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Some Generic Principles for Solving Mathematical Problems in the Classroom

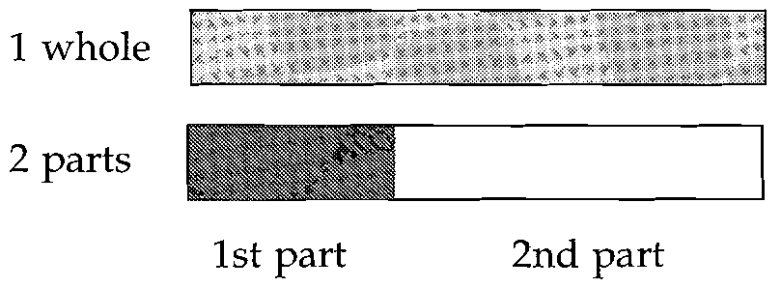
Fong Ho Kheong

Using heuristic processes in problem solving has been an emphasis in mathematics education (Hatfield *et al.* 1997; Reys *et al.* 1995). It is also the focus of mathematics pedagogy in Singapore, where acquiring and mastering appropriate strategies for problem solving are considered key factors to success in solving non-routine problems (Ministry of Education 1989). In the process of helping learners to acquire appropriate strategies, however, mathematics educators may have neglected or overlooked a crucial factor, which is helping learners to understand, and then master 'generic' principles which are implicit in many commonly encountered mathematical problems.

The objective of this article is to show that acquiring generic principles, where these exist, will enhance learners' ability to solve mathematical problems with facility, thereby enabling them to be more effective problem solvers. The following sections discuss two principles to illustrate the point made above.

Part-whole principle

The part-whole principle is fundamental to problem solving. This principle is frequently used by problem solvers, but some do not fully appreciate the fact that it is essential to solving many mathematical problems. In fact, learners may have difficulties because they are not fully aware of this principle and how to apply it to solve the problems presented to them. This principle is often represented as a part-whole model.



The part-whole principle has been built into simple 1- or 2-step word problems. For example:

Ann spent $3 on food.
She spent another $2 on a book.
Then she found that she had $4 left.
How much money had Ann at the beginning?

This problem can be conceptualized using a simple diagrammatic model to represent the three parts: $3 on food, $2 on the book and $4 left. The whole represents the amount of money Ann had at the beginning.

The part-whole principle is also often used in constructing fraction problems. For example:

Mrs Li baked some biscuits.
She gave 1/2 to her friends.
Of the remainder, she gave 1/4 to her children.
Then she had 24 biscuits left.
How many biscuits did she bake?

The 1 whole can be conceptualized as the total number of biscuits Mrs Li baked. The parts are: 1/2 to her friends, 1/4 of the remainder and 24 biscuits left. Besides whole numbers and fractions, the part-whole principle is also applicable to topics such as ratios and proportions, and speed (Fong 1996).

Completing-whole principle

Another generic principle that underlies many mathematical problems is the 'completing-whole' concept. The underpinning idea is to convert a specific value to a standard reference, such as 1 whole or 100, because this standard value will help to simplify computation.

In mental computation, we often apply this principle in addition and subtraction. For example, when we add two numbers, of which one is near hundred, we need to reformulate this number as 100 in the mind, plus or minus an appended number. For example:

Find the number in the blank.

245 + 98 = _____

To solve the problem mentally, add 100 to 245 to make 345. Since we have added two more (because we added 100 instead of 98), we need to subtract 2 from the result, 345. Thus, the answer is 345 − 2 = 343.

This 'completing-whole' principle is also used to compute time intervals. We round off a given time to hour/s. Then we work from there to add or subtract minutes. The result is then subtracted or added depending on the context of the problem. For example:

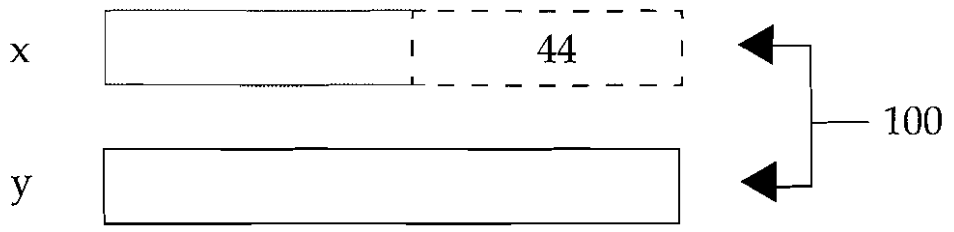
A bus left Singapore at 10.20 am and reached Kuala Lumpur at 5.00 pm on the same day.
Find the time taken to complete the journey in hour and minutes.

Using the 'completing-whole' concept, we treat the starting time as 10.00 am. The period between 10.00 am and 5.00 pm is 7 hours. But since the bus started 20 minutes earlier, you need to subtract 20 minutes from your result of 7 hours. Thus, the time taken is 7 hours – 20 minutes = 6 hours 40 minutes.

Another problem that involves the use of the 'completing-whole' principle is solving a simultaneous equation using the model approach.

The sum of two numbers is 100.
The difference between them is 44.
Find the two numbers.

One approach is to use the 'completing whole' principle instead of using simultaneous equations. The diagram below shows the two numbers.



In the above figure, the shorter bar is extended (as shown by the dotted lines) to make it the same length as the longer bar (i.e. to complete the whole).

Now, let the longer bar be 1 unit. Then from the figure, notice that

2 units	→	100 + 44 = 144
1 unit	→	72
1 unit − 44	→	72 − 44 = 28

Thus, the two numbers are 28 and 72.

Discussion

The part-whole and completing-whole principles were discussed to illustrate two generic principles, among many, for solving mathematical problems. Because they are generic, it is important for learners to master the underlying principles so as to enable them to fully understand and apply them to solving specific problems presented to them. Teachers, however, may assume that pupils have already acquired the underlying principles or they may not be aware of the importance of overtly teaching these principles. In addition, because the principles are generic and implicit, they are not typically spelled out in school mathematics syllabuses, unlike the way it is done for strategies for solving mathematical problems (e.g. the guess-and-check and the making-a-list strategies). A case may, therefore, be made for educators to seriously consider including generic principles in the mathematics syllabuses.

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