An analysis of Singapore secondary students' performance on open-ended tasks in mathematics[1]

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Open-ended tasks are one mode of alternative assessment used in mathematics. This study is part of a larger research project on integrating alternative assessment strategies into mathematics daily teaching and learning. It investigates secondary one students' performance in solving open-ended problems in Singapore context. Over three hundred secondary one students from one high-performing school and one non-high performing school participated in this study. The test consists of three open-ended tasks, covering both arithmetic and geometry. The openness of the problems includes two aspects – multiple approaches, which allow students to begin working on the problems using different approaches, and multiple acceptable answers (i.e., more than one correct answer). The results reveal that the majority of students adopted "trial and error" as the only strategy, which limited the number of possible solutions that most students can obtain. The students also experienced much difficulty in organizing their solutions. The results suggest that more variety of problem solving strategies need to be taught and documentation is another important skill that needs to be paid attention to in problem solving.

Key words: Open-ended, alternative assessment, mathematics, problem solving, secondary

Introduction

In the year 1995, fourth and eighth grade students of Singapore were ranked first in mathematics in the Third International Mathematics and Science Study (TIMSS) (Beaton et al., 1996). The results prompt many researchers to examine factors that may help to explain why students from this small island perform so well in mathematics. In the years that followed, Singapore students continued with their outstanding performance in the TIMSS 1999 and TIMSS 2003 (Gonzalez et al., 2004; Mullis, Martin, Beaton, et al., 1997; Mullis, Martin, Gonzalez, et al., 2000), which generate greater interest and effort by both local and international researchers to look into Singapore education system and investigate the key factors underlying their success in the TIMSS. However, some researchers argued that the test items used in the TIMSS may not measure students' ability in higher-order thinking (e.g., Bracey, 2000; Wang, 2001). They questioned the effectiveness of the instrument, as most of the test items are multiple-choice questions, which are only helpful in measuring procedural knowledge and routine problem solving skills. This implies that the well publicized results may only indicate that Singapore students have acquired a good foundation in the basic mathematical knowledge and are proficient in applying routine procedures and manipulation skills. As for higher-order thinking skills, these studies do not provide sufficient evidence to draw any inference about the Singapore students' performance in solving challenging problems. According to Cai (1997), in order to fully understand students' mathematics ability, especially about problem solving, the investigation must be in a multidimensional manner, inclusive of both routine and non-routine tasks.

Open-ended tasks are one such form of non-routine problems, which allow a variety of correct responses and at the same time help to elicit the thinking processes involved in solving the tasks. This study intends to investigate the performance of Singapore students on open-ended mathematics tasks. According to Takahashi (2001), there are two types of open-ended problems: problems with only one solution but diverse approaches and problems with multiple correct answers. In this study, all the tasks are designed to have more than one acceptable answer and multiple approaches. Besides providing

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readers with research evidence on how Singapore students perform in solving open-ended tasks, the study also intends to explore the implications and offer suggestions for the improvement of students' ability in problem solving.

Methods

To investigate Singapore secondary students' performance in solving open-ended problems, the study uses three open-ended mathematics tasks, covering both arithmetic and geometry in content. A total of 315 secondary one students participated in the study and the answer scripts were collected and analyzed.

Participants

In Singapore, all students are placed in either the Special (SP), Express (EXP), Normal Academic (NA), or Normal Technical (NT) course in the secondary schools, based on their Primary School Leaving Examination (PSLE) scores. The SP and EXP courses adopt the same framework, as it forms the basis of mathematics learning and teaching for all students in secondary schools. The mathematics curriculum for the NA course is designed as a subset of the mathematics syllabus for the SP/EXP course (Ministry of Education [MOE], 2000). The syllabus content for the NT course is comparable to that of the NA course with differentiation in the sequence of topics and rigor in the mathematics content.

Due to the fact that the percentage of students taking the NT course in each year is no more than 15% (MOE, 2004) and not all secondary schools offer the course, this study has excluded this group of students from its research focus. Students involved in this study were selected from the SP, EXP, and NA courses.

Two schools, a high performing school and a non-high performing school took part in this study. Two classes from different courses in each school were involved. School A is identified as high performing school, as it was ranked as one of the top 50 best performing schools, based on the GCE "O" Level Examination in the year 2001 and 2002; School B is a neighborhood school and was not ranked among the top 50 best performing schools based on the same criteria and hence was deemed a non-high performing school. A total of 315 secondary one students from the two schools participated in the study with one SP and one EXP class from school A and one EXP and one NA class from school B. Table 1 gives a profile of the participating students from these two schools.

Table 1
A Profile of the Participating Students

	School A				School B			
Class	1	2	3	4	5	6	7	8
Course Type	SP	SP	EXP	EXP	EXP	EXP	NA	NA
No. of Students	38	38	39	40	40	40	40	40

Instrument

The main purpose of this study is to investigate how Singapore students perform in solving open-ended problems. Three mathematics tasks were specifically designed for this study with each of the tasks having more than one acceptable answer. Due to the fact that the participating students had just began their secondary education when the study was conducted, the tasks used in the study only covered either arithmetic or geometry, the two main topics in the Singapore primary mathematics curriculum. Hence, the test consists of one geometry task, *Cube Arrangement*, and two arithmetic tasks, *Filling Numbers*, and *Movie Selection*.

To assess students' performance in working on the open-ended tasks, task-specific rubrics were designed for each task by the two researchers. The rubrics mainly look into three particular aspects described as follows:

? *Mathematics approach/procedure* (A): Decisions on how to approach the problem. Strategies, skills, and concepts involved in solving the task;
? *Problem solution* (S): The answer(s) to the question(s);
? *Presentation* (P): Documentation/Explanation of how the problem was solved.

Thirty-five secondary one students (EXP: 18, NA: 17) from another school participated in the first pilot test on 19 February 2004. During the pilot study, each student was assigned to two open-ended problems, 18 students with the tasks on *Cube Arrangement* and *Movie Selection* and 17 students with the tasks on *Filling Numbers* and *Movie Selection*. The pilot study showed that students were generally able to comprehend the tasks except for some terms (e.g., “proposal”). The two tasks were completed within 20 minutes. A 10-minute informal interview with the students after the test reveals that most students did not have any prior experience in solving these types of tasks before but they expressed interest in attempting more of such tasks.

Based on the first pilot results, the researchers made necessary modifications for the three open-ended tasks, including further simplification of phrases that appeared in the task (e.g., change “proposal” into “combinations of movies”). On 1 March 2004, a second pilot study was carried out with 36 NA students from a fourth school. Students were asked to work on all the three refined open-ended tasks within 20 minutes. The results show that all the students had little difficulty in understanding the tasks. However, a timeframe of 20 minutes was too short for students to complete all the tasks. Based on the feedback for the second pilot, it appears that a more reasonable timeframe for the three tasks is a class period of 30 minutes. Besides, students expressed difficulty in working on the task, *Filling Numbers*, and had spent too much time working on it thereby did not have sufficient time for the other two tasks. Hence, the final test was set in the order of *Cube Arrangement*, *Movie Selection*, and *Filling Numbers* (see the appendix).

Data collection

The final test papers containing the three open-ended mathematics tasks were distributed to all eight secondary one classes in the two schools in March 2004 and collected from 315 students in April 2004, with a response rate of 99.4%.

Data processing and analysis

Based on the pre-designed task-specific rubrics, the two researchers first marked 10 random selected students’ answer scripts individually to concur on the levels for each aspect. A fair agreement was reached (ICC = 0.68)[2]. To enhance the inter-rater reliability, the researchers held discussions about the rubrics and made further refinements followed by a second round of independent marking. The average ICC increases to 0.96. Only one researcher continued with the scoring of the tasks based on the finalized rubrics. Students’ marks were then analyzed mainly using quantitative methods, such as frequency and percentage, to depict how students perform on open-ended tasks and the three aspects as stated in the pre-designed rubrics.

Results and Discussion

The results on students’ performance in solving open-ended problems are reported in the following three aspects as outlined earlier on: the use of effective strategies, completeness of students’ answers, and presentation of the solutions.

The use of effective strategies

Using effective strategies is a very important aspect in problem solving. The three tasks in the study can be approached using different strategies, including “trial and error”, “making a systematic list”, and “looking for a pattern”. Although all these strategies could possibly lead to the correct answers, the latter two are more effective and efficient, in comparison to the strategy on “trial and error”, as the strategy “trial and error” is dependent on the number of trials and students who use this strategy may miss out on some answers. According to the first aspect, use of effective strategies, the corresponding rubrics assigned to the tasks *Cube Arrangement* and *Movie Selection* have five different levels of attainment, and six levels for the task *Filling Number*, arranged in a hierarchy from the lowest where no attempt or no approach is evident to the highest level where the most effective strategies are adopted.

The data show that the students depended heavily on the strategy of “trial and error”. In particular, about 76% of the students used this strategy in solving the task *Cube Arrangement*, 65% in *Filling Numbers*, and 49% in *Movie Selection*. None of the students received full marks in the task *Cube Arrangement,* in the aspect of using effective strategies. In other words, no student demonstrated a

[2] ICC stands for Intraclass Correlation Coefficient.

systematic way of arranging the cubes in their answer script, such as moving only one cube while keeping the other three stationary and then viewing the arrangement from another perspective from plane to solid. Out of 135 students, only 29 arranged the cubes in a partially systematic way, that is, fix three and move one.

For the task *Filling Numbers*, it is also found that none of the students received full marks in terms of using effective strategies. The highest level is assigned to those who are able to recognize either of the relationships that

- the sum of all the numbers and twice the central number is divisible by 3;
- the sum of all numbers minus away the central number is divisible by 3.

From the data, we gathered that 44 students (19%) systematically switched the surrounding numbers (either clockwise or anti-clockwise) and 3 (1%) used effective strategies to find different central number(s) but failed to make the above mathematical observation in a more general way.

In solving the task *Movie Selection*, two students (6%) managed to find that the combinations of any five movies satisfy the task requirement; three students (10%) made a systematic list to search for the possible combinations of the movies.

In addition, the study also finds that some students made some effort to approach the challenging tasks but failed to adopt effective strategies (*Cube Arrangement*: 12%, *Filling Numbers*: 27%, *Movie Selections*: 16%). The findings reveal that many students adopted the approach of “trial and error”. It appears that the local students needed more exposure to a variety of problem solving strategies as well as non-routine and open-ended problems which require students to adopt different approaches.

Completeness of students’ answers

The key feature of the open-ended tasks used in this study is that every task has more than one acceptable answer. In particular, the task *Cube Arrangement* has seven distinct[3] correct answers; *Filling Numbers* has two different set of answers (central numbers in both answers are different from 1); *Movie Selections* has 56 correct combinations. Regarding the aspect, completeness of the answers, five levels ranging from no attempt or solution given being incorrect (Level 0) to full answers being obtained (Level 4) are assigned for all the three tasks in the task specific rubrics.

The results reveal that majority of the students provided only partial answers. In the task *Cube Arrangement*, about 15% of the students stopped after obtaining one answer; 69% obtained more than one answer but the answers were incomplete; and only 10% (three students) managed to find all the seven distinct answers. In fact, it is only possible for those who did not limit the cube arrangement in a plane but also in a solid to attain all the seven answers. However, many students were unable to get the point.

In the task *Movie Selection*, about 28% of the students just provided one combination of five movies; 14% managed to find more than one possible combination; and 29% were able to find all 56 correct combinations with two students (6%), as reported earlier, further summarizing that the combinations for any five movies were acceptable.

The findings in this study are consistent with the two pilot studies. Students in the main study experienced the most difficulties in solving the task, *Filling Numbers*. The data show that 39% of the students were not able to provide any correct answer while 23% only managed to find one central number but were not able to provide a complete answer. Some were not able to obtain a complete answer because they had randomly switched the surrounding numbers without changing the central number; 37% were able to find one complete answer with a different central number from the one given in the task (i.e., 1); and only 10% of the students (3) gave full answers.

The results reveal that a number of students stopped after obtaining one answer, which reflects their mindset that mathematics problems have one and only one solution. According to Schoenfeld (1992), this perception of one solution mathematics problem is a common misunderstanding among students. The findings that many students failed to find all acceptable answers although they were able to find one solution, to some extent, show that students were not skillful in handling tasks with more than one acceptable answer. One possible explanation is that they were not aware of other effective strategies to

[3] The arrangement cannot be obtained from other combinations merely through simple transformations, such as rotations.

solve the problem or they did not know what strategies to adopt in approaching such open-ended tasks. It may also mean that local students lacked the experience in solving open-ended tasks in their mathematics learning and need more attention from their teachers in this area.

Presentation of the solutions

The presentation of the solutions is another important aspect in problem solving, as one cannot effectively communicate ideas to others, without clear, logical, and organized presentation. In the view of the researchers, a well organized presentation is essential and five different levels are assigned in the rubrics under this aspect of presentation for all the three open-ended tasks. The five levels range from the lowest level where "no attempt is evident" (Level 0) to the highest level where "the documentation is clear and greatly adds to the reader's understanding of the procedures" (Level 4).

The analysis shows that many students' presentations were either irrelevant, not organized, or the working procedures was not clearly visible. In particular, more than 60% of the students' presentations in the answer scripts only attained the first three levels of the rubric (*Cube Arrangement*: 85%, *Movie Selection*: 95%, *Filling Numbers*: 61%). The students performed slightly better for the last task *Filling Numbers* which probably is attributed to the fact that "blank figures" have been provided in the answer scripts. Hence students only need to fill in appropriate numbers in the empty circles (see the appendix). The findings is also consistent with the two pilot studies carried out earlier on where, *Filling Numbers* is the only task that students were able to obtain the maximum level in the aspect of presentation (two students). About 15% of the students in the task, *Cube Arrangement*, 5% in *Movie Selections*, and 38% in *Filling Numbers* presented a relatively clear solution where partial working procedure can be observed.

The students' writings show that many merely included computational procedures or geometrical figures without detailing the necessary labels or explanations. Therefore, readers who are not given the original tasks would not possibly be able to comprehend the students' solutions. Even the two researchers, who designed the tasks, also encountered much difficulty in understanding the students' writings and the procedures. Students' limitations in presenting a well organized solution need to receive more attention from their mathematics teachers in daily classroom instruction.

Summary and Implications

This study intends to investigate the performance of Singapore students in solving open-ended tasks. In this research, open-ended tasks are defined as non-routine problems with more than one acceptable answer or multiple approaches in solving the problems. More than three hundred secondary one students from one high performing and one non-high performing school were assigned to complete two arithmetic and one geometry open-ended tasks in the study. The study reveals that most students adopted the approach of "trial and error", which might be one reason in helping to explain why students were unable to obtain full solutions to the tasks. The study also finds that quite a number of students stopped at obtaining just one answer, which probably reflects the mindset that mathematics problem has only one acceptable answer. Furthermore, it is found that students in this study experienced much difficulty in presenting their solutions in a clear, logic, and organized way.

The findings suggest that Singapore students are able to approach such challenging mathematics tasks to some extent though they appear to lack the experience and exposure to solving such problems. It is important for school teachers to be aware of the limitations of students in this domain. More opportunities in working on non-routine and open-ended tasks should be offered to students in their daily mathematics learning, which has been emphasized in the Singapore mathematics syllabus for both primary and lower secondary students (MOE, 2000a, 2000b). The researchers believe that the exposure would not only help students to improve their ability in problem solving, but also foster their higher order thinking in the learning of mathematics. Furthermore, with more exposure to open-ended problems, students may develop a more positive view towards mathematics and mathematics learning. Secondly, students need to acquire a variety of problem solving strategies and be provided with more learning opportunities to use these various strategies. Thirdly, the ability to communicate effectively is another important domain which requires more attention from school teachers. Besides, communicative skill has been highlighted as one important area in the Singapore mathematics syllabus (MOE, 2000a, 2000b). In conclusion, the researchers believe that with substantial exposure and careful planned classroom instructions, students would improve their ability in solving open-ended mathematical problem solving and at the same time establish a more positive perspective toward mathematics and mathematics learning.

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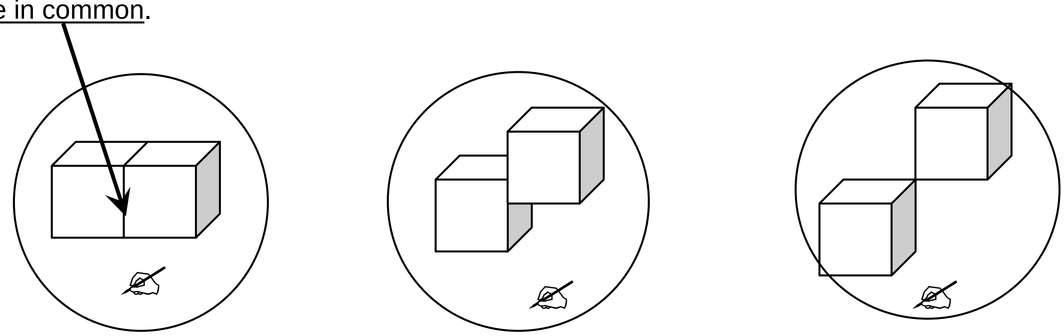
Appendix

Open-ended Test Paper

CUBES

There are four cubes of the same size. Put them together and make as many 3-D shapes as you can. **HOWEVER** you must make sure that any two cubes next to each other have to have one face in common.



Your arrangement of the cubes

MOVIE SELECTIONS

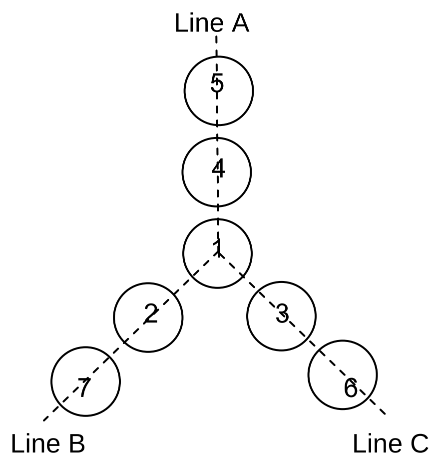
The teachers in the school have decided to celebrate Children's Day with the screening of several movies. The celebration will continue from 7.30am till 3.30pm.

A	Lilo and Stitch	1h 25 min
B	The Lion King	1h 27min
C	Beauty and the Beast	1h 30min
D	The Prince of Egypt	1h 30min
E	The Grinch	1h 45min
F	Finding Nemo	1h 21 min
G	Toy Story 1	1h 17min
H	Toy Story 2	1h 32min

The teachers want to screen as many movies as possible within the given duration (7.30am till 3.30pm). You are supposed to help the teachers to come up with all the possible combinations of movies that can be screened within this time limit.

FILLING NUMBERS

The figure below shows 7 numbers in circles.



The sum of the three numbers in Line A (4 + 5 + 1 = 10), Line B (7 + 2 + 1 = 10), and Line C (6 + 3 + 1 = 10) is the same. Using the seven numbers (1, 2, 3, 4, 5, 6, 7) without repetition, find other possible arrangements such that the sum of the numbers in Line A, Line B, and Line C is equal.