacher.

The following are some instructional strategies that have been found to be effective:

a) Mathematics log writing (Kjos et al., 1994 & Lester et al., 1989), whereby students use writing activities to develop understanding of mathematical concepts and their metacognition.

b) Effective questioning techniques (Fortunato et al., 1991; Lester et al., 1989 & Mevarech et al., 1997), whereby teachers establish an environment in which both teachers and students continuously ask questions with regard to the problem-solving process so as to better understand, monitor and direct their cognitive processes.

c) Identification of structural properties of problems (Santos, 1995), whereby teachers consistently ask students to identify similarities and differences among methods of solution and structural properties of problems that involve different contexts.

d) Pair problem-solving method (Lester et al., 1989 & Narode, 1985), whereby students develop metacognition and conceptual understanding by working in pairs, reasoning aloud and interviewing each other so as to understand the thought processes of the problem solver.

METACOGNITION AND STUDENTS' ABILITY

The relationship between metacognition and giftedness has long been of interest in the area of gifted education (Shore, 1986; Shore & Dover, 1987). In fact, Cheng (1993) pointed out that "It is speculated that the outstanding abilities of gifted individuals might be partly understood in terms of metacognitive functioning", and that "Experimental studies designed to single out metacognition for investigation have repeatedly found the performance of gifted children superior in comparison to that of their average peers".

In a study of over 1000 Singapore secondary and pre-university students from 12 schools, Wong (1992) found that "students need guided instruction in the use of metacognitive strategies for (mathematical) problem solving". He also observed that "students from the Normal stream seem to use strategies on metacognition less frequently".

As we enter into the new knowledge-based economy with increasing levels of globalization, we need to raise the educational level of the masses. Given the importance of metacognition in mathematical problem solving, it is crucial that mathematical instruction for the weaker students should include guidance on the use of metacognitive strategies for problem solving.

Problems Encountered by Students in Problem Solving

Davidson, Deuser, and Sternberg (1994) stated that all problems contain three important characteristics: givens, a goal, and obstacles. The givens are the elements, their relations, and the conditions that compose the initial state of the problem situation. The goal is the solution or desired outcome of the problem. The obstacles are the characteristics of both the problem solver and the problem situation that make it difficult for the solver to transform the initial state of the problem to the desired state. They see problem solving as the active process of trying to transform the initial state of a problem into the desired one, and metacognition helps the problem solver to:

(1) recognize that there is a problem to be solved,

(2) figure out what exactly the problem is, and

(3) understand how to reach a solution.

They also listed the following four metacognitive processes that are important contributors to problem solving performance across a wide range of domains:

(1) identifying and defining the problem,

(2) mentally representing the problem,

(3) planning how to proceed, and

(4) evaluating what you know about your performance.

It is thus not surprising that Newman (1993) found that difficulty in problem solving may occur at one of the following points:

(1) Reading

(2) Comprehension

(3) Strategy know-how

(4) Transformation

(5) Process skill

(6) Solution

In a study of Singapore students, Kaur (1995) found that the students encountered the following difficulties in mathematical problem solving:

(1) Lack of comprehension of the problem posed

(2) Lack of strategy knowledge

(3) Inability to translate the problem into a mathematical form

THE PROBLEM WHEEL

A scheme that assists weaker students in monitoring their comprehension of the problem and their regulation of the use of their resources, in terms of the knowledge and skills that they learnt will thus increase the chances of success in these students' attempts at solving the problem. Lee et al. (1998) used Richard Paul's Reasoning Model which has such a scheme in a variety of instructional approaches in the mathematics classroom for a group of secondary gifted students. The Reasoning Model used focuses on eight elements:

(1) Purpose, goal, or end in view

(2) Question at issue (or problem to be solved)

(3) Points of view or Frame of reference

(4) The empirical dimension of our reasoning

(5) The conceptual dimension of our reasoning

(6) Assumptions (the starting points of reasoning)

(7) Inferences

(8) Implications and consequences (where our reasoning takes us)

The eight elements of the Reasoning Model are depicted as a Wheel of Reasoning, reflective of the iterative and non-sequential nature of the process. Using the Wheel of Reasoning as a guide, students and teachers then engaged constantly in self and mutual questioning to monitor and self-regulate their problem solving processes.

However, being non-native speakers of the English language and feeling overwhelmed by the amount of intellectual information to be absorbed, the weaker pupils often find such a sophisticated scheme more of a hindrance than a help. Taking the language handicap into consideration and tapping on the scheme used by professional mathematicians, Chang, Yeap and Lee (2000) revised the Reasoning Wheel and presented the following Problem Wheel (Figure 3) for use by weaker students:

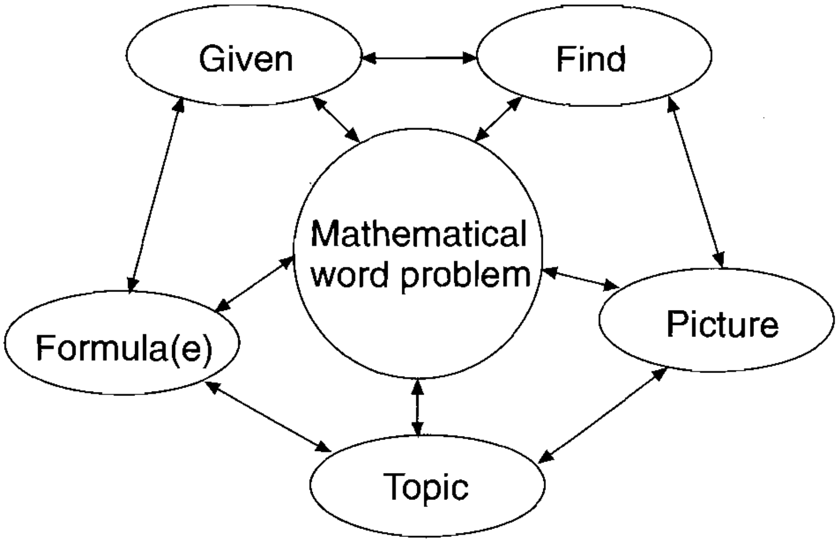

Figure 3. Problem Wheel

Comprehending the information in a word problem involves making sense of the structure of a mathematical word problem. A mathematical word problem generally consists of two components — the known ("given") and the unknown ("to be found"). Hence, by getting our pupils to list down the given information and to state what is to be found would aid them in gaining a better understanding of the problem situation. As for translating information in a word problem into mathematical concepts, it requires pupils to make an effective connection between the understood information and the repertoire of mathematical knowledge that they have acquired from their mathematics learning. As the saying goes "a picture tells a thousand words", so getting pupils to draw pictures/diagrams could help them to make sense of and see relationships between the knowns and unknowns in the problem situation. This could serve as a bridge for them to then discriminate between and select the mathematical skills and knowledge necessary for tackling the word problem — which are the topic(s) that I could draw concepts, ideas and skills from and what are the strategies, if any, that could assist me in solving the problems?

The ideas are depicted as a wheel with double headed arrows to convey the message that the steps are not to be perceived linearly, though sometimes they may occur as such. More often than not, pupils go through the various components of the wheel non-sequentially. They may possibly go back to earlier components of the wheel to revise the information gathered and translated as they move round the "Problem Wheel" to gain a better understanding of the word problem and try to translate the information into mathematical concepts. The interactivity of the various components reflects the dynamics involved in solving word problems.

SOME FINDINGS

A study was conducted on four secondary two pupils who had been identified, based on their secondary one results, as weak in mathematics. The lessons, conducted over a period of 12 weeks, with students meeting once a week for the duration of one and a half hours, took the form of an organized remedial tutorial class. The instructional approach basically used questioning techniques and pair problem solving to encourage students to engage in monitoring and self-regulating their problem solving processes. The Problem Wheel provided a scheme for both teacher and students to engage in class and pair questioning.

The students also sat for a pre- and post-test. The tests were parallel, consisting of four non-routine problems based on the primary mathematics syllabus. This was to minimize the possible effect of a lack of content knowledge rather than the ability to solve problems. A problem that required knowledge beyond what the students had been taught was also included in the tests. This would provide an insight into how students handle problems that require resources beyond their means.

As the problems in the tests were to provide contexts to examine the thinking processes that are engaged in by the students during problem solving, the students were required to answer the following questions after attempting each of the problems:

(1) What are the questions that you asked yourself as you solved the problem? What do you know about the problem, and what do you need to find out more to help you to solve the problem?

(2) What are the difficulties that you encountered?

(3) Did you manage to solve it? If yes, after how many different attempts? If no, when and why did you give up?

(4) Write down the steps that you took as you tried to solve the problem.

(5) If you have solved the problem, how do you feel about your answer?

A qualitative analysis of the students' responses to the first question for each problem, reveals that the metacognitive approach, through the use of the Problem Wheel, has helped the students to better comprehend the problem posed. This is in particular reflected by the students' ability to identify the actual question posed in each of the problems. A typical response in the pre-test, for example, is "Is it easy or hard?", while a typical response in the post-test, such as "What is it I'm finding?", reflects a more focused approach towards the comprehension of the problem. In fact, it was observed that the students' normal reaction to a non-routine problem was to stare blankly into space. On the other hand, when they were prompted to activate the Problem Wheel through appropriate questioning, there was greater interest and a higher level of perseverance in making an attempt to solve the problems.

It was also observed that the students' responses to question 5 reflected a more realistic and accurate judgement of their answers to the problems. A response to question 5 with an erroneous approach to a problem in the pre-test such as "I am very happy about the problem", was usually accompanied by a more realistic or accurate evaluation of the students' erroneous attempts at a problem in the post-test, such as "Satisfied" or "Not sure". The post-test also indicated an attempt by some to evaluate the reasonableness of their answers, such as "Satisfied because I have 'tested' it".

However, in terms of the students' responses to the problem that requires resources beyond what they have learnt, there were no significant differences between the pre- and post-test. Students generally assumed that they were expected to be able to solve all the problems, to the extent that mathematical rules were bent and ignored to help the students obtain an answer to the problem. None of the four students mentioned anything about lack of resources or the possibility that the problem could not be solved.

Discussion and Conclusion

The treatment period and the sample size do not provide any concrete evidence for the effectiveness of a metacognitive approach towards mathematical problem solving among weaker students. However, a qualitative study of the cases does indicate a greater level of success among these students in handling problem solving. In fact, these students reported that they felt more confident in mathematical problem solving during an interview after the treatment period. The motivational effect of increased confidence on enhancing students' problem solving ability could be further investigated. The students' belief that all problems posed must be solvable also needs further examination if we are to prepare them to handle real life problems.

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