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Understanding and Overcoming Pupils' Learning Difficulties in Mathematics

Jessie Ee and Wong Khoon Yoong

Abstract

This article uses the Singapore mathematics curriculum framework as a heuristic device to summarise the learning difficulties in mathematics experienced by primary school pupils. It suggests several general instructional strategies that might bring about a strategic approach to learning mathematics. This approach aims to give pupils a stronger awareness and control of their learning so that they can achieve a better understanding of mathematics at their level.

Introduction

There is a need for educators to be constantly conscious of understanding not only how pupils process and construct mathematical knowledge but also the difficulties that they face. A clearer understanding of how pupils think and learn can be formed by drawing from developmental psychology, information processing and metacognitive theories (Borkowski, 1992; Piaget, 1952; Vygotsky, 1986). The quality of instruction is vital for each stage of pupil's mathematical development. Pupils may improve or deteriorate in their capacity to retain what they have learnt. As mathematical learning progresses from the concrete to complex and abstract structures, greater individual differences are likely to be observed as pupils perform mathematical tasks. There is a need to help pupils acquire and integrate new knowledge pertaining to mathematics within and across different developmental stages. As instruction is elaborated at each stage, pupils need to be made more aware of their mathematical declarative, procedural and conditional knowledge so that they can gain better awareness and control of their mathematical learning.

The first step toward realizing this challenging task is for teachers to have an understanding of the difficulties encountered by pupils when they learn mathematics. These difficulties will be discussed under the five components of the Singapore mathematics curriculum framework. This will alert local teachers to the

benefits of reflecting on their mathematics teaching based on this framework. It should be pointed out at the outset that not every pupil would exhibit all the characteristics of learning difficulties mentioned below. Nevertheless, teachers need to observe carefully to what extent their pupils show signs of one or more of these "deficits" and take the necessary steps to help them overcome the deficits observed.

Concept-Related Deficits

Concepts are the foundations of mathematics. However, many primary pupils are found to lack basic concepts such as place value, area, and volume. Concepts are abstract entities in the mind and they have to be represented externally in various forms. For example, the concept of "four" can be represented in the enactive (concrete) mode, the iconic mode, and the symbolic mode in an instructional sequence as expounded by Bruner (1964); see Figure 1. Besides, examples of "not four" should be discussed to help pupils identify the defining characteristics of the target concept.

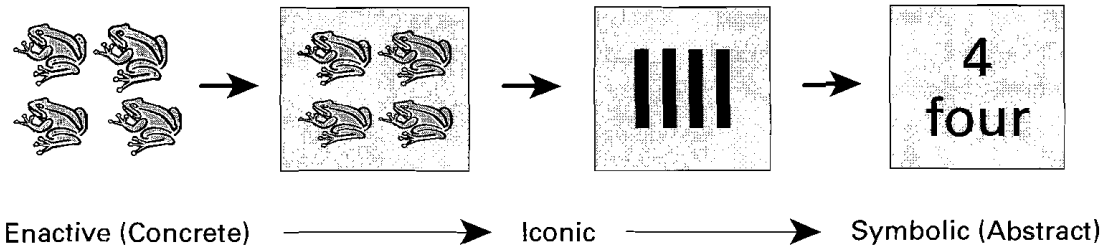

Fig. 1. A sequence from the enactive to the iconic to the symbolic mode.

Establish linkage using simple language

According to Hiebert (1984), it is not just the use of manipulatives that improve mathematical understanding but rather the explicit construction of links between actions on the concrete objects and the related symbolic procedures. In the above example, the pupils must be made aware of the aim of working on the manipulatives and drawing pictures. Without this awareness many pupils may not be motivated to carry out the activities as given by the teacher. In communicating this awareness to the pupils, the teacher should use simple language, for example, "real objects", "pictures", and "numbers" instead of the more sophisticated terms used by Bruner. In a similar vein, Chang, Yeap and Lee (2000) have used simplified terms such as "given" and "find" in their Problem Wheel model to help weak secondary school pupils develop skills in problem solving.

The translation from one mode to another (see example in Figure 2) often requires a multi-sensory approach besides mere language. This will enhance pupil's transfer of learning as the pupil perceives the concrete object and relates it

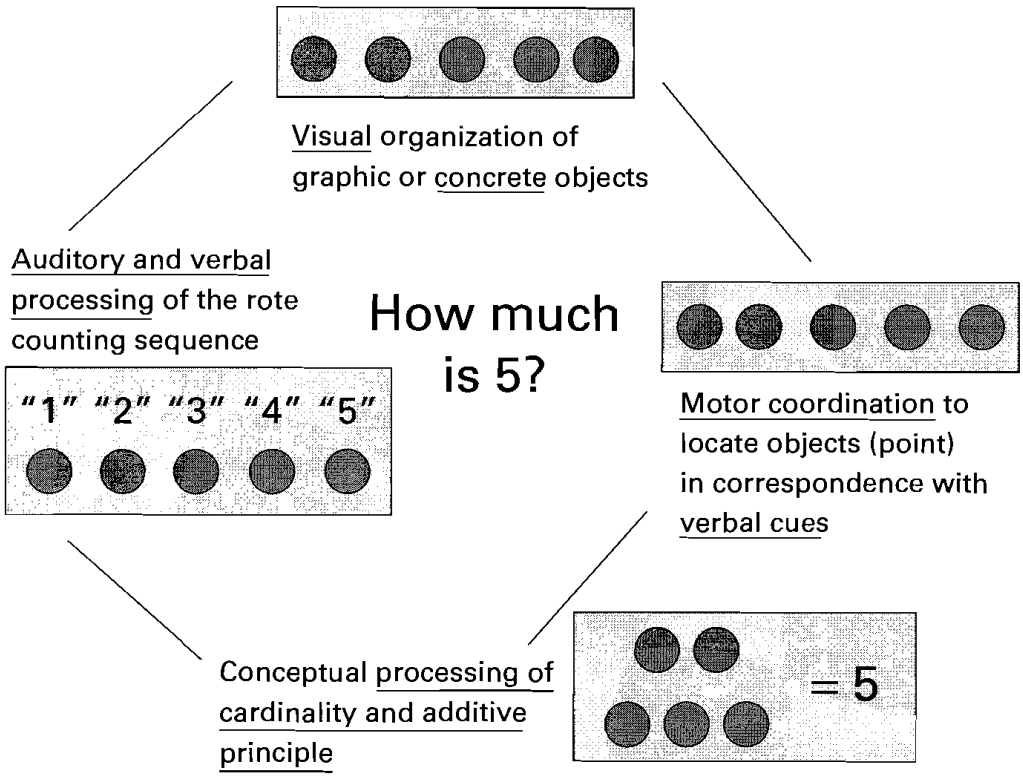

Fig. 2. Enhancing translation and transfer of the concept 5

to the verbal cues and mathematical symbols. Furthermore, as pupils perceive the relationship from one mode to another, commonly used words such as "more", "less", "early", and "late" will denote specific mathematical operations that communicate the concept and its associated properties. Pupils who have difficulty in reading and understanding the language of mathematics will face an uphill task to master the concepts. A further difficulty arises when the medium of communication is English, which is not the mother tongue of the pupils. To partially alleviate this problem, several strategies may be attempted.

(a) Get pupils to write their own glossary of commonly used mathematical terms, using voice input into computer to enhance motivation.
(b) Encourage pupils to talk about mathematics in various situations such as shopping or counting scores at games.
(c) Encourage pupils to think aloud when they solve problems so that they become more aware of their own thinking process and the thinking of their peers. This feature of scaffolding instruction could be very important for pupils to learn from more capable peers.
(d) Provide comparison of the meanings of words used in daily living and in mathematics.
(e) Motivate pupils to read and write stories that include mathematics (Bebout, 1993).

Information processing deficits

Another cluster of deficits that affect the development of concepts relates to information processing. Miller and Mercer (1997) have provided a comprehensive list of some of these deficits. Of particular interest are the following that relate to the visual-spatial aspect:

(a) difficulty in reading small prints;
(b) tendency to lose place on a worksheet;
(c) difficulty in differentiating between numbers, e.g., 6 and 9;
(d) difficulty in writing across the paper in a straight line;
(e) difficulty in relating to directional aspects of mathematics, e.g., alignment of numbers or left and right or up and down especially with computation algorithm steps.

About 5% of lower primary pupils may be afflicted by the digit reversal deficit (Wong & Koay, 2001), for example, $4 + 9 = 31$. This could develop into an early form of dyslexia (Edwards, 1998) if left unattended. Other motor deficits include writing numbers illegibly, slowly or inaccurately, and difficulty in writing numbers in small spaces (Smith, 1998).

Most of the information processing deficits mentioned above require one-to-one or small group coaching conducted by patient and caring teachers. When the problems become more chronic, clinical testing and remedies may be required. This underscores the need for mathematics teachers to work cooperatively with paraprofessionals in other fields to deal with these deficits.

Skills Deficits

A large proportion of classroom time in mathematics lessons is devoted to practicing skills about learned algorithms. The mastery of these skills often becomes hindered by the way the skills are presented to the pupils in isolated fashions without any linkage to the underlying concepts. When there are many rules to learn in a rote manner, pupils mix up the steps of different rules and this is often attributed to deficits in memory. These include: inability to retain mathematical facts or new information; forgetfulness in maintaining procedural steps; difficulty in reviewing past learning, for example, multi-step word problems (Zentall & Ferkis, 1993). Memory may be enhanced when what is to be memorized is meaningful and has many associations, especially when they are salient ones. For example, help pupils to relate the formula for the area of a rectangle to meaningful activities such as counting squares, covering surface with tiles, and finding patterns. Teach pupils how to use paint and draw programs to prepare attractive posters about mathematics results, and post them at prominent places around the classrooms or at home (by enlisting the support of the parents). Alternative ways of remembering the same algorithms and facts should be dealt with. For example,

learning the 9 times table by verbal rehearsal, finger mathematics, and patterns (e.g., $9 \times 7 = 63$, where 6 is one less than 7 and, $6 + 3 = 9$).

Another reason for certain skills deficits is that pupils create their own rules by generalizing from poorly or incompletely understood ones. These are referred to as misconceptions in the literature (Olivier, 1989). For example, about 20% of Primary 1 pupils reported in Wong and Koay (2001) made this mistake: $\begin{array}{r} 41 \\ -\ 18 \\ \hline 37 \end{array}$. This is the well-known misconception of "always subtract the small number from the big number." These misconceptions are difficult to deal with. To understand the sources of these types of deficits, the teachers have to interview the pupils to probe for the reasons behind their answers, for example, using Piagetian clinical interviews. Teachers who think that the pupils have made "careless" mistakes are more likely to deduce that they have attention deficits in not being able to maintain attention on several steps. However, this "careless" interpretation may lead to wrong remediation being given to the pupils. To overcome misconceptions requires the use of meaningful activities and diagnostic teaching (Bell, 1994).

Processes Deficits

The Singapore Primary Mathematics syllabus lists eight thinking skills and 11 heuristics for problem solving under the process aspect of mathematics learning. Twelve examples are already given in the appendices of the syllabus. Working through these and other examples found in textbooks in a systematic way is no simple task for most teachers, especially those teaching pupils with learning disabilities. Our belief is that these pupils are able to master most of these thinking skills, and the challenge is to experiment and find out which strategies might work effectively. For example, the teacher may need to do "guess and check" before "drawing a diagram" or vice versa. Unfortunately there is no local research that we can draw on to even tentatively answer this question.

On the other hand, we wish to offer the following suggestions for experimentation by teachers.

(a) Provide ample opportunities for pupils to apply the thinking skills by giving them examples of graded difficulty level.
(b) Celebrate small successes made by pupils. Avoid giving the impression to pupils that they cannot "think" by using only very simple examples.
(c) Use more hands-on experiences such as "acting out" the problem using blocks, Cuisenaire rods, and daily objects such as ice-cream sticks and buttons.
(d) Encourage pupils to solve the problems using their own methods first. It is not necessary to "rush" to cover all these thinking skills and heuristics.
(e) Conduct school-based action research to find out which strategies work and why.

Metacognitive Deficits

Metacognitive strategies are processes that require pupils to become aware of their own thinking processes and through such awareness, take steps to plan, monitor and regulate their thinking. Pupils with learning difficulties not only use ineffective strategies but they have metacognitive deficits. They fail to monitor and regulate their learning. Even though metacognitive strategies have gained much attention in recent years in several countries, their implementations in the local mathematics classrooms are not yet widespread. However, Montague (1997) maintained that this is a promising approach for pupils with learning difficulties especially those who have learned basic mathematical knowledge but cannot apply them successfully when solving mathematical problems.

Some metacognitive questions relevant to mathematics problem solving are given below.

(a) What information is given and what is to be found?
(b) What strategies do I have and how are they relevant to the problem?
(c) Can I carry out these strategies?
(d) Does what I am doing bring me closer to the intended solution?
(e) How do I know I have found the correct answer?

In introducing such strategies in primary mathematics teaching, the teacher needs to make these strategies explicit. One way is to display posters that describe these self-monitoring questions. Pupils can also be encouraged to make their own note cards consisting of these questions. Teachers must also model the application of these strategies by thinking aloud as they solve problems in front of the class. They can also encourage peer evaluation by getting pupils to ask questions like "What do you think of Jane's answer, John?" or "Why do you say he is right, Mary?" Pupils must be given time to think and respond. In the event when no one has the correct answer, the teacher will provide supportive and constructive feedback on pupils' attempts. During discussion, teachers can brainstorm and help pupils to see that there are many solutions to a problem as well as encourage pupils to clarify their answers for the benefit of the entire class, using prompts such as "Please tell us how you get this" or "Please tell us more about ..." Pupils' feedback on these questions will provide clues about their understanding as well.

Teachers may also assist pupils to visualize or draw diagrams to encourage understanding and retention. This is related to the iconic mode suggested by Bruner (1964). A series of small goals can be set to help pupils reach the final goals. In ensuring that pupils monitor and self-regulate their learning, they must be taught to use various ways to check their answers and to consider alternative strategies. A good example is the "restaurant" problem given on page 135 of the Singapore primary mathematics syllabus (shown in Figure 3).

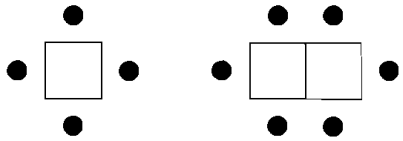

Fig. 3. A "restaurant" problem.

A square table can seat 4 people. How many such square tables, arranged to form a long table, are needed to seat 30 people?

For pupils with learning difficulties, they should act it out by arranging desks and sit according to the problem situation. As the pupils are doing this, explain to them that the purpose is to understand the problem (monitor the thinking). Subsequently, allow them to count the number of desks and pupils and put these values on a given list. Pupils may be encouraged to answer the question "Why do we want to make a list?" This can be followed by one of the three methods given, with questions at each key step. Once the problem has been solved, ask for alternatives. In the recapitulation phase, pupils may be encouraged to suggest similar problems that can be solved using the methods learned. For example, "what if we wish to seat 98 people or 100 people?" For each extra question, encourage pupils to check by asking, "How do we know we have the correct answer?" The context of the question can also be changed, for example, instead of "seating people" change to "How many toothpicks are required to make these patterns?" and so forth. For this extension part, probe with a question such as, "What is the purpose of doing problems similar to this one?" Through these questions, pupils perceive the steps that are modeled and will be encouraged to emulate the skills learnt in their own problem solving.

As Singapore primary pupils spend more time learning mathematics in computer-based lessons, metacognitive questioning should be included as suggested by Healy (1999). The thinking process should be made explicit when demonstrating the use of a software through questions such as, "I need to make a copy and this icon shows a double page. What does it suggest?" or "What do I need to do first after highlighting the data if I am to draw a chart?" When pupils are practising, teachers need to observe what they are doing, and quietly probe them to access their level of understanding with questions such as, "I wonder what that icon does?" or "I noticed sometimes you got answers with a few decimals and sometimes answers with many decimals, could that be important?" According to Healy, "children of any age can eventually begin to internalize this sort of dialogue from which they gain self-control and problem-solving skills" (p. 248).

This metacognitive approach to learning can also be integrated with games and hands-on activities. When used in this way, it supplements the predominant practice of drill and practice. This type of learning will take more time. The literature in cognitive strategic instruction emphasizes explicit instruction in these strategies through careful modeling, guided practice, corrective and positive feedback,

which are desirable in most forms of teaching practice. Other cognitive strategies include mnemonic skills (Wood, Frank & Wacker, 1998), mind-mapping, organizational skills, and elaboration strategies (Herrington, Wong & Kershaw, 1994).

Affective Deficits

Pupils with learning difficulties in mathematics often experience greater difficulty than their peers without disabilities. Many of them have encountered frequent mathematical failures that result in the development of learned helplessness in mathematics (Ee & Chan, 1994; Parmar & Cawley, 1991). Their repeated failure and lack of mathematical understanding lead to dependency on the teacher or their peers for help. Ee, Moore and Atputhasamy (2001) and Ee and Chan (1994) indicated that pupils with learning difficulties are more likely to have work avoidance tendencies and attributional beliefs that things are not within their personal control. Their maladaptive motivational orientations, attributing success to luck and failure to lack of ability, and work avoidance tendencies are detrimental to learning. This may also result in mathematics anxiety at a later stage.

Groteluschen, Borkowski and Hale (1990) maintained that, besides strategy-based instruction, pupils require attributional retraining. Borkowski (1992) stressed that teachers need to focus on using success-oriented dialogues aimed at coping with failure, such as dialogues that link attributions to performance (effort) or strategy. Thus, pupils would be more likely to attribute their success to effort, strategy use or improved ability, rather than to luck or easy task, and to attribute their failure to lack of effort or strategy use, rather than to lack of ability, bad luck, or task difficulty. Teachers can also provide opportunities for success to enhance pupils' self-competence and self-esteem.

One way to promote a more positive attitude toward mathematics is to show applications of school mathematics to real life situations. Common examples include the use of money in purchasing, measuring heights and weights, depicting information in pictorial form, and finding one's location in maps. More challenging applications include using mathematics to play strategic games, creating beautiful designs using geometrical shapes, and using real-life data for project work (Bishop, 1988). The tasks to be given must be age-appropriate and preferably related to national education issues in Singapore.

Conclusion

This brief paper attempts to relate overseas research on special needs in mathematics to the Singapore mathematics curriculum framework. Before the strategies discussed above are applied, it is imperative that proper diagnosis be undertaken to understand more about the pupils' difficulties. As each child is unique, no one method or approach is suitable for everyone. Teachers must understand their

students and consider their prior experience and knowledge before implementing remediation plans and approaches with the use of appropriate strategies to promote learning and understanding.

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