Using journal writing to understand primary three students’ perception of multiplication and division: Case studies

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Primary school students’ ability to communicate their mathematical understanding is one of the main objectives in mathematics learning in schools. This study investigates how journal writing can be used to examine primary students’ perception and their ability to explain their understanding of multiplication and division based on two case studies. Two journal writing tasks are given respectively to 158 and 36 primary three students in two Singapore schools. The writing tasks specifically aim to uncover how young students perceive the learning of division and multiplication. An analysis of students’ writings reveals that the use of appropriate journal writing prompts gives students the opportunity to explain their understanding and perception of mathematics learnt. The results from students’ journal writing help teachers to understand their students’ thinking and take the necessary measures to clarify the mathematical concepts involved in the topics of multiplication and division. Moreover, students’ writings serve as a source of reflection for teachers to re-think and improve their instructional strategies. The results suggest that teachers can gather a lot of information about students’ thinking in their writings and journal writing can be an effective and alternative way to increase teachers’ understanding of their students learning in mathematics. Given the appropriate writing prompts, journal writing can be a source for students to reveal to teachers their understanding of mathematics concepts and learning. Students need more exposure and opportunity to explain mathematics explicitly and substantially in writing. The researchers believe that through writing, students will learn to develop and foster higher ability to communicate their mathematical ideas and understanding.

Key words: journal writing, mathematics, Singapore, primary, multiplication, division.

Introduction

The Singapore mathematics syllabus for primary level emphasizes that students’ ability ‘to communicate mathematically...to be able to illustrate, to interpret, to explain and to discuss mathematical ideas and experiences in doing mathematics’ (Ministry of Education [MOE], 2000, p. 17) is an important communication aspect in learning mathematics. In other words, part of mathematics learning is to develop students in using mathematical language to explain mathematical ideas and understanding precisely, concisely and logically. However, research has been done to show that in the primary mathematics classrooms, the textbook exercises and problems in the worksheets that students work on are usually structured in a way merely meant for drill and rote practice with not much sustained writing expected (Yeo & Zhu, in press). Most of the time, worksheets are given to students for consolidation of topics learnt and students assimilate what teachers teach, in a seemingly procedural fashion. In addition, getting the correct answers seems to be the main indicator of students’ understanding of mathematical sums and problems. Students should be given the opportunity to write substantially even in mathematics classrooms and it is believed that the use of alternative strategies such as journal writing could help in developing students’ writing aspect.

In the recent years, there has been an increasing interest in the use of journal writing as an alternative assessment tool in mathematics classrooms (Badger, 1992; Drake & Amspaugh, 1994; Pugalee, 2001). There are studies that attempt to classify and explain how students’ writings in mathematics classes convey their understanding and learning of mathematics to teachers through the use of different writing activities (e.g. Pearce & Davison, 1988; Shield & Galbraith, 1998) and studies that assert the potential benefits of using writing to develop students’ understanding of mathematics (e.g. Dougherty, 1996; Bagley & Gallenberger, 1992). Although the use of journal

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writing in mathematics learning is not prevalent in the context of our local schools, some small-scale structured research work in the mathematics classrooms such as Yazilah and Fan (2002) on journal writing with primary five students, both Kurup (2003) and Ng (2003) working with primary four to six and five students respectively in using journal writing as an alternative assessment, and Yeo’s (2001) study in using journal writing in the junior college level seem to suggest the increasing recognition of developing students’ mathematics learning through the use of writing.

This study intends to investigate how journal writing can be used to examine Singapore primary three students’ perception and their ability to explain their understanding of multiplication and division based on two separate case studies. A journal writing task is a piece of work that a student writes through to explain and describe their knowledge and understanding about a particular mathematical concept or process depending on the given writing prompts. It is hoped that journal writing will help bring out the communication aspect of the learning of mathematics at primary three. The researchers also believe that through journal writing, students will be given the opportunity to express themselves and to connect cognitively and aesthetically with their teachers about mathematics. In addition, it is also believed that teachers, too, will be able to make better progress in understanding the young learners and hence enhance their own delivery of lessons.

Participants

As indicated earlier, this study is part of a large research project on integrating new assessment strategies into daily mathematics classroom teaching and learning. In particular, two primary schools participate in this present research. One is a high-performing school and the other, a non-high performing school. In order to have more representative samples from each school, two classes of primary three students are selected such that one has high-ability (HA) and the other mixed-ability (MA) students. A total of four classes or 158 students and four teachers are involved in the study. The school that is identified as a high performing school ranked as one of the top nine best performing schools based on the Primary School Leaving Examination Results in year 2003.

Data processing and analysis

To investigate how primary three students can use journal writing to express their understanding of mathematics to their teachers, two specific journal writing tasks are examined. One case study looks at students’ writings based on a journal writing task relating to the learning of multiplication and the other case study examines students’ writings on the topic on division. In the first case study, the journal writing task is designed by the researcher and all the four classes of participating students work on the task and this task is completed by 155 students. Since journal writing is to be integrated in the normal teaching curriculum time with teachers having the autonomy to set appropriate journal tasks for their own students, the journal writing task in the second case study is attempted on an individual basis by a participating teacher in finding out how her students perceived division at the point when division is taught in class. This second task is completed by 36 MA students.

Teachers integrate the journal writing tasks into the normal daily teaching curriculum and both the journal writing tasks are completed during curriculum time. In fact, the journal writing tasks are infused into students’ daily learning as part of their everyday tasks they would have attempted. On completion, students’ writings scripts are collected and marked.

Results and Discussion

Task 1:

You have learnt about multiplication. A new pupil is going to join you and your teacher wants you to explain to your new friend what this means:

\[ 6 \times 4 = 24. \]

You can use pictures to explain too. Write down how you are going to go about it. Tell as much as you can about \( 6 \times 4 = 24 \).

Remember you ARE the teacher now.

The results of analyzing students’ writings in this task show that out of 155 students, 105 (68%) students are able to conclude that \( 6 \times 4 = 24 \) represents either 6 groups of 4 or 4 groups of 6 in diagrammatic forms. In particular,
at least 70 of the 155 students actually use the phrase ‘6 groups of 4’ in their explanation. The following are three examples from students’ writings demonstrating the above notation:

Eg 1
6 x 4 means 6 groups of 4

Eg 2
6 x 4 means six fours or four sixes. Or you can count all the apples on the plates.

Eg 3
Say that there are four groups and in each group there are six marbles. To find out the numbers of marbles we can use multiplication. Multiplication is quite similar to addition. If you do addition your number statement will look like 6+6+6+6 which means 6 plus 6 plus 6 plus 6. For multiplication, your number statement will look like 6 x 4 or 4 x 6 which means 4 times of 6 or 6 times of four. No matter you do 6 x 4 or 4 x 6 you will always get the same answer, 24!

It is also noted that in students’ writings, the symbol ‘x’ seems to have been used consistently as ‘times’. Hence 6 x 4 = 24 to the students means 6 times of four. There is also evidence from the students’ writings showing that most of the students are able to decipher multiplication as groups of items. Students’ writings also reveal that they tend to use 6 x 4 and 4 x 6 interchangeably. Although the calculation to 6 x 4 is the same as 4 x 6 which yields 24, the idea however is different. It is therefore beneficial to emphasize the difference during teaching.

Furthermore, the results also show that there is a small proportion of students (12%) who write about their ideas of the difference between 6 x 4 and 4 x 6 using diagrams as shown in example 4.

Eg 4
6 x 4 is 6 groups of 4
4 x 6 is 4 groups of 6

In particular, a student explains in his writing explicitly using the words ‘but the concept is different...’ as shown in example 5.

Eg 5
6 x 4 = 24 means 6 groups of 4.
4 x 6 also = 24 but the concept is different. It means: 4 groups of 6

It is interesting to note how another student explains using a scenario to show when 6 x 4 and 4 x 6 can be used interchangeable and when it is not appropriate. This is illustrated in example 6.

Eg 6
6 x 4 means 6 groups of 4... 'the correct way of saying six times four is six multiply by four'... an example is

Q1. John has 6 bags. In each bag, he puts 4 sweets. How many sweets does he have?

<table>
<thead>
<tr>
<th>Number of groups</th>
<th>number of items</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 4 4 4 4 4</td>
<td></td>
</tr>
</tbody>
</table>

This question you must write it as 6 x 4, if there is no question, you can either write 6 x 4 or 4 x 6...
The results of the analysis of students’ writings also reveal that only 31% of students use repeated addition as another way of expressing multiplication. In fact, $6 \times 4 = 24$ can be interpreted as 6 multiplied by 4; hence 6 appeared 4 times repeatedly instead of 6 times 4 which could then be interpreted as 4 appearing 6 times. Mathematics teachers would understand that symbols themselves are meaningless. However, symbols can acquire meanings when contexts are provided. It is therefore important that mathematics teachings involve using appropriate language and context to illustrate and give meaning to an otherwise seemingly meaningless symbol ‘$\times$’. Because mathematical symbols are abstractions, the words and phrases used to discuss them invariably shape how individuals think about them. Such words and phrases then could be associated with the symbol of multiplication. Hence, if words such as ‘multiplied by’ and ‘times’ are used consistently over a period of time, students will be better able to see the subtle differences in $6 \times 4$ as 6 multiplied by 4, i.e. $6 + 6 + 6 + 6$ and $6 \times 4$ as 6 times 4 which can be represented as a repeated addition of $4 + 4 + 4 + 4 + 4 + 4$. Cases where students are able to show successfully the relationship between multiplication and the use of repeated addition are demonstrated in the following examples 7, 8 and 9. Again, it is assumed that the students use the relational word of the symbol consistently to effect the repeated addition statements shown.

**Eg 7**

- 6 row
- 4 row

$6 \times 4 = 24$ mean 4 group of 6 (as in the script)
Also mean $6 + 6 + 6 + 6 = 24$
Mean $4 + 4 + 4 + 4 + 4 + 4 = 24$

**Eg 8**

$6 \times 4$ is $4 + 4 + 4 + 4 + 4 + 4 = 24$ or
$4 \times 6$ is $6 + 6 + 6 + 6 + 6 = 24$

*It means that you add 4 six times or add six 4 times. The total number is the answer for $6 \times 4 = 24$ because the meaning is the same.*

**Eg 9**

a) $6 \times 4$ is 6 groups of four
b) $4 \times 6$ is 4 groups of six
Example : a) $6 \times 4 = 6 + 6 + 6 + 6$
Example : b) $4 \times 6 = 4 + 4 + 4 + 4 + 4$
Example: a) $6 \times 4$ = to the same number as $4 \times 6$
Example: b) $4 \times 6$ = to the same number as $6 \times 4$

To summarize, results from analyzing students’ writings on the first journal writing task show those students generally have a good understanding of $6 \times 4$. The use of diagrams by students to illustrate their understanding of the mathematical expression $6 \times 4 = 24$ seems to suggest that they understand the concept of multiplication as an ‘expansion’ of groups and repeated addition. However, some of the students’ writings do not concretize their ability to decipher the subtle difference between $6 \times 4$ and $4 \times 6$. Students should be given more opportunities to practice in explaining their ideas and thinking through the use of writing. Students’ writings also consistently reveal that although some of them are able to say that $6 \times 4$ yields the same value as $4 \times 6$, these students do not quite successfully explain that they have understood the commutative property of multiplication. However, a few are able to articulate that $6 \times 4$ is the same as $4 \times 6$ by using diagrams to help them in their explanation.

**Task 2 :**

You have learnt to do simple division. What is division? Give examples of some simple division sums. Write all that you know about division. You may draw pictures/models to illustrate your understanding."

The results of analyzing the 36 MA students’ writings show that all of them are able to identify the symbol (+) with division. In their attempts to explain their understanding of the task, students are able to create their own
numbers to using the division sign for example ‘24 ÷ 4 = 6’ and go on to illustrate what they mean by the division statement.

From the students’ writings, about 47% of the students perceived division as ‘related to or opposite of multiplication’. This is a very interesting analysis; are students regurgitating what is being said and taught in class and possibly explain the high incidence of students’ writings reflecting division as being ‘opposite of multiplication’? Moreover, 27% of the students insist that knowing their multiplication tables would inadvertently lead to better division sense! The following examples are some of the students’ writings illustrating the above notions:

Eg 1  It is easy to learnt division if I know multiplication. Division is like an upside down of multiplication like this:

Division :  \[ 25 \div 5 = 5 \]
Multiplication :  \[ 5 \times 5 = 25 \]

Division” means share or grouping
Division is also means share equally.

Eg 2  The way you write a division statement is the opposite way you write a multiplication statement. For example 5 x 10 = 50 and the division statement is 50 ÷ 10 = 5 or 50 ÷ 5 = 10 I can get the answer easily if I know my multiplication table.

Eg 3  Division is to put equal numbers into each groups and It is opposite the way you do multiplication.
Example: 24 ÷ 6 = 4

Eg 4  When you do an division sum you must know your multiplication for example, there are 15 sweets how do I share them equally among 5 children.
You can do skip counting as well.

Eg 5  Division and multiplication are related how 6 x 10 = 60 then division will be 60 ÷ 10 = 6 or 60 ÷10 = 6 division is gonna be hard if you do not know your time tables example 60 ÷10 = 6 is 6 groups then you have put thing inside the circle’s equally like this

and then the answer is inside the circle 10.

The results also indicate that 86% of the students mention in their writings about division as having to do with being “equal, in groups or shared”. The analysis of students’ writings gives an insight into students’ thinking of division being associated with not only multiplication but also the concepts of grouping, sharing and equality in sharing. A particular student’s writing in example 6 demonstrates the above point taken:

Eg 6  Division is a sum that you must divided it in to groups. I will show you one sum that that it must show that it is equal to all children.
example:

There are 100 cookies in a box. If there are 4 children, how many cookies will each child get?

Cookies  \[100 \div 4 = 25\]

You must draw four circles first. You must put one in each circle, then you non-stop putting till you count to 100.
Although not mentioned in the problem posing of ‘equalness’, in the workings, it is quite obvious that the above student takes for granted the necessity of having equality in distribution.

In addition, results show that eight of the 36 students resort to problem posing to illustrate their understanding of division. This could be due to the fact that they find telling stories easier to relate to. This is not difficult to comprehend as most mathematics teachers, when first tackling the topic on division, would generally create a scenario for equal sharing before introducing formal division with symbols. This could have created an impact in these students of using the same approach to explain division in what they would then deem as the simplest and most effective way. It seems to suggest that using ‘problem scenarios’ before embarking on the ‘mechanical’ and procedural learning of some mathematics topics could lead to better retention in students’ learning.

Furthermore, results of the analysis also indicate that almost all the 36 students use a big number divided by a smaller one to explain the idea of division. To illustrate, see example 7. It is indeed a concern as to whether these students assume that in division statements, the biggest number would ALWAYS be the dividend. Do students really understand that in division, the dividend need NOT always be the larger number? The occasion may not have arisen for these students to wonder the case as most of the mathematical problems they work with tend to be ‘safely’ presented such that the larger number is the dividend. However, the researchers feel that it is imperative for teachers to be aware of the above situation.

Eg 7

Division is like the big number divided with a small number equals to a small number e.g. 48 ÷ 8 = 6.
Division is like dividing in a group. Division is putting the biggest number in groups the second number is the number of groups than follow by the answer. 35 ÷ 5 = 7

The results in analyzing students’ writings also enable the teacher to find out that students have a weak understanding of the number placing and the idea of the position the number takes in a given mathematical division statement. It is evident that these students cannot accurately comprehend the concepts of the dividend and divisor, and the position of the numbers when using the mathematical notations. They seem to think that as long as there are three ‘terms’, a division sign and an equal sign, the statement is therefore acceptable! Examples 8 and 9 clearly spell out the above notions:

Eg 8

division is just dividing things into groups, here is one example. If you don’t now how to draw a working I will show you how to draw.

4 ÷ 56 = 14

Eg 9

12 x 2 = 24
12 ÷ 24 = 2
24 ÷ 2 = 12
11 x 7 = 77
11 ÷ 77 = 7
7 ÷ 77 = 11
12 x 6 = 72
72 ÷ 6 = 12
6 ÷ 12 = 72
6 ÷ 72 = 12

To summarize, in general, multiplication concepts are well received and explained by the students. It would augur well if students could be given the opportunity to decipher the subtle difference between 6 x 4 and 4 x 6 on a more detailed basis. The journal writing prompts are able to tease out what students actually think about certain learnt mathematical issues. Their responses and beliefs could spur on further journal writing prompts to elicit more mathematical understanding in them. Students’ writings show that most of the participating students are able to relate the meaning of the symbol ‘÷’ to division. Moreover, students are able to demonstrate their understanding about the relationship between division and multiplication in their writings. However, teachers may want to place more attention in teaching the understanding of dividend, divisor and quotient. For example, how would they know which number is the dividend and divisor, given a particular multiplication statement?
Summary and conclusions

Multiplication and division form the main topics of mathematics learning at primary three level. A large portion of curriculum time is apportioned to the learning of Multiplication and Division at this level. Moreover, multiplication and division are important concepts in mathematics with very broad application purposes in our daily lives. Results from analyzing students’ writings show that students’ ability to perform and compute using multiplication and division do not necessarily mean the ability to explain and interpret concepts. Students seem to find it easier to show teachers their computational skills than to explain in their own words the concepts or ideas or their thinking. Students’ writings also seem to reflect their rigid thinking about thoughts and views about multiplication and division. The researchers believe that to improve students’ ability to communicate in writing about mathematics should be emphasized in the normal daily teaching and learning in the classroom. The exposure and the given opportunity for students to express their thinking, ideas and knowledge to teachers and peers will encourage reflection on their own learning and thus develop further depth of understanding.

Journal writing has a place in the learning of mathematics in primary schools. As a teacher comments: “Through journal writing, I am able to have a better understanding of my students’ weaknesses”, journal writing can help enhance the teaching strategies of the teacher. Journal writing helps to communicate to teachers that what is taught may not be learned in the same way by all students. It also provides an avenue for teachers to reflect on their teaching strategies as can be seen in the majority responses from their students. Teachers can then ask themselves: “Are my students thinking or are they just regurgitating procedural aspects of mathematics learning?”

Reference