INTUITIVE THINKING AND PROBABILITY

Review by Chan Wai Soen

INTRODUCTION

Thinking about uncertain or chance events is nothing new to many people. Various card games, playing the lottery or even deciding on whether it will rain during a picnic has an element of uncertainty. Intuitive thinking or making predictions based on insufficient knowledge plays a large part in problem solving. The technique of trial and error or 'guess and check' relies heavily upon this. When we make a guess, we are performing an activity that involves chance or probability.

Many of us have intuitive ideas about chance events. Given two bags each with an equal number of durians, one would intuitively guess that the chances of randomly picking out a good durian must be higher for the bag with more good durians. For this situation, one's intuition is correct. But take another case. I asked students this question: *"A fair coin is tossed five times and a Head appears each time. What would be the result of the sixth toss?"* This seemingly simple question elicited three common responses. Some students said that it is equally likely to be a Head or a Tail as the coin is a fair one. A second opinion was that it should be a Tail as the coin is a fair one and five Heads have already appeared. The third response was that it should be a Head as five Heads have already appeared. This is a situation where intuitive thinking has resulted in some misconceptions.

Intuitive thinking when applied to probability creates a great deal of problems. What follows is a brief look at the development of intuitive thinking about probability, its effects and implications for teaching.

REVIEW OF RESEARCH

Piaget and Inhelder (1975) were the first researchers to study the development of the idea of chance in children. According to them, the concept of probability as a formal set of ideas develops only during the formal operational stage, which occurs about twelve years of age. By that age, children can reason probabilistically about a variety of randomising devices. In an experiment to demonstrate that children have an intuitive understanding of the law of large numbers and that intuitive thinking about chance events starts even before they are taught, they used a game with pointers which were stuck onto cards divided into various sectors and then spun. They found the children could predict that, in the long run, the pointer would fall onto every region marked on the card.

Other researchers have looked at the effect of instruction. Fischbein (1987) explored the foundation of intuitive thinking as precursors to learning probability in mathematics.. He asserted that there were primary intuitions which were related to personal experiences that appeared prior to instruction and secondary intuitions which appeared through instructional influence. He found that intuitive ideas, whether primary or secondary, often resulted in fallacies. Information processing theorists, like Scholz (1991) and Borovcnik and Bentz (1991) claimed that fallacies in thinking about chance could be explained by a lack of memory space or a lack of specific cognitive structures. Cognitive psychologists, Kahneman, Slovic and Tversky (1982) extended the study of fallacies in thinking about chance by looking at students' misconceptions in probability. They found that because of the lack of memory space or cognitive structures, students tended to use two kinds of judgmental heuristics which enabled them to summarise large amounts of data and rapidly come to a conclusion. Such heuristics are called ***the representative heuristic*** and ***the availability heuristic*** which often give rise to misconceptions in learning probability.

The representative heuristic

The representative heuristic is the tendency to overestimate the likelihood of events based on how well an outcome represents some aspect of its parent population (Kahneman, Slovic and Tversky 1982). For example:

When considering sequences of boys(B) and girls(G) being born in a family, the sequence BBGGBG is usually rated as more probable than the sequence BGBBBG. The reason being that a family should have an equal number of boys and girls.

If there are 7 heads out of 10 tosses of a coin, then it is equivalent to obtaining 70 heads from 100 tosses. Such misconception arises because students tend to believe that the sample should reflect the distribution of the parent population.

The availability heuristic

When people estimate the likelihood of events based on how easy it is for them to recall particular instances of the event, Kahneman *et al* (1982) consider them to be employing the availability heuristic. This heuristic can influence significant bias based on one's own narrow experiences and personal perspective. For example, if we were to experience bad service once at a restaurant, we may decide not to visit that restaurant a second time.

In their study Kahneman et al (1982) found statistically naive students rated the percent of people that had had a heart attack and were over 55, higher than the percent of people that just had a heart attack. One possible explanation was that the subjects had acquaintances who had had heart attacks and were also over 55.

THE OUTCOME APPROACH

The above heuristics explain how people arrive at a probability judgement which is incorrect. However another researcher, Konold (1989) was interested *in the manner* that people incorrectly interpreted a question about probability or a probability value. He called the outcome of such erroneous interpretations, the **outcome approach** which is based on a model of informal reasoning under conditions of uncertainty. For students who use such an approach, the tendency is to predict the outcome of an single trial not the probability of its occurrence . Such predictions usually take the form of yes or no decisions on whether an outcome will occur on a particular trial. These predictions are then evaluated as having been either right or wrong. This is a deterministic view of an uncertain situation. In the outcome approach, subjects also tend to rely more on causal explanations rather than explanations due to chance and variability.

For example, when asked to interpret a weather forecaster's prediction of a 70% chance of rain on a particular day, in order to plan for a picnic, many college students took the statement to mean that it would rain. Students tended to use anchor values of 0%, 50% and 100% as corresponding to "no", "don't know" and "yes" respectively in a probabilistic situation. Hence 70% was taken to mean that it would rain. In fact, a 70% chance of rain would simply mean that in 100 days, for example, 70 of them would have rain. There would be no indication whether there would be rain on that day. Thus, whether one should cancel the picnic is essentially a personal judgement.

CONCLUSION

It is perhaps important to remember that students' misconceptions are deeply rooted. Instruction may not change the student's views. Garfield and Ahlgren (1988) maintain that even with all the results of research, teaching a conceptual grasp of probability still appears a very difficult task. Students have preconceived ideas of chance, randomness and probability. These ideas will conflict with the mathematics that teachers are trying to impart to them. There was an instance when I asked a student to toss a die for 10 times. She obtained a large number of sixes. I asked her to compare her data with the data I generated for 1000 tosses. The two, of course, were quite unlike each other. I remembered her remark well. She said: *"...the dice tossed by me, not by you...!"*

IMPLICATIONS FOR TEACHING

Researchers have suggested a number of implications for instruction to overcome students' difficulties in learning a subject like probability that tends to be influenced by erroneous intuitive thinking.

1. ***Garfield and Ahlgren (1988) contend that before the teaching of probability, students must have an understanding of ratio and proportion.***
   Students must be able to function at the formal operational level. They must have the necessary skills in dealing with abstractions.

2. ***Teachers need to recognise and confront common errors in students' probabilistic reasoning.***
   To recognise them, researchers like Fischbein (1987) and Konold (1991) advocate the use of in-depth interviews. It is important to make students aware of how beliefs and conceptions can affect probabilistic judgements. Through interviews with a few of my students I found that predicting the occurrence of an event could mean asking whether an event was sure to happen, a result similar to the use of the outcome approach. All these interpretations I might not have found out had I used a pencil and paper test.

3. ***Students must acquire adequate comprehension skills*** (Borovcnik and Bentz 1991, Scholz, 1991).
   Terms like *random*, *certain, possible* and *at least* can cause confusion. I have found that for some students a 100% chance of rain is not a certain event. It only means that it is very likely to rain. Their reason was that it was never possible to be certain about the weather.

4. ***Concrete classroom activities are useful in giving students an idea of chance.***
   Gathering physical data gives students a feel for probability. For example, collecting data on the tossing of three coins and throwing a coin three times may help students to abstract the identical mathematical structure from different experimental situations. Konold (1991) has asserted that empirical data could also be used to challenge students' beliefs. In my experience, students trust their data more than data supplied by the teacher. I gave data for 1000 tosses of a die and asked students to predict the result of ten throws. Initially they used the data but after eight throws,

they tended to rely on their results to predict the outcome of the next two throws. Thus, any data used for teaching is best when the data is obtained by the students themselves.

5. ***Teachers also need to create situations requiring probabilistic reasoning that correspond to the students' views of the world.***
Scholz (1991) notes the distinction between a textbook and a social judgement paradigm. A textbook or a problem solving type of question would allow for a mathematical model of the situation, where the information is precise and accepted to be objectively assessed. In such a case, a unique solution is usually possible. However in a social judgement paradigm, the content is realistic and the information may not be precise and even redundant. There is no exact answer, but an estimate based on expertise and experience. Everyday situations, for instance, the chances of waiting for more than ten minutes for a bus, may be used to give a sense of realism to the topic. Instances of the use and misuse of probability would also be useful for students to see the applicability of theory to actual situations.

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