TYPES OF MATHEMATICAL PROBLEM POSING TASKS

Review by Yeap Ban Har

INTRODUCTION

When students engage in mathematical problem posing, they generate new problems, or reformulate existing ones (Silver, 1994). In mathematics education, problem posing has long been recognized to be an important component of mathematical ability (e.g. Polya, 1954; Brown & Walter, 1970). Polya (1954) described it as a self-directed activity fundamental to doing and making mathematics.

More recently, Silver (1994) argued that students should be provided with opportunities to engage in problem posing because it is:
- a means of improving students' problem solving;
- a way to improve students' dispositions toward mathematics;
- a window into students' mathematical understanding;
- a prominent feature of mathematical activity;
- a feature of inquiry-oriented instruction; and
- a feature of creative activity or exceptional mathematical ability.

Recent reforms in mathematics curricula around the world have advocated that problem posing activities be a more prominent feature in the mathematics classroom. This is evident in the revised mathematics curriculum for Singapore schools. The curriculum calls for opportunities for pupils *"to extend and generate problems"* (Ministry of Education, 2000, p. 17). One of the reasons offered is that problem posing helps students become sensitive to mathematical relationships in problem situations. Mayer and Hergarty (1996) found that problem solvers who pay attention to mathematical relationships in problem situations tend to be more successful than those who do not.

This review aims to describe some problem posing tasks that mathematics teachers can use as part of instruction, as homework tasks and during assessment.

REVIEW OF RESEARCH

There is a wide range of problem posing tasks, each not necessarily the same as the others. Silver (1994) emphasizes that problem posing does not necessarily have to precede problem solving. Some problem solvers generate sub-problems *during* the solution of a larger problem. It is also not uncommon practice among mathematicians to think of related problems *after* solving a problem.

Stoyanova (1999) classifies problem-posing tasks as free, semi-structured or structured. A problem-posing task is described as free when students are simply asked to pose a problem from a situation, contrived or realistic. A semi-structured task requires students to formulate a problem that would draw on their previous mathematical knowledge, skills and concepts. Structured problem posing tasks are those that require students to pose a problem based on other problems. Stoyanova (1999) provides a comprehensive list of examples in each category.

Yeap (2000) classifies problem posing tasks according to the nature of numerical information in the task. Numerical information in a problem-posing task may be in concrete form, pictorial form or symbolic form. A problem-posing task may require students to pose problems during a visit to the zoo or to pose problems based on some fruit the teacher brings to class. The former situation is open while the latter is contrived. In such tasks, the numerical information (such as the number of lions) is in concrete form. Alternatively, students may be asked to pose problems based on a photograph of some animals or a pie-chart describing the number of animals belonging to the cat family that the zoo has. Both contain numerical information in pictorial form but with differing degree of abstractness. In contrast, most of the tasks used in research studies to date only contain numbers in symbolic (numeral or word) form. Yeap (2000) illustrates each category with examples for the primary grades.

Some problem posing tasks are entirely open. Students are asked to pose mathematics problems but are not provided with any numerical information. Ellerton (1986), in her landmark study involving 10,500 secondary school students, created this task:

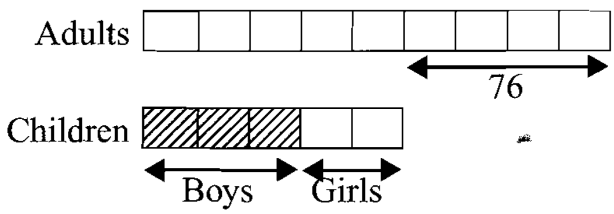
> *"Make up a short problem that you think would be quite difficult for a friend of yours to solve. Show how you would work out the answer."*

Mathematicians Lowrie and Whitland (2000) asked third grade children to design mathematics problems for children in second grade and fourth grade. Van den Heuvel-Panhuizen and associates (1995) similarly required fifth grade children to pose easy and difficult problems on percentage.

Other problem posing tasks are less open. Students are given numerical information but without a context. In an investigation involving fifth grade students, English (1997) used tasks that required students to pose problems for given symbolic expressions such as *27+38, 96-24, 11x3* and *24÷3*.

Other problem posing tasks require students to generate a problem given a solution or answer. Yeap and Kaur (1997) suggested an activity that required students to write a question that has an answer of *x=3*. Ho, Lee and Yeap (2000) asked Primary 5 pupils to write a ratio problem during an examination. The test item is shown below.

"Write a ratio problem sum for this model."



In other problem posing tasks, a situation containing some numerical information is given. In his classic book, Krutetskii (1976) used problems with unstated questions to determine mathematical abilities of school children. He gave tasks such as: *"Twenty-five pipes of lengths 5 m and 8 m were laid over a distance of 155 m."* He was interested in whether the respondent could perceive the relationships implicit in the situation and hence formulate sensible questions such as

"How many pipes of each kind were laid?"

Silver and Cai (1996) asked middle school students to write questions that could be answered based on information in a given passage. Students were asked to write three different questions that could be answered using this information:

> *"Jerome, Elliot, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles."*

Yet other problem posing tasks require students to pose problems similar to the ones they have solved. English (1997) asked fifth grade children to pose problems that had a similar structure to a combinatorial problem and a patterning problem. A patterning problem was given:

> *"Sam delivered pamphlets to earn some pocket money. On the first day, he delivered 150 pamphlets. On the second day, he delivered 165 pamphlets, and on the third day, he delivered 180 pamphlets. If he continues to deliver pamphlets in this pattern, on which day will he deliver 210 pamphlets?"*

Children were then asked to pose a similar problem.

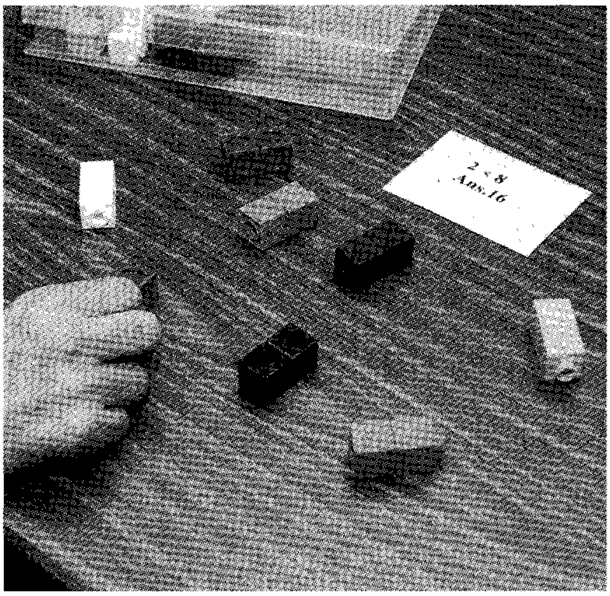
Brown and Walter (1970, 1983) had earlier written about the **'what-if-not'** strategy. Their extensive work with pre-university and undergraduate students emphasized the generation of new problems from previously solved ones by varying the conditions or goals of the original problems. For example, when students have solved a trigonometric equation $2\sin x - \cos x = 0$, they can generate more equations by varying the conditions of the original one. Students may pose equations:

$$2\sin x - \cos x = 1$$
$$2\sin^2 x - \cos x = 0$$
$$2\sin 2x - \cos x = 0$$

by systematically varying one condition each time.

CONCLUSION

Three ways of classifying problem posing tasks have been described in this review. Firstly, problem posing activities can be classified according to whether they occur before, during or after problem solving. This classification allows teachers to recognize the role of problem posing in problem solving. Secondly, they can be classified according to the nature of numerical information they present. This classification allows teachers to select tasks that embed mathematical concepts in concrete, pictorial or symbolic forms. Thirdly, problem posing tasks can be classified according to how open the tasks are.



William Oh

IMPLICATIONS FOR TEACHERS

Mathematics teachers can use problem posing tasks to achieve a variety of instructional objectives:

1. To improve students' comprehension of word problems.

The use of problems with unstated questions enables students to focus on the content of the passage. They can demonstrate their comprehension level by posing suitable questions that can be answered using information in the passage. In response for example, to this passage: *"There are 3 more girls in Primary 4A than in Primary 4B. There are 15 boys in each class,"* one child wrote *"How many boys are there in the two classes?"* while a second child wrote *"How many girls are there altogether?"*. It is clear that the second child was unable to comprehend the passage to infer that the question he posed cannot be answered.

2. To develop students' abilities to perceive the structure of a mathematical problem.

The construction and deconstruction of word problems helps students see the structure of a problem beyond its superficial features. The use of ill-structured problems posed by students facilitates rich discussions that can help students construct rich problem structure schemas.

3. To improve students' attitudes towards word problem solving.

The writing of one's own problems gives students a sense of achievement. Teachers may compile problems posed by students into homework assignments.

4. To develop flexible thinking.

The use of the 'what-if-not' strategy encourages students to consider possibilities and the potential each offers.

5. To identify misconceptions in mathematical concepts.

The problems posed by children serve as a window on their thinking. A child who was asked to pose a problem that requires the calculation of 2 x 8 wrote *"Ali has 2 stamps. Siti has 8 more. How many stamps do they have altogether?"* It is clear that this child has not developed the meaning of multiplication.

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