An Analysis of Singapore Secondary Students’ Performance on One Authentic Open-Ended Mathematics Task

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Abstract

Authenticity and open-endedness are two important factors that are being greatly emphasized in mathematics teaching and learning nowadays. This study investigates how Secondary one student in Singapore solved an open ended mathematics problem, River Cruise, set in an authentic context. Over three hundred secondary one students from a high-performing school and a non-high performing school participated in this study. The results reveal that the majority of the students only take into account one condition in coming up with their solution, when it involves more than one possible condition. Working within the framework of that assumed condition, most students merely present one correct answer, when there exist multiple correct answers. The majority of the students do not adopt a systematic approach in solving the problem and the solutions presented are not well organized. However, it has been observed that students enjoyed working on the problem. The results suggest that students need greater exposure to authentic and open-ended tasks in mathematics learning. The exposure will not only help them to develop their problem solving ability and foster their mathematical thinking but also encourage them to make connections between mathematics and the world around us.

Keywords: authentic, open-ended, mathematics, Singapore, secondary.

Introduction

Singapore students were ranked first in the world in mathematics on the Third International Mathematics and Science Study (TIMSS) series from 1995 to 2003 (Beaton et al., 1996; Gonzalez et al., 2004; Mullis, Martin, Beaton, et al., 1997; Mullis, Martin, Gonzalez, et al., 2000). Their success in the TIMSS arouses the interest of many researchers in the Singapore education system, both locally and internationally. Much effort has been invested to examine the system, including instructional materials and pedagogy. However, some researchers argued that the test items used in the TIMSS may not measure students’ ability in higher-order thinking (e.g., Bracey, 2000; Wang, 2001). They questioned the effectiveness of the instrument as most of the test items are multiple-choice questions, which may be only helpful in measuring procedural knowledge and routine problem solving skills. This implies that the well publicized results may only indicate that Singapore students have acquired a good foundation in the basic mathematical knowledge and are proficient in applying routine procedures and manipulation skills. As for higher-order thinking skills, these studies do not provide sufficient evidence to draw any inference about the Singapore students’ performance in solving challenging problems According to Cai (1997), in order to fully understand students’ mathematics ability, especially about problem solving, the investigation must be in a multidimensional manner, inclusive of both routine and non-routine tasks.

This study intends to investigate the performance of Singapore students on one authentic and open-ended problem, River Cruise. The authenticity, according to Assessment Standards for School Mathematics (National Council of Teachers of Mathematics, 1995), is the degree to which activities are faithful, comprehensive representations of the contexts and complexity found in important, real-life performances of adults that are non-routine yet

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meaningful and engaging for students. It is believed that tasks with this feature are intended to assess students’ ability to apply standard-driven knowledge and skills to the real-world challenges. Open-endedness in this study refers to problems with more than one acceptable answer. Compared to routine problems, open-ended problems might be a better measure of students’ higher-order thinking skills (Cai, 1997). It allows students to explore and present their unique answers in accordance to their problem solving ability. Hence it provides more opportunities for varied ability students to demonstrate their mathematics ability (Wu, 1994). This study aims to provide readers with the research evidence of Singapore students’ mathematics performance in an open-ended performance task, a much less researched area in the local context. It also explores the implications and offers suggestions for the improvement of students’ ability in solving open-ended problems.

**Methods**

**Participants**

In Singapore, all students are placed in either the Special (SP), Express (EXP), Normal Academic (NA), or Normal Technical (NT) course in the secondary schools at the end of their primary education, based on their Primary School Leaving Examination (PSLE) scores. The SP and EXP courses adopt the same framework in mathematics instruction, as it forms the basis of mathematics teaching and learning for all students in the schools. The mathematics curriculum for the NA course is designed as a subset of the mathematics syllabus for the SP/EXP courses. (Ministry of Education [MOE], 2000). The syllabus content for the NT course is comparable to that of the NA course with differentiation in the sequence of topics and rigour in the mathematics content.

Due to the fact that the percentage of students taking the NT course in each year is no more than 15% (MOE, 2004) and not all secondary schools offer the course, this study has excluded this group of students from its research focus. As a result, students involved in this study were selected from the SP, EXP, and NA course.

Two schools, a high performing school and a non-high performing school took part in this study. Two classes from different courses in each school were involved. School A is identified as high performing school, as it was ranked as one of the top 50 best performing schools based on the GCE “O” Level Examination in the year 2001 and 2002. School B is a neighborhood school and was not ranked among the top 50 best performing schools in the two years and hence was deemed a non-high performing school. A total of 145 secondary one students from the two schools participated in the study with one SP and one EXP class from school A and one EXP and one NA class from school B. Table 1 gives a profile of the participating students from the two schools.

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<tr>
<th>School</th>
<th>SP</th>
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<td>Class 4</td>
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**Instrument**

The aim of this study is to investigate Singapore students’ performance in solving one authentic and open-ended problem. A mathematics worksheet, *River Cruise*, set in an authentic context, including one open-ended problem, was specifically designed for this study. The topic involved in the worksheet is “algebraic expression”, covering four sub-topics: (1) using letters to represent numbers, (2) expressing basic arithmetical processes algebraically, (3) substituting numbers for letters in formulae and expressions, and (4) manipulating simple algebraic expressions. Working on the basis that student may not be familiar with solving such challenging problems, a number of “warming up” exercises (targeted on basic understanding of the concepts and simple manipulations) are designed before they attempt the open-ended task. These “warming up” exercises are intended to familiarize students with
the context of the problem as well as to review the basic concepts and skills involved (i.e., algebraic expression), as the main purpose of the study is to examine how students approach open-ended problems.

The scenario in this open-ended problem is about a school teacher who wanted to take her class of students for a river cruise. Two sizes of boats are available with differences in capacity and rental fees. The teacher needs to make a decision on the number of boats to rent to meet the criteria in each of the four sub-questions: (1) at a minimum cost, (2) with a minimum number of boats, (3) with a minimum number of empty seats, and (4) the most reasonable plan (see Appendix).

To assess students’ performance in working on the open-ended problem, a task-specific rubric is designed. Therubric looks into four particular aspects, described as follows:

- **Mathematics approach/procedure (A):** Decisions on how to approach the problem. Strategies, skills, and concepts involved in solving the task;
- **Problem solution (S):** The answer(s) to the question(s);
- **Presentation (P):** Documentation/Explanation of how the problem was solved;
- **Connection (C):** Statements indicating recognition of interconnected mathematical ideas within or beyond the solution.

**Data Collection**
The open-ended problem is sequenced after a number of “warming up” exercises in the worksheet meant for daily instructional classroom practice. The timeframe assigned by teachers on the task varies across the four classes. Teachers use the worksheet only when they are teaching the particular topic on algebraic expression or to review the topic. Hence all the three classes with the exception of the NA class completed the worksheet in late April, 2004, while the NA class implemented it in early August, 2004.

The worksheet was designed to be completed within two class periods (i.e., 60 minutes). Teachers were instructed to go through the “warming up” exercises with their students in the first 30 minutes and allocate the other 30 minutes for students to work on the open-ended problem. However, due to the different abilities of the students, the time used to complete the “warming up” exercises varied across the classes. Despite such difference, students were given sufficient time to work on the open-ended problem in the class. After 60 minutes, the scripts were collected and returned to the researchers, with an overall response rate of 91.8%, ranging from 72.5% to 100% across the four classes.

**Data Processing and Analysis**
The two researchers marked the students’ solutions based on the pre-designed task-specific rubric independently. The inter-rater reliability between the two researchers on the four sub-questions across the four marking aspects ranges from 0.596 to 0.904, with an average of 0.783, indicating a moderate agreement. Descriptive statistics, such as mean and percentage, is used to delineate how students perform in the open-ended problem in four particular aspects as stated in the pre-designed rubric.

2 The criterion is only applicable to Qn3.

**Results and Discussion**
In this section, the results of this study are reported in the following sequence: use of strategies, “openness” of students’ answers, presentation of the solutions, and connections within or beyond the solutions, parallel to the sequence of the four categories identified in the pre-designed task-specific rubrics.
Use of Strategies
According to Pólya (1971), there are four stages involved in problem solving: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. This aspect examines how students go about solving the “river cruise” problem. In the other words, we examine the approaches or strategies adopted by students to solve the problem. In general, this problem can be solved either in a systematic way such as making a systematic list or by trial and error. Although both methods can plausibly lead to the correct answers, the systematic approach, in the researchers’ view, is deemed to be more effective and efficient in solving this problem. In accordance with the rubric, four different levels (level 0 to level 3) are assigned to this aspect arranging from “no attempt or no approach is evident” (level 0) to “a systematic list is made” (level 3).

The data show that the majority of students are able to adopt strategies which could lead to the correct answers (Qn1: 77.2%, Qn2: 77.2%, Qn3: 66.9%, Qn4: 67.6%). However, in most cases, the strategies used by students are “trial and error” (Qn1: 60.7%, Qn2: 69.7%, Qn3: 54.5%, Qn4: 55.9%). It appears that students are able to solve open-ended problems but not completely and improvement on their problem solving skills is still deemed necessary. The strategy “trial and error” is considered to be useful for problems with relatively simple situations or when small numbers of trials are involved. In more complex situations, the strategy “trial and error” may not lead to a complete solution without sufficient number of trials and could be time-consuming. The researchers do not intend to devaluate the usefulness of “trial and error” as it is may be useful and effective at the initial stage of problem solving. However, a variety of approaches or more reliable strategies should be adopted to solve open-ended problems.

In addition, the analysis also reveals that some students do not make any attempt to solve the problems and the percentages of students not attempting the questions increases when the question numbers increase, ranging from 10.0% on Qn1 to 28.3% on Qn4. We believe that this is not a positive sign, as no attempt may indicate that students are either not interested in solving such challenging problems or they have no idea on how to approach the problem. If the inherent reason is the former, teachers then need to be aware of the affective component of the students and provide encouragement and support when they are solving such problems. More exposure to open-ended problems and instruction on a variety of problem solving strategies would be helpful to those who have no idea on the approach.

“Openness” of Students’ Answers
To solve this open-ended problem, students need to take into account two different conditions: (1) the presence of the school teacher as a part of a group in one of the boat trips, and, (2) the school teacher does not commute with the any of the groups in the boat cruise. However, the solutions submitted reveal that the majority of students merely consider only either one of the above conditions. About 15.9% of the students reflect both conditions in their solutions for Qn1, 9.7% for Qn2, 12.4% for Qn3, and 6.9% in Qn4, respectively.

Under the broad framework of the two conditions, there are several plans which students can work out that satisfy the specific criteria (applicable to Qn2 and Qn3). The data show that many students stop at presenting just one suitable plan, irregardless of whether only one or both conditions are taken into consideration. In particular, 55.2% of the students present only one plan for Qn2 and 54.5% for Qn3, with most of them, as reported earlier, considering only one condition.

In accordance with the pre-designed task specified rubric, the highest level is assigned when solutions include both conditions and consider all appropriate renting plans. However, for each sub-question, overall, there are less than 10% of the students being accorded the highest level for solution. None of the students presented all the nine possible renting plans for Qn3 in their solutions.
The results, to some extent, reveal the underlying perception among students that there is only one answer for all mathematics problems. According to Schoenfeld (1992), it is a very common misunderstanding among students to believe that all mathematics problems have one and only one solution. The students’ performance in this study may reflect that they lack the experience in solving such open-ended problems or even the exposure to such problems and henceforth held on to such a belief. This is further supported by an analysis on problem types in a popular Singapore mathematics textbook series for the lower secondary, which reveals that 99.1% of the problems in the textbooks are close-ended (i.e., problems with only one correct answer) (Zhu, 2003). More exposure of students to open-ended problems is very much desired.

**Presentation of The Solutions**

Communication is one of the important mathematics skills specified in the framework of Singapore mathematics curriculum (MOE, 2000). Without clear, logical, and organized presentation, one cannot effectively communicate ideas with others. Therefore, the pre-designed rubric includes presentation of the solutions as one important aspect of students’ performance in problem solving. Five levels are prescribed in the rubric, from level 0 where “no attempt is evident or provide irrelevant work” to level 4 where “presentation of work is clear and greatly adds to the markers’ understanding of the working procedure”.

The data reveal that most students lack organizational and communication skills. In each sub-question, more than half of the solutions is marked at the first three levels in this aspect (Qn1: 63.4%, Qn2: 75.2%, Qn3: 68.3%, Qn4: 51.7%). That is, the solutions are either irrelevant, hard to be read, or no working procedure is visible. Very few students about four or five document their solutions in a highly logical and clear way for Qn1 to Qn3. The corresponding number of students who are able to document their answer for Qn4 with clarity is relatively higher (32), as students are required to examine all plans which satisfy all the criteria in the previous three sub-questions and select the most appropriate plan with justification. However, the researchers deem that among 145 participants is a relatively small number.

The data collected reveal that many solutions only include computational procedures and readers who are not given the original problem would not possibly be able to comprehend the solution. Even the two researchers, who crafted the questions, encountered much difficulty in understanding the students’ solutions. Having a well organized documented solution with logical and clear presentation adds not only to the markers’ understanding, but also helps the problem solvers to metacognize when they reflect on their own working. Students with relatively low ability in presenting solutions need to receive more attention, such as more guidance from their mathematics teachers in the daily classroom teaching.

**Connections Within or Beyond Solutions**

Some mathematical connections can be drawn out from the solutions of Qn3 through careful examination. The mathematical connections deduced by the two researchers for Q3 are: (1) the numbers related to the plans are all odd numbers for one condition and even numbers for the other condition (level 1 or 2, depending on the number of conditions involved), and (2) the numbers of boats to be rented can be expressed by a mathematical rule (level 3 or 4, depending on the number of conditions involved).

Although to a problem solver, obtaining a solution is important, making connections to mathematical knowledge within or beyond the problem is also very meaningful. Referring to Pólya’s problem solving model, one can easily find that it does not merely stop at “carrying out the plan” but includes the fourth stage on “looking back”. In the last stage, problem solvers need to reflect on the original problem, the working procedure, as well as the solution. Many discoveries and theories have been made through reflections. Unfortunately, the data show that none of the participants in this study made
any attempt to look for mathematical ideas beyond the solutions. The findings may reflect that the students do not have the habit of looking beyond the solutions and one of the reasons could be the demands of the examination-oriented education system, where students are accorded full marks as long as they provide the solutions that satisfy the requirements of the question. However, the inherent value of problem solving can be lost if students are only concerned with solutions because new knowledge can be gained via problem solving. Besides the essence of problem solving does not just lie in the solution but the experience garnered in working on the problem. Hence classroom teachers may consider including this aspect in their daily teaching so as to help the students develop this positive attitude and habit in problem solving.

**Implications and Suggestions**

This study intends to investigate the performance of Singapore students on one authentic and open-ended problem, *River Cruise*. The study involves 145 secondary one students from two local schools. The data show that the majority of students are able to adopt strategies which could lead to the correct answers. However, the strategies adopted by most students are not systematic but random “trial and error”. While the majority of students are observed to enjoy solving such problems, some do not make any attempt in their answer scripts. The results reveal that the majority of the students only take into account one condition in coming up with their solution, when it involves two possible conditions. Working within the framework of that assumed condition, most students merely present one correct answer, when there are multiple correct answers. On the whole, the students’ presentation of their solutions is not well organized. In addition, the study also reveals that students do not make any attempt to explore and make connections beyond the required solution.

The findings suggest that Singapore students are able to solve such challenging mathematics problems but lack the experience and exposure in solving open-ended problems. School teachers need to be aware of the limitations students faced and provide more opportunities for students to work on such problems. It is also detailed in the mathematics syllabus that problem solving should include “a wide range of situations from routine mathematical problems to problems in unfamiliar contexts and open-ended investigations …” (MOE, 2000, p. 10). The researchers believe that exposure to such problems will not only help students develop their problem solving abilities, but also foster higher-order thinking. Secondly, efforts should be made to help students’ develop their communication skill, which is specified in the skills component of the pentagonal framework of the Singapore mathematics curriculum. Thirdly, school teachers should change their mindset of merely requesting for the correct answer(s) from students but stretch their minds to explore new knowledge, look for generalized patterns or rule, discover theories, and new experience through solving open-ended problems. This, in the view of the researchers, is an essential aspect of problem solving and should be emphasized in the teaching and learning of mathematics. In conclusion, we believe that with substantial exposure and carefully planned instruction, students would improve their ability in problem solving, as well as develop positive attitudes toward mathematics.

**Reference**


Appendix

The Open-Ended Problem Used in the Study

Mrs Lim intends to take her class of Sec 1 students for a river cruise on the Singapore River. There are two sizes of boats: the big and small ones. The big boat can carry 6 people and the fare is $10 per boat. The small boat is able to transport 4 people and charges a fare of $8 per boat. Given that Mrs Lim's class consists of 50 students. Assuming that you are Mrs Lim and you want to organize the river cruise for your students, can you come out with all the possibilities of renting the boats?

(1) At a minimum cost;
(2) With minimum no. of boats;
(3) With minimum few empty seats;
(4) Choose a most reasonable renting plan and explain clearly about your reasons for your choice.