Mental Arithmetic in the Teaching of Primary Mathematics

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INTRODUCTION

Mental arithmetic is one of the basic skills recommended by the National Council of Teachers of Mathematics (NCTM 1980). It is one of the most useful skills which could be used by children and adults in their daily lives. With the introduction of technology, however, some educators are of the view that calculators could replace mental arithmetic because it is laborious to learn the skills associated with mental computation and not all children can master these skills, even with the teaching of appropriate strategies. Opponents of this stand, however, are of the view that recall computation is not the only objective in mental computation.

There are many other good reasons for children to learn mental arithmetic. One reason is that mental arithmetic may help stimulate children's thinking because it involves processing, recalling, and relating information. Another reason is that it is not always feasible to have a calculator with us at all times, no matter how small it is. Thus, if they have acquired this mental skill, children will be able to apply it whenever it is required. The other apparent reason for learning mental arithmetic is that it can help pupils speed up their work when a calculator is not permitted during examinations. Readers should be aware that at the Primary Six Leaving Examination (PSLE) in mathematics and the first paper of the 'O' level mathematics papers, candidates are not allowed to use calculators. Under such circumstances, mental arithmetic will be of great help to them. This paper discusses some of the essential mental skills and strategies which children need to learn in order to master mental arithmetic.

CHANGING MONEY

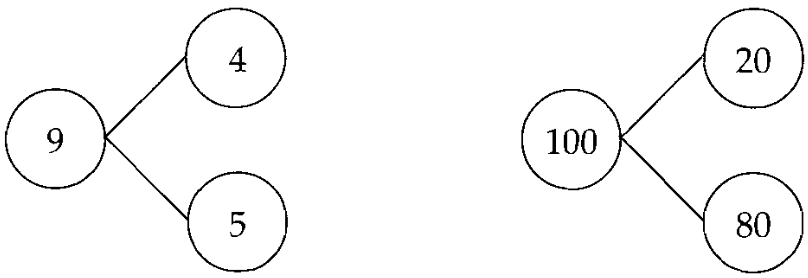
One of the most useful skills in mental arithmetic is the changing of money in daily life. We often travel by taxi and pay taxi fares in the form of loose change. Occasionally, when we do not have loose change, we have to pay taxi drivers using big notes such as $10 and $50. Usually, we do not carry calculators along with us to check if we have received the right change. This is the time mental computation can be useful. For example, if the taxi charges for a certain trip is $5.20 and you pay the taxi driver $10, how much change will you get back? The following algorithm is usually used by us when we work out the problem on paper:

$$\begin{array}{r} 10.00 \\ -\ 5.20 \\ \hline 4.80 \\ \hline \end{array}$$

You first rename the $10 to $9 and 100 cents. The figure 80 cents is obtained by subtracting 20 cents from 100 cents. $5 is then subtracted from $9 to give $4. The answer is $4.80. This is the paper-and-pencil method which requires you to write down the solution step-by-step. If you attempt to store this series of steps in your memory, you may have difficulties recalling and using the earlier figures you have obtained. This is due to the fact that the steps involved may interfere with recalling the earlier facts which have been stored.

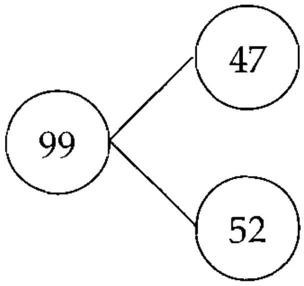
Mental computation usually applies a simpler strategy to solve the problem and the algorithm involves only a few simple steps. In the example above, to apply mental computation, you need only to examine the figure 5.20. For the $5, you subtract it with $9 and you get $4. Register the figure 4. Next, for the 20 cents, you subtract it with 100 cents and you get 80 cents. Recalling the earlier number 4, you get $4.80. Notice that the algorithm is much shorter and the only requirement is to apply number bonds for 9 and 100.

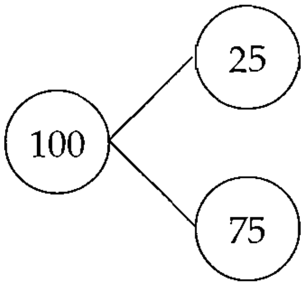
The current primary mathematics curriculum requires children to master number bonds at the primary level. For example, the following number bonds are used to compute the mental sum described above:



From this example, teachers should see the implication of mastering number bonds with respect to its application in mental computation in the changing of money.

You can also apply the same strategy to solve a more difficult mental problem. For example, you may purchase a bigger item which costs $47.25. As before, you may not have the loose change and pay $100 to a cashier. How much change will you get back? Applying the same strategy using number bonds, do you think you can get the answer mentally without using a calculator? First, you need to subtract $47 from $99, that is, $52. Next, you subtract 25 cents from 100 cents and you get 75 cents. Recalling the earlier figure $52, your answer is $52.75. In this problem, you need to master the number bonds for 99 and 100 as shown in the solution above.





LEFT-TO-RIGHT STRATEGY

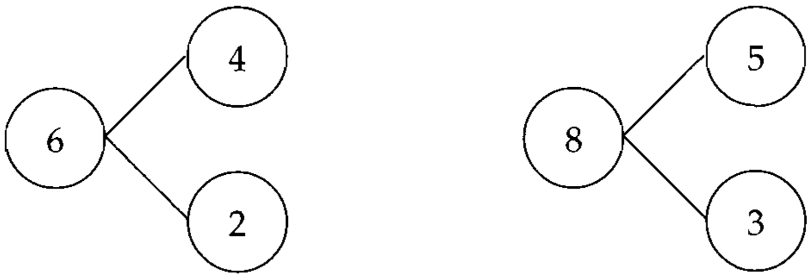
The left-to-right strategy enables children to compute addition operations especially when the number of digits is small. In the traditional textbook, children are usually taught to apply an algorithm which starts to work from left to right. This textbook algorithm is illustrated below.

Step 1	Step 2
$\begin{array}{r} 45 \\ +\ 23 \\ \hline 8 \\ \hline \end{array}$	$\begin{array}{r} 45 \\ +\ 23 \\ \hline 68 \\ \hline \end{array}$

First, you add the ones digits (i.e. 5 and 3) and you get 8. This figure is written down below the ones column (Step 1). Next, you add the tens digits (i.e. 4 and 2) and you get 6. Similarly, this result is written down below the tens column (Step 2). This strategy is not suitable for mental computation when you have more columns and the steps requiring regrouping. Under such circumstances, children may have difficulty recalling the numbers which have been registered earlier.

However, the left-to-right strategy requires you to add or subtract numbers from left to right. For example, in the earlier addition sum, you first add the tens-column numbers to get 6. This is then followed by adding the ones-column numbers which give you 8. Then you read 68 which is the result of the addition sum.

From the above example, a teacher should be able to figure out the skills or concepts required by children to handle this mental problem. The following are two essential number bonds:



A mental computation addition problem which requires regrouping of numbers in the ones column is shown below.

Step 1

$$\begin{array}{r} 48 \\ +\ 39 \\ \hline 8 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} 48 \\ +\ 39 \\ \hline 87 \\ \hline \end{array}$$

Before you begin to add the numbers, you first make a survey of the ones-column numbers to check whether regrouping of numbers is required. In this example, 8 and 9 in the ones column make 17. Thus, regrouping is necessary. When making a survey to check the necessity to do regrouping, you do not need to remember the figure 17. In this case, what you need to remember is to add a ten to the tens-column number later on. Next, at the tens column, you add 1 to 4 and 3 to make 8 (i.e. 4 + 3 +1). Note that you have added 1 to the tens column because from the survey, you have registered a "1" in the tens column. Then you add the digits from the tens column and you get 7 (i.e. 8+9=17). Note that you have ignored the digit 1 as it has already been added to the tens column from the previous step.

The following is another example which requires you to add two three-digit numbers:

Step 1	Step 2	Step 3
$\begin{array}{r} 267 \\ +\ 385 \\ \hline 6 \\ \hline \end{array}$	$\begin{array}{r} 267 \\ +\ 385 \\ \hline 65 \\ \hline \end{array}$	$\begin{array}{r} 267 \\ +\ 385 \\ \hline 652 \\ \hline \end{array}$

First, you make a survey to check if regroupings are required in the tens column and the ones column. Here, notice that in both the tens column and the ones column, regroupings are needed. Thus, you need to remember to add one more to the hundreds-column and tens-column numbers. In Step 1 above, 2 + 3 + 1 = 6 and you register the digit 6. Next, in Step 2 above, 6 + 8 + 1 = 15 and you register 5 only. Mentally, you should rehearse the number 65 before you move on to the next step. Then, in Step 3 above, 7 + 5 = 12, and you need only to register the digit 2. Combining with 65, you get 652 which is the answer to the sum of the two numbers.

If you have a memory good enough to register and recall numbers, you may be able to add two 4-digit numbers which require regrouping. You may try to add the following numbers mentally:

$$\begin{array}{r} 4\,567 \\ +\ 7\,654 \\ \hline \end{array}$$

ADDITION AND SUBTRACTION OF NUMBERS NEAR 100

The current emphasis in primary mathematics includes the use of number bonds to recall number facts and compute the addition and subtraction of numbers. When children have mastered number bonds, they are able to compute addition and subtraction sums mentally, especially when one of the addends is near 100.

(a) *253 + 98 =*

In this addition, one of the addends, that is, 98, is a number which is near to 100. You may add the two addends by first treating 98 as 100. The number 100 added to 253 gives 353. You should read 353 in digital form that is, 3...5....3 instead of "three hundred and fifty three". Notice that when you register the number in digital form, it takes up less memory space as compared to registering the number in verbal form. Reading in digital form helps to speed up computation. Next, you need to subtract 2 from 353 and you get 351, which is the answer. The reason you need to subtract 2 is that you have added two more when the number 100 (instead of 98) is added to 253. You will realize that using this mental method is more efficient than using the textbook algorithm in the addition of numbers, of which one of them is near 100.

(b) *253 + 104 =*

Similar to the method described above, you treat the second addend 104 as 100. The number 100 added to 253 gives 353. Since you are supposed to add 104 instead of 100, you need to add 4 more to 353 and the answer is 357 (i.e. 353 + 4 = 357).

(c) *253 – 98 =*

In the case of subtraction, you apply the same strategy by treating 98 as 100. By subtracting 100 from 253, you get 153. Next, you add 2 to 153 to get 155. Using a similar argument, you need to add 2 to 153 because you have subtracted 2 more from 253 when the number 100 was subtracted from 253. The answer is 155. If you attempt to use the textbook algorithm to subtract 98 from 253, you have to apply a more complicated renaming concept to get the answer. Usually, it is more difficult to apply the renaming concept to carry out subtraction mentally.

(d) *253 – 104 =*

In the case where the subtrahend (i.e. 104) is more than 100, you first subtract 100 from 253 and you get 153. Next, you subtract 4 again from 153 and you obtain 149. You should be able to figure out the reason for applying the above procedure for this particular case.

The above examples show the four different cases of addition and subtraction of numbers where the subtrahend is near 100. The strategies described are designed for these specific cases. In addition, there are some general steps which learners should have at their disposal when the mental computation strategies are applied. First, they have to learn and to remember the strategies for mentally solving this type of problems. Second, they have to master and apply number bonds to speed up computation. For example, they have to know 8 + 9 = 17 and 14 - 9 = 5 immediately instead of using counting to get the answer.

MULTIPLYING NUMBERS MENTALLY

MULTIPLYING A 2-DIGIT NUMBER BY A SINGLE DIGIT NUMBER

In order to multiply numbers mentally, children should first learn to master the 100 multiplication facts. In other words, they are required to recall the number facts immediately. For example, children should be able to recall $7 \times 8 = 56$. If children do not have the skill to recall the 100 facts, they are not likely to be able to work on complicated multiplication facts such as those numbers involving tens and hundreds values.

The strategy for computing more complicated multiplication operations is to apply the distributive law in addition. The following distribution operation shows the steps involved in the multiplication of two numbers [i.e. a and (a+b)]:

$$\begin{aligned} a(b+c) &= a \times b + a \times c \\ &= ab + ac \end{aligned}$$

The main feature of this law is to express the second multiplicand into a sum of two numbers. For example, if you want to multiply 36 by 8, you first express 36 = 30 + 6 mentally. Next, you multiply 30 by 8 which gives you 240. You only need to register 24 in this case because the last digit always ends in 0 but you must be aware that 4 stands for 40 and 2 stands for 200. Then you multiply 6 by 8 to give 48. Adding 48 to 240, you get 288.

MULTIPLYING A 2-DIGIT NUMBER BY A 2-DIGIT NUMBER

We apply a different strategy when we multiply a 2-digit number by another 2-digit number. The strategy requires you to work from left to right and register numbers which you have operated. In this strategy, there are 3 steps to follow:

$$\begin{array}{r} 26 \\ \times\ 37 \\ \hline \end{array}$$

Step 1: Multiply the tens digits of both numbers. $2 \times 3 = 6$. This number is registered as 600 (in fact you are multiplying 20 by 30)

Step 2: Multiply 6 by 30 and you get 180. Multiply 7 by 20 and you get 140. Add them up and you get 320. At this stage, you must add 600 to 320 and you get 920. You must register this new number and forget the rest.

Step 3: Multiply 6 by 7 and you get 42. Add 42 to 920 and you get 962.

Basically, the concept used above is based on the distributive law in addition. The steps used are summarized as follows:

$$(a + b)(c + d) = ac + (ad + bc) + bd$$

MULTIPLIERS INVOLVING 25 AND 50

When the multipliers are 25 or 50, the strategies used may be much simpler. The underpinning principle is to convert a difficult multiplication fact into a simple division fact. For example, the product 76 x 50 can be expressed as 76 x 100/2 = 7600/2. Dividing 7600 by 2, you get 3800.

Similarly, if 76 is multiplied by 25, you can express the multiplication statement as follows:

$$\begin{aligned} 76 \times 25 &= 76 \times 100/4 \\ &= 7600/4 \\ &= 1900 \end{aligned}$$

76 × 25 is converted to 7600/4 and dividing 7600 by 4 you get 1900. Notice that in this particular case, children need to master the division facts by 4 in order to apply this method.

We can also apply this strategy when the multiplier is 125. We know that 125 x 8 = 1000, so this fact can be applied to solve the following multiplication problem mentally.

$$\begin{aligned} 76 \times 125 &= 76 \times 1000/8 \\ &= 76000/8 \\ &= 9500 \end{aligned}$$

However, children need to master the 8 facts in division if they wish to apply this strategy.

Repeated Numbers Ending in Fives

An efficient method in multiplication can be applied when you multiply a number by a repeated number ending in five. For example, when we multiply 25 by itself, the answer is 625. The following steps show the method to obtain the result:

$25 \times 25 = \_\_\_$

Step 1: The answer always ends with the number 25.

$25 \times 25 = \_\_\_\ 25$

Step 2: Add 1 to one of the tens digit. You get 3.

$2 + 1 = 3$

Step 3: Multiply 3 by the tens digit of the other multiplicand.

$3 \times 2 = 6$

Therefore, $25 \times 25 = 625$

Using this strategy, you should be able to work out a more difficult multiplication problem such as 115 x 115.

Step 1: The ending digits is 25.
Step 2: Add 1 to 11, i.e. 11 + 1 = 12
Step 3: Multiply 12 by 11. 12 x 11 = 132.
The answer is 13225.

Conclusion

The previous paragraphs show some strategies for computing numbers mentally. Each strategy possesses unique features in computation. For example, the multiplication problem requires children to apply the distributive law in addition while the addition computation requires children to use the concept of regrouping. In all cases, they are expected to master number bonds in addition and multiplication. Children are expected to have already mastered the number bonds before some of these strategies are introduced to them. For example, in the case of changing money, children need to know the number bonds for nines, tens, ninety nines and hundreds.

Mental computation involves processing, recalling, and relating information. The underlying principle to apply in mental computation is to minimize the information to be stored and processed in the working memory. Thus, in some cases, children are expected to read numbers in digital form instead of verbal form. For example, 235 should be read as 2....3.....5 instead of "two hundred and thirty five". Children should apply this method for storing information so that additional memory space may be used to process other information.

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