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President's Message



The Mathematics Teachers Conference (MTC) is a conference jointly organized by the Association of Mathematics Educators (AME) and the Mathematics and Mathematics Education Academic Group at the National Institute of Education (NIE). The inaugural conference was held in 2005. To date the following MTCs have been organized for mathematics teachers and mathematics educators.

Date	Theme
2nd June 2005	Assessment
1st June 2006	Enhancing Mathematical Reasoning
1st June 2007	Mathematical Literacy
29th May 2008	Mathematical Problem Solving
4th June 2009	Mathematical Applications and Modelling

The MTC has a unique one-day programme which comprises of keynote lectures and workshops. Every effort is made on the part of the organizing committee to make the conference a relevant, enriching and enjoyable one. All our past conferences have been very well attended. We look forward to welcoming you to MTC 2009.

Berinderjeet Kaur
President,
AME (2008 – 2010)

Mathematics Teachers Conference

Theme : Mathematical Applications and Modelling

2009

4th June 2009 National Institute of Education 1 Nanyang Walk Singapore

Jointly organized by:

- Association of Mathematics Educators
- Mathematics and Mathematics Education Academic Group, National Institute of Education, NTU

Keynote Lectures

Primary

- Mathematical modelling in school - Examples and experiences by Professor Gabriele Kaiser (University of Hamburg, Germany)
- Models and modelling in mathematics at the primary level by Dr Jaguthsing Dindyal (National Institute of Education, Singapore)
- Using ICT for applications of mathematics by Mr Barry Kissane (Murdoch University, Western Australia)

Secondary & Junior College

- Mathematical modelling in school - Examples and experiences by Dr Gabriele Kaiser (University of Hamburg, Germany)
- Implementing applications and modelling in secondary school: Issues for teaching and learning by Dr Gloria Stillman (University of Melbourne, Australia)
- Using ICT for applications of mathematics by Mr Barry Kissane (Murdoch University, Western Australia)

Concurrent Workshops (Participants to attend only one)

Primary

- P1 - Collaborative problem solving as modelling in the primary years of schooling by Dr Judy Anderson (University of Sydney, Australia)
- P2 - Mathematical modelling in the primary school by Ms Dawn Ng Kit Ee (National Institute of Education, Singapore)
- P3 - Infusing mathematical applications in the primary mathematics lessons by Dr Joseph Yeo Kai Kow (National Institute of Education, Singapore)
- P4 - Experiencing mathematical modelling in a PBL setting: Task features and design by Mr Eric Chan Chun Ming (National Institute of Education, Singapore)
- P5 - Supporting students' mathematical communication within modelling activities by Professor Glenda Anthony (Massey University, New Zealand)
- P6 - Mathematical modelling for primary school by Dr Yeap Ban Har (National Institute of Education, Singapore)

Secondary & Junior College

- SJ1 - Mathematical modelling in lower secondary level by Professor Gabriele Kaiser (University of Hamburg, Germany)
- SJ2 - Designing and using modelling tasks at the secondary school level by Dr Gloria Stillman (University of Melbourne, Australia)
- SJ3 - Mathematical modelling in upper secondary school level by Bjorn Schwarz (University of Hamburg, Germany)
- SJ4 - Developing students' understanding and interpretation of tables of data using a five step framework by Dr Marian Kemp (Murdoch University, Western Australia)
- SJ5 - Mathematical modelling using hand-held technology by Mr Paul Poh (Yishun JC, Singapore) & Dr Soon Yee Ping (Mathlodge)
- SJ6 - Applications of mathematical content and processes in real life by Mr Joseph Yeo Boon Wool (National Institute of Education, Singapore)
- SJ7 - The computer lab in the classroom: Using Texas Instrument (TI) Nspire as a hand-held manipulative for teaching and learning secondary school mathematics by Dr Rosalind Phang and Dr Tang Wee Kee
- SJ8 - Connecting secondary school mathematics with the real world by Ms Pushparani d/o K. Nadarajah, Master Teacher (West 3) (Ministry of Education, Singapore)
- SJ9 - Mathematical modelling: Using mathematics to make decisions by Dr Toh Tin Lam (National Institute of Education, Singapore)
- SJ10 - Enhancing mathematics teaching through hand-held graphing technology by Dr Ng Wee Leng (National Institute of Education, Singapore)
- SJ11 - GenSing: Generative activities using technology: Hands on activities used in Singapore classrooms by Dr Sarah Davis (National Institute of Education, Singapore)

Inter-Schools Fastest Fingers Graphing Calculator Competition 2008

Yvonne Chin - Ho Wang Yi
Meridian Junior College



The Fastest Fingers Graphing Calculator Competition (FFGCC) was conceptualized in 2007 by Meridian Junior College's Mathematics department to popularize the use of graphing calculator among our students. It aimed to engage students in a whole new world of Mathematics with the graphing calculator incorporated into the new Advanced Level curriculum. The competition with an element of fun injected into the learning of Mathematics proved to be a subtle yet effective platform to promote the use of the graphing calculator.

To reach out to more students, the FFGCC 2008 was extended to all JCs, CI and IP schools. A total of 10 institutions joined us in embarking on this journey of thrill and fun. With much speed and accuracy, the teams from Catholic Junior College, Jurong Junior College, Serangoon Junior College, Tampines Junior College as well as our very own Meridians sailed their ways through to the finals held on 6 August 2008.

We were honored to have Ms Bobbie Baird, Vice President of Texas Instruments as the Guest of Honor for the event. During the competition, teams pitted their skills against each other, attempting to

out-wit their opponents to clinch the top group prize of \$500. The teams used their graphing calculators to solve the posted questions as a team, hoping to send in their answers via the Texas Instrument (TI) Navigator system in the shortest possible time. Meridian students did us proud by emerging as champions. Top five individuals from the preliminary round also won themselves \$50 TI vouchers and thumb drive each.



The audience was not left out of the excitement. During the intervals of the competition, student audience from the participating institutions attempted specially crafted questions on their graphing calculators to win themselves attractive tokens sponsored by TI.

With positive feedback from all, the Fastest Fingers GC Competition aimed to continue to establish a new frontier in engaging more students in becoming experts in their use of GC in Mathematics education.

A Systematic Approach to Solve Word Problems

Song Shu Fang
Tanjong Katong Primary School

A primary goal of the Singapore School Mathematics Curriculum is to help pupils develop the ability to solve a wide variety of mathematical problems (Ministry of Education, 2006). The Principles and Standards for School Mathematics of the National Council of Teachers of Mathematics (NCTM) too places emphasis on problem solving as the 'hallmark of mathematical activity and a major means of developing mathematical knowledge' (NCTM, 2000, p.116). However, many of my primary pupils continue to experience difficulty in solving word problems, even though they have had much exposure to word problems in the class. The following extracts typify some of the problems faced by a group of Primary 5 pupils in solving mathematical word problems.

Pupil A: *My problem is I don't know how to start after reading the questions and I don't know how to draw the models.*

Pupil B: *My problem is that when I see a problem sum I don't understand it very well, but when I see it I don't read it once through again instead I guess and do it without understanding. When I do it finish, I don't read the questions again to make sure I do the right thing. I am also careless. I don't check my work.*

Pupil C: *Teacher, I need you to teach me step by step. I need more help in problem sums. I look at the problem sums it is like very hard to me.*

Pupil D: *If I face difficult questions, I will try to understand but I will get confused.*

Pupil E: *I get confused when I see a lot of numbers.*

This paper documents my process of helping pupils overcome their difficulties to solve mathematical word problems. To Polya (1971), teaching pupils to think is a key focus of problem solving. However, care must be taken so that efforts directed to teach pupils 'how to think' in solving word problems do not get transformed mechanically into merely teaching them 'what to do', resulting in another form of procedure. Most of all, I would like to emphasize the importance of cognition and metacognition in my study to help pupils successfully solve word problems.

1. Reasons for inability to solve word problems

An interview and a simple survey questionnaire were administered on 16 Primary 5 pupils in my attempt to learn more about the difficulties pupils faced in solving word problems. The following reasons were gathered from the two instruments:

A. Difficulties in representing the problem

The pupils did not know how to represent the given information in the word problem. They were confused and did not know how to begin with the numbers presented in the problem.

B. Difficulties in understanding the problem due to language barriers

The pupils did not understand the question and consequently were not able to come up with the right approach to solve the problem.

C. Do not know the 'goal' of the problem

The pupils did not understand the demands or expectations of the question. They adopted the right approach but stopped working on the solution prematurely because they were either not aware that they had already obtained the expected answer or how to proceed further to obtain a complete solution.

D. Lacking in conceptual understanding

The pupils were able to solve word problems in their workbooks but when the word problems were modified, they failed to identify the appropriate computation strategy to solve the problem.

E. Lacking in cognitive processes

When the answers were discussed in class, the pupils were more concerned about documenting the procedures rather than analyzing the underlying processes in the solution.

2. Rationale for the study

In the past, I used to spend a substantial portion of my lesson on demonstrating step by step procedures to solve word problems. I worked on the assumption that if each step was clearly explained before proceeding to the next, pupils would be imbued with the skills needed to solve similar word problems. However, when word problems differed from those discussed previously, even though the underlying concepts required were the same, the results were far from satisfactory.

Why is this so? Upon reflection, I realized that pupils were not cognitively engaged in the learning process. Explicit teaching in the classroom had not helped to imbue pupils with the skills to solve those word problems. They were merely passive followers of the procedures shown on the whiteboard. When the structure of the word problems differed, the pupils were stumped, unable to proceed.

I decided to embark on an action research to engage my pupils cognitively in the process of solving word problems. A systematic structure to serve as a guide in the thinking process was introduced to the class. Questioning techniques aligned to the proposed structure was used during problem solving in the mathematics classroom to help students internalize the structure. In addition, the proposed structure was printed on all mathematics worksheets to help them recall the steps.

Through this study, I hope to engage pupils cognitively and inculcate in them a systematic approach to solve word problems. In the process, I hope this develops their confidence to become independent and self-directed problem-solvers.

3. Methodology

The following proposed 7-step structure was used to guide the pupils in their thinking process and their reflection during the problem solving process:

Step 1 Read the question

Pupils read the word problem at least once to determine any important information

Step 2 Highlight the information

As they read, pupils were then required to highlight or underline the essential information.

Step 3 Identify the goal

Pupils must then identify the intent of the problem sum. For example: "Tom has **32** stamps. Peter has **twice** as many stamps as Tom. How many **stamps** have **they altogether?**"

Step 4 Analyze the given information

Pupils break the word problem into smaller parts and note the 'hidden' information. They must then formulate a relationship to connect the parts from the given information. For example:

$\frac{1}{9}$
"A school has 1060 pupils. When 40 boys and of the girls are absent, the number of boys present is equal to the number of girls present. How many boys are there?"

The 'hidden' information culled from the above question is:

- The total number of children is 1060. Hence, for each part, the number of boys or the number of girls should not exceed the $\frac{8}{9}$ whole which consist of 1060 children.
- $\frac{1}{9}$ of the girls is present.
- The rectangular model that represents the number of girls present in the school is the same as the model that presents the number of boys present.

Step 5 Devise a plan

The pupils are required to reflect on the following questions:

- What to do with the gathered information?
- What other information is necessary and how to obtain this information?
- How to make use all the information gathered to work towards the goal?

By answering the above questions, the pupils devise a plan to solve the word problem.

Step 6 Carry out the plan

The pupils carry out the computations to solve the word problem.

Step 7 Check the reasonableness of the answer and the achieved goal

The pupils are required to verify the accuracy and reasonableness of the solution of the word problem.

4. Integrating the structure in the mathematics classroom

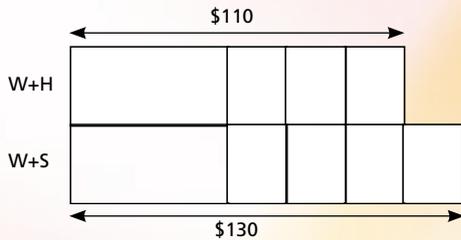
I modeled the use of the devised structure to solve word problems and adopted the strategy of 'thinking aloud' in the mathematics lesson during explicit classroom instruction. Indicated below is a short extract from a lesson when I demonstrate how to solve a mathematical word problem:

Question:

"A watch and a handbag cost \$110. The same watch and a pair of shoes cost \$130. The ratio of the cost of the handbag to that of the pair of shoes is 3:4. Find the cost of the watch."

- "I read the question once and highlight the essential information."
- "Then I repeat to find out the "the goal of the question" and at the same time note the keywords in the goal".

- "For the above question, it refers to 'the cost of the watch'. Base on the given information, I will now formulate the relationship.
 - watch and handbag - \$110
 - watch and shoes - \$130
 - handbag: shoes = 3 : 4
- "The first two parts show that the watch is the common item".
- "The relationship of the handbag and the shoes is 3:4. It means that the cost of the pair of shoes is 1 unit more than the handbag. In other words, a total of 7 units represent the cost of both the handbag and the shoes. This information is hidden or inferred from the word problem."
- "I will now represent the information in the model".



- "When I compare the models, what do I get? I will find out the value of 1 unit."
- "What does 1 unit represents and what can I find out next? I can find out what 3 units represent and the price of the handbag".
- "After obtaining the price of the handbag, what can I do to find out the price of the watch? I can subtract the price of the handbag from \$110 to get the price of the watch".
- "Will I be able to get the same answer if I find out what 4 units (price of the shoes) represent instead?"

Pupils were requested to model after the way the researcher "think aloud" and deliberated in the process of problem solving.

5. Observations

Initially, the pupils worked in pairs or threesomes to practise using the structure to solve word problems, adhering to the steps closely. However, they were weak at self-questioning and found it much easier to question their peers on their plan and approach. Their peers would then respond to the questions posed. This act of peer-questioning and answering gave rise to rich dialogues amongst the pupils and provided opportunities for clarification of thoughts and even peer teaching. The pupils found these practices less stressful and enjoyable. Some even experienced a sense of achievement.

Most importantly, the pupils developed the habit of questioning and were engaged cognitively when solving word problems.

When the pupils had internalized the structure, they worked on the word problems individually, adhering to the steps in the structure. The pupils were then called upon to share their solutions. From their presentations, I gathered rich insights into the pupil's thinking and the underlying processes.

An interview and the same survey questionnaire were administered on the same 16 pupils at the end of the year. All the pupils responded very positively to the use of the devised structure for solving word problems. By adhering to the proposed 7 steps in the structure, the pupils acknowledged in the interview that they spent more time thinking through the processes and in the midst, learnt to engage in self-questioning and self-monitoring. Through the interview, I gathered that the pupils had become more confident at solving word problems because the proposed structure had helped to engage them cognitively and metacognitively. Samples of the pupils' work are shown in Appendix A.

6. Conclusion

This study is my initial attempt to carry out an action research in my class. The process of formulating a structure to guide the pupils in their thinking has been very enriching and rewarding. I will be extending my action research to another class. Most importantly, this study offers an avenue for teachers to support and engaged weak and average pupils in the process of mathematical problem-solving.

7. References

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8. Acknowledgement:

- I would like to express my sincere appreciation to Dr Tay Boon Hou for his guidance in carrying out this action research.
- I would also like to thank Mdm Foo Kum Fong, Master Teacher (Maths) E5 Cluster for her guidance and inputs in writing this paper.

Appendix A

Name: _____ Date: 25/3/09

1. On tree planting day, 40 pupils planted some trees. Each boy planted 3 trees and each girl planted 2 trees. The boys planted 30 more trees than the girls. How many boys were there?

Step 1: Read the question.
 Step 2: Highlight the information.
 Step 3: What is the goal?
 Step 4: What information is given?
 Step 5: Plan
 Step 6: Work it out.
 Step 7: Check if the answer is reasonable.

Goal: How many boys were there?

Information: 40 pupils, each boy planted 3 trees, each girl planted 2 trees, boys planted 30 more trees than girls.

Plan: use guess and check

2. A tank was $\frac{1}{3}$ full. When another 700 ml of water was poured into the tank, it became $\frac{2}{3}$ full. Find the capacity of the tank.

Step 1: Read the question.
 Step 2: Highlight the information.
 Step 3: What is the goal?
 Step 4: What information is given?
 Step 5: Plan
 Step 6: Work it out.
 Step 7: Check if the answer is reasonable.

Goal: Find the capacity of the tank

Information: 

Plan: Find the tank - what fraction of the tank.

Work it out:

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\frac{1}{3} = \frac{700}{x}$$

$$x = 2100$$

The capacity of the tank is 2100 ml.

2. Kim had a sum of money. After giving away $\frac{1}{5}$ of it to his mother and $\frac{1}{3}$ of the remainder to his father, he had \$700 left. What was the sum of money he had at first?

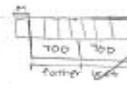
Step 1: Read the question.
 Step 2: Highlight the information.
 Step 3: What is the goal?
 Step 4: What information is given?
 Step 5: Plan
 Step 6: Work it out.
 Step 7: Check if the answer is reasonable.

Goal: To find the sum of money Kim had at first.

Information: Goal: \$700
 Remainder: $\frac{2}{5}$ gave $\frac{1}{3}$ of it to his father, divide it equal parts for his father (200)

Plan: Find the remainder (700). Find (1u). Find (6u).

Work it out:



Remainder \rightarrow 700 + 700 = 1400

$$7u \rightarrow \$1400$$

$$1u \rightarrow \$200$$

$$6u \rightarrow \$1200$$

He had \$1200 at first.

Why do We Teach What We Teach in Schools?

Lee Peng Yee
National Institute of Education

1. A question in mathematics

There is a mathematics syllabus in Singapore. It spells out what we should teach in schools. Then there is an institute of education. It trains us how we might teach it. We do not often ask the question why we are teaching what we teach. In other words, we ask what and how. But we do not ask why.

For example, do we know how many formulas there are for finding the area of a circle? One, two, or more? There are at least three. Let A be the area of a circle with radius r , diameter d , and circumference c . Then we have

$$A = \pi r^2,$$

$$A = \frac{\pi}{4} d^2$$

$$A = \frac{1}{2} cr$$

The second formula says: the area A is roughly three quarters of the square that circumscribes the circle. The third formula also has a meaningful geometrical interpretation. Think of cutting an orange sideways and eating it. Then in the process we would find a rectangle with base $c/2$ and height r and having the same area as the circle. My question is: why do we teach to our pupils the poorest formula among the three? I am not advocating a change in the syllabus. My real question is why we do not ask such questions.

We have a new syllabus 2007. We introduced calculators as a mathematical tool at the primary level. Why? We dropped transformation geometry at the secondary level. Why? We introduced box-and-whisker plots but not scatter plots. Why? The area of a triangle is half of its base times height. Why do we need another formula $\frac{1}{2} bc \sin A$? Why do we teach trigonometric identities though we no longer do so? The list can go on and on.

Do we have answers to the above questions? Do we ask such questions? Why should we ask such questions? Let us describe the historical development of mathematics from the point of view of school mathematics. What happened in the past led to what school mathematics is today.

2. Euclidean geometry

Once upon a time, mathematics in the west is nothing but Euclidean geometry. It was the geometry as expounded in Euclid's Elements around 300 BC. For a long period of time, professor of mathematics was called professor of geometry. The changes did not come along until the sixteen century. From 1600 to 1900, it was Qing dynasty in China. In Europe, that was the time of renaissance in Italy, reformation in Germany, revolution in France, and industrial revolution in England. It culminated in colonization and finally dominance of the world by the West. This is roughly what happened to the world during the past 400 years.

As far as geometry is concerned, three major events took place. The first was the collapse of Euclidean geometry. Euclidean geometry was based on axioms including the parallel axiom. By axioms we mean something we accept to be true without questioning. Given a line and a point not on the line, we can draw only one line parallel to the given line. This is an axiom. However mathematicians in early days tried to prove it. They failed. As usual, when we failed to prove something,

we like to think that maybe it is not true. Indeed, mathematicians constructed models to show that there are geometries other than Euclidean geometry.

Imagine that you and I both travel northward from two different cities on the equator. I go first and I shall end up at the North Pole. You go next. You will also end up at the North Pole. We are supposed to travel in parallel, and yet we meet at the North Pole. In the language of mathematics, we cannot draw two parallel lines on the globe. So the parallel line axiom has to remain as an axiom. We cannot prove it.

This discovery caused a severe blow to Euclidean geometry. As a result, Euclidean geometry no longer reigned supreme. Later on people found that actually Euclidean geometry describes the world of Newton and it does not describe the world of Einstein.

- **Spherical and other geometries spelled the fall of Euclidean geometry.**

The second major event was that geometry went algebraic. René Descartes (1596 – 1650) invented coordinates, now called Cartesian coordinates. A point in geometry is a pair of numbers in the Cartesian coordinates. A line in geometry is a linear equation via the Cartesian coordinates. To find the intersection point of two lines is equivalent to finding the solution of two linear equations. In other words, Descartes provided a way to convert a problem in geometry to a problem in algebra. Solve it in algebra and convert the answer back to geometry. So a geometrical problem can now be solved algebraically. For computation, it is easier to do it in algebra than in geometry.

There are many geometries. Is there a unified approach? Felix Klein in one of his lectures gave such an approach. Thereafter it was known as Erlangen Programme. He said that what geometry is depends on what transformations we consider. For example, if we consider only transformations that do not change shapes and sizes, then we have Euclidean geometry. In Euclidean geometry, we study the properties under which certain geometrical figures do not change shapes and sizes. If we consider enlargements, under an enlargement a figure changes size but not shape. Then we have projective geometry. Any property that does not change under given transformations we call an invariant property. In the language of Felix Klein, geometry is a study of invariant properties of geometrical figures under a given set of transformations. Again it is easier to work with transformations expressed in terms of algebra, in this case, matrices. Again working with matrices we are solving a geometrical problem algebraically.

- **Cartesian coordinates served as a bridge for the migration to take place from geometry to algebra.**

The temple of Euclidean geometry collapsed. The massive migration took place from the land of geometry to the land of algebra. It was not the end. Euclidean geometry was still alive. Then came the third event.

Euclidean geometry was not rigorous according to the way Euclid presented it. In other words, Euclidean geometry was not axiomatic as Euclid intended. By axiomatic we mean we assume certain conditions

without proof and call them axioms. Based on the axioms and nothing else, we prove results. Then we use the axioms and the results, we prove further results. This is called axiomatic proof. Euclid did not give enough axioms so that he could prove theorems using only those axioms. The person who saved it was David Hilbert (1862 – 1943). He gave a series of axioms and put Euclidean geometry on a sound foundation. Hence we say he saved it. His book the Foundations of Geometry is still in print after all these years. As a result, Euclidean geometry no longer makes sense as it was. Euclidean geometry is dead. Hence we say Hilbert killed Euclidean geometry.

- **David Hilbert saved and killed Euclidean geometry.**

Hilbert did not only kill Euclidean geometry. He killed also the teaching of Euclidean geometry in schools. Now geometry is not taught in the way I learned it when I was in schools. The approach we use in schools now was suggested by G. D. Birkhoff in his book Basic Geometry (1941). Basically, we assume various conditions for the congruence of triangles without proof. Then we proceed from there proving other results. There was an attempt in the 60s to replace classical geometry by transformation geometry or finite geometry. None worked out. We often associate proof with geometry. It is rigour that we want to impart to our students. Proof is a good way to get to rigour. Rigour is everywhere in mathematics, not just in geometry alone. So is geometry everywhere in mathematics. We should always look at algebra geometrically and geometry algebraically. In a way, Van Hiele levels no longer describe accurately the geometry we teach in our schools.

3. Al-jabr or algebra

Al-jabr or algebra is an Arabic word. We can fairly say that algebra came from the Arab world. The first book on algebra was written in Arabic. It is interesting to note that Euclid's Elements was translated into Latin also from Arabic. The presentation of algebra in those days was rhetorical. That is, everything was described in words and no formula. In those days, algebra was nothing but solving polynomial equations. Linear and quadratic equations were classified into many special cases. Then a solution was found for each case. They did so because they did not recognize the existence of negative numbers.

It is interesting to note that some Italian mathematicians Cardano and his rival Tartaglia actually made a living out of solving cubic equations in the sixteen century. Complex numbers were invented due to solving cubic equations, and not quadratic equations. When the degree of a polynomial equation increases, it becomes almost an impossible task to solve it. Some of you may remember the days when we learned elementary symmetric functions like etc. They are used to solve cubic and quartic equations. Solving cubic equations was in the textbook in the 50s.

It was a dead-end to solve polynomial equations by means of elementary symmetric functions. The best result thereafter was by Gauss (1799). He proved that there exists a solution for every polynomial equation. It is so important that the result is now called the Fundamental Theorem of Algebra. He did not show us what the solution is. He simply said there was one. This was not an isolated event. In astronomy, an Englishman and a Frenchman predicted independently that there was a planet beyond Uranus before it was found. Consequently, a German astronomer found the eighth planet Neptune in 1846. Similarly, an American astronomer Clyde Tombaugh found the ninth planet Pluto in 1930 as predicted. I met the scientist

and had lunch with him in Las Cruces in 1992. There was a similar incident in the prediction of a missing chemical element in 1869. It is a powerful method. It is known as existence proof in mathematics. Nowadays polynomial equations and also differential equations are solved mainly by numerical methods. The solution of a cubic equation can be found in calculus, though not in school mathematics.

- **Algebra came from the Arab world and in time dominated school mathematics.**

In the nineteenth century, algebra took a different turn. It went structural and numerical. These are the two major events in modern algebra. There was a big story at the time. One of the three famous construction problems is how to trisect an angle by ruler and compasses. The problems are dated back to the time of Greek. They did not ask whether. Instead they asked how because they believed that it could be done. It was not until Galois (1811 – 1832) who showed that it could not be done. The tool he used was group theory in abstract algebra. It may be the first time that advanced mathematics was used to prove or disprove a simple problem, at least we thought it was simple. Definitely, it was not the last time as we can see many such examples later. Galois was also famous for dying at the age of 21 in a duel. The duel was with pistols at twenty-five paces.

The number 3 in 3 coconuts is an abstract concept. It was made abstract for good reasons. Similarly, algebra was made abstract also for good reasons, not just for solving the three construction problems. Other than group theory, there are other algebraic structures. Collectively, they are called abstract algebra. The key concept is structure, algebraic structures and different kinds of algebraic structures.

If we look at linear algebra carefully, it is nothing but geometry or geometry without pictures. Some concepts in linear algebra like eigenvalues and eigenvectors are introduced to simplify computation. Abstract algebra is not abstract at all. It is down to earth and has practical applications. A more recent example is linear codes. Telecommunication cannot do without linear codes. There is no need to say more about algebra going numerical.

- **In the years 1600 to 2000 geometry turned algebraic and algebra went structural and numerical.**

We inherited school mathematics from the West. There was also mathematics in the East. For example, Chinese remainder theorem in abstract algebra was discovered in China 2000 years ago. In China, it was called Sunzi method. It was known to Arabs as Chinese method, and only more than 1000 years later to the West as Chinese remainder theorem.

Looking at it from Europe, there were two great dynasties in China over a period of 2000 years. They are Han Dynasty (206BC – 220 AD) and Tang Dynasty (618 – 907 AD). During Han Dynasty, it was Roman Empire in Europe. During Tang Dynasty, it was Dark Ages in Europe, though a golden age in the Arab world. Two Chinese classics in mathematics have been translated into English and other western languages. They are Jiu-zhang Suan-shu (Nine chapters of arithmetic) and Shu-shu Jiu-zhang (Book of mathematics in nine chapters). The above translation of the titles is one that is closer to the Chinese original. The books consist of collection of practical problems in farming, construction, business etc. They represented the achievement in mathematics during Han Dynasty and Tang Dynasty respectively.

All problems are stated in context. No computation was given and no formula. The solution of a problem was obtained by going through a process. The emphasis is on process. Recently, the West rediscovers the Chinese approach to mathematics.

Qing Dynasty declined after the Opium War (1839 – 1842). Chinese did not really proceed from numbers to symbols. In the early twentieth century, Chinese imported school mathematics from the West. There was also mathematics, rich in content and approach, in other Asian countries in ancient time.

- **Chinese learn mathematics differently and they learn how before they learn why.**

School mathematics is dominated by geometry and algebra. We described the major events in the past relating to geometry and algebra. It is only by going through the past we learn why we are doing what we do today. We must also look into the future to learn what we should do today.

4. Mathematics today

School mathematics that we are teaching today is mathematics of the last century. What is mathematics of this century? Here we are interested only in those having a potential of getting into school mathematics, and we shall be brief. However we make an exception for differential geometry, as this is the hottest topic in town.

As we mentioned above, equations are solved mainly by numerical methods. The use of calculators and computers makes a great impact on mathematics and mathematics teaching. At first, it was numerical computation. Lately, it was symbolic manipulation. The prediction is that we need both for doing research in mathematics that requires computation. In the 70s, there was a movement to replace calculus by finite mathematics in the first year of undergraduate study. The rationale was that students would then be better prepared for using computers. It did not succeed. At that time, it was also predicted that algorithm, a popular tool in computation, would become an important item in school mathematics. It did not happen. Computation, numerical or otherwise, requires a totally different kind of approach. The answers are approximate rather than exact as we are so used to for years. The process is iterative rather than of a finite number of steps. Sometimes being able to solve a problem is not good enough. We must solve it within a time frame or by making use of a suitable model. In short, computation is an essential component of mathematics. In time to come, some of it will get into school mathematics.

- **The fourth milestone in mathematics education after Euclidean geometry, calculus, pure mathematics is computation.**

It is unlikely that differential geometry or more technically manifolds will be part of school mathematics curriculum. It is taught in the university, though not necessarily a core module. This is an area of active research in the past 50 years. Six mathematicians were awarded Fields Medal for their work in manifolds. Fields Medal is the equivalent of Nobel Prize in mathematics. In 2006 at the International Congress of Mathematicians in Madrid, Spain, a Russian mathematician Perelman was awarded Fields Medal for proving the final step of Poincaré conjecture.

What is Poincaré conjecture? If you live on a circle, locally it is a straight line. If you live on a sphere or the surface of the Earth, locally it is a flat plane. Suppose we can construct a four-dimensional sphere. It is

hard to visualize it, but not so hard to express it algebraically. Note that a two-dimensional sphere is a circle, and a three-dimensional sphere is the sphere as we know it. The question asked is: if you live on a four-dimensional sphere, what is the geometrical structure locally? The conjecture says that on a four-dimensional sphere, a good local structure (called manifold) looks like a three-dimensional sphere. What I have described can be made precise mathematically. Some 50 years ago, half of the people believed that it was true and the other half not. Now most people believe it is true. The tool used to prove the conjecture is Ricci flow in partial differential equations. Finally, the long-standing conjecture has been proved. Ricci flow is now a hot topic for research.

- **The marriage of geometry and calculus gave birth to differential geometry.**

In a sense, mathematics is nothing but modelling. We use a rectangle to model a table top. We use quadratic functions to model the free fall of an object. After modelling, we solve the corresponding mathematical problem by exact methods, approximate methods, or probabilistic methods. In schools, we teach mostly exact methods.

In the sixteen century, calculus was used to design a better steam engine. During and after World War II, many problems we encountered were discrete in nature. For example, allocation of resources for war effort cannot be solved by calculus. The method invented was later used in engineering or management after the War. For example, the design of telephone network cannot be solved by traditional mathematics. These are problems in discrete mathematics. The famous four-colour problem is also a problem in discrete mathematics. It says: we need not more than four colours to colour the countries on a given map. The problem was posed in 1852. It was proved in 1890 for five colours. It was solved for four colours in 1976 using the brutal force of computers. It is a fact that discrete mathematics is coming to schools. It is not difficult to pose a problem in context at the school level involving discrete mathematics.

- **Mathematical models are no longer restricted to physical sciences.**

Again in 2006 at the International Congress of Mathematicians in Madrid, Spain, Kiyoshi Itô from Japan was awarded the Gauss Prize for his contribution to making progress in mathematics that has significant implication in other fields. Itô sent his daughter to receive the prize. He said what he did was pure mathematics. He had no intention to apply it elsewhere but others did it for him. The contribution is stochastic differential equations. The other fields are engineering, finance among others. In particular, Black and Scholes won the Nobel Prize for applying stochastic differential equations to derivatives in finance. Different types of problems require different kinds of tools. Some problems are so big like the design of a dam. Some problems are so fluid like derivatives in finance. Computation has to be done using probabilistic or stochastic methods. There are discrete models like telephone lines, and there are also stochastic models like derivatives in finance. Randomness is an important concept in stochastic methods. It is already in mathematics curriculum in some countries.

- **Mathematical tools go from exact to approximate and further to stochastic.**

The ten story lines given above are the topics covered in ten lessons

of a module MME802 Fundamental Concepts in Mathematics for graduate students in mathematics education at the National Institute of Education, Singapore. In each lesson, we introduce certain fundamental concepts with related skills, in addition to the story lines. For example, the first three lessons on geometry are: three theorems on triangles in hyperbolic geometry via Poincaré model, representation of translation, rotation etc by 3×3 matrices, and an axiomatic proof of $(-1) \times (-1) = 1$. It is a module in mathematics. The connection with school mathematics curriculum, in particular curriculum in Singapore, is also given. For more information on the school curriculum in Singapore, see the articles in References.

5. Why we teach what we teach

We taught Euclidean geometry because it was an academic pursuit. We taught algebra because it was a more efficient tool to do computation than to do it with geometry. We taught statistics because we thought it was useful.

When we review the events in the past, we note that many topics were in and out of the syllabus. For example, transformation geometry was introduced into the syllabus after Euclidean geometry was cut. Eventually transformation geometry was gone. It is not in the new syllabus 2007. Algebra was not taught in primary schools. Then it was and then it was not. Now it is in the syllabus 2007. We wonder why we teach certain topics and why we do not. I make a list below. It is by no means a complete list.

5.1. Rich in content and rich in examination questions.

Why was Mozart a prodigy in music? He was born and brought up in a rich environment, rich in music. We learn more if the topic we are learning is a rich topic. Mechanics is rich. So is Euclidean geometry. However they lost their place in school mathematics due to not satisfying other items in the current list. We shall not elaborate here. In Singapore context, the topic must also be rich in exam questions. Numerical methods was at one time in the A level syllabus. After a while, we ran out of exam questions. So the topic was dropped from the syllabus.

5.2. For computation and for rigour.

If we check the verbs used in PSLE (Primary School Leaving Examination) and in the O level exam papers. The most commonly used word is "find". What is involved is to compute. So we teach and test computation. Another equally important, if not more important, topic that we teach and test is rigour. The value of mathematics is in rigour. Proof is part of it. If we want a baby to grow, we must feed the baby with solid food. Rigour is solid food for mathematics students. Some employers prefer mathematics students because they somehow believe that mathematics students have been trained in rigour.

5.3. For assessment though not assessment alone.

By all means, we teach for assessment. There is nothing wrong to teach for assessment. Assessment can be a negative factor in learning. It can also be positive if we play it right. It is a fashion now to talk about assessment of learning, for learning and finally as learning. The key is not to teach for assessment only.

5.4. For knowledge and for the use of knowledge.

We always learn a piece of mathematics and learn how to use it. The difference is that now we make it explicit. If we learn a concept and there is no way to use it in the syllabus. Then we wonder why we teach it in the first place. Suppose we do use it. Then the next question is: how do we test the use of knowledge?

5.5. What we can relate to.

We do not mean solving problems in context in the sense that the problems are authentic or realistic. Sometimes it could be difficult or quite meaningless to do that. We learn better and faster through association with something we are familiar with. Hence it is a good idea to include things that we can relate to. Those things could be what students can do and can learn from. They may have nothing to do with our daily life or the environment around us. The key is to be able to relate to. Being authentic or realistic helps but it is not a major issue.

5.6. Statistics is a misfit.

What we are teaching in schools is exam statistics and, strictly speaking, not statistics. Statistics is a misfit in the mathematics syllabus. There has been suggestion that we may wish to teach statistics differently or even assess it differently. Maybe we need to ask a totally different set of questions concerning the teaching of statistics.

5.7. Certain concepts must be taught early.

It is my belief that if you want your children to eat certain kind of food, you better feed them before they were five years old. Certain things you have to learn from young. Some students learn statistics late in their school days. They forever have a problem thereafter. If we want our students to understand randomness, we better teach them when they are in the primary schools.

5.8. For workplace.

Most people get a job after schooling or university. If all they see in their workplace is computers, there is no reason why we do not use computers in schools. If team work is important, then we should start having team work in schools. After all, we educate our students for work.

I have taught mathematics at the university level for the past 45 years. Some students came back to see me after 30 years. They said they were my students. They could not remember when they were in my class. They could not recall the title of the course they took. To prove that they were my students, they told me a story I told them. I can recognize my own stories. So I know they were indeed my students. Apparently, they do not remember what I taught them. However they do remember a little bit of how I taught mathematics. The point I want to make is that teaching mathematics is not teaching content alone. Also, I am not saying that content is not important.

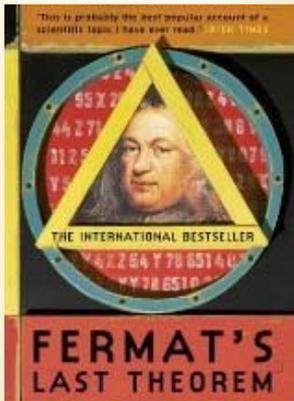
In the same way, we may not need to know why we teach what we teach. But it is my strong belief that some of us should know. We rely on these people to revise the syllabus perhaps another ten years from now. Without knowing why, we shall not be able to design a good syllabus.

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Fermat's Last Theorem: The Story of a Riddle that Confounded the World's Greatest Minds for 358 Years

Book Review by Leong Chong Ming
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Two hundred mathematicians were packed in an auditorium in the Sir Isaac Newton Institute in Cambridge University, waiting for the unfolding of what was considered to be the most important mathematics lecture of the 20th century. The lecturer, an English gentleman, walked in and started writing with great fluidity without stopping on the boards. What the audience saw on the three blackboards were dense mixture of mathematical and Greek symbols and

highly sophisticated Algebra belonging to the domains of **Elliptic Equations** and **Modular Forms**.

The lecturer was Professor Andrew Wiles, an Englishman who studied in Cambridge and took up professorship at Princeton University. As the lecture progressed, the audience were drawn closer and closer to the edge of their seats. When the lecturer finally wrote the last line of his long chain of arguments, the audience unanimously gave a standing ovation. Professor Andrew Wiles had just proved Fermat's Last Theorem (FLT), the age-old mathematical conundrum that had baffled the greatest mathematical minds for 358 years!

Wiles had been fascinated with problem-solving since his childhood days. At the young age of 10, Wiles would devour books on scientific conundrums and mathematical riddles. However, there was one book, "The Last Problem" by E.T Bell, which contained a problem known as FLT that would hold his attention for the next 30 years.

Pierre de Fermat (1601 – 1665) was born on 20 August 1601 in France. As a child, he showed precocious ability in mathematics but took up a career in the civil service instead. He was appointed a councillor at the Chamber of Petitions, to assess if the petitions made by the locals merit the king's attention. His job did not allow him to mingle with the general public but gave him the chance to pursue his interest – Mathematics.

Fermat came across Pythagoras's theorem and the Pythagorean Triplets. He was attracted by the sheer quantity of the Pythagorean Triplets. Fermat extended the equation, " $x^2 + y^2 = z^2$ " to the more general equation " $x^n + y^n = z^n$ ($n > 2$)" and wondered if there existed integral solutions. He jotted down a note which would baffle the finest of all mathematical minds for more than three centuries.

He wrote: "It is impossible for a cube to be written as a sum of 2 cubes or a fourth power to be written as the sum of 2 fourth powers, or in general, for any number which is a power greater than the second to be written as a sum of 2 like powers."

In other words, Fermat meant that the general equation, " $x^n + y^n = z^n$ ($n > 2$)" has no integral solutions. This was the famous FLT. Fermat wrote an additional comment which would haunt generations of mathematicians to come.

He wrote: "I have a truly marvellous demonstration of this proposition which this margin is too narrow to contain."

Fermat died on 9 January 1665 without leaving a proof for his famous theorem. Fearing that his father's mathematical legacy would be lost forever, the eldest son of Fermat, Clement-Samuel Fermat undertook the task to publish all of Fermat's work, including the famous marginal note containing FLT.

For 350 years, FLT was regarded only as "Fermat's Last Conjecture" because to professional mathematicians, a mathematical statement is deemed a conjecture until a complete proof has been sought. Although Fermat did not give a complete proof of his conjecture, he did however, provide a proof for the special case when $n = 4$ by employing a technique he developed, called **The Method of Infinite Descent**.

After Fermat's death, over three and a half centuries, great mathematicians such as Euler, Germain, and Cauchy had tried to prove this theorem, but every attempt ended in failure. The 17th century problem was about to die a natural death until 1993.

Proving FLT had always been Wiles's obsession since his childhood. Wiles, a Mathematics Professor at Princeton University decided to work in isolation, in his quest for the proof of FLT. After six years of research, Wiles thought that he was beginning to see a ray of light at the other end of the tunnel and decided to seek a second opinion. His choice went to an old friend and his colleague, Professor Nick Katz. Wiles revealed to Katz that he was working on the Kolyvagin-Flach technique and he needed Katz to affirm his calculations. Katz decided to set up a formal lecture course opened to the department's graduate students, titled "Calculations on Elliptic Curves". Upon the completion of the series of lectures, both Wiles and Katz put their brains together and refined the Kolyvagin-Flach technique. It did not take long before Wiles was able to prove that all the families of elliptic equations were modular, except one particular family which refused to yield to the technique.

One day in May, Wiles was casually browsing through a paper written by Barry Mazur. In the midst of those dense algebraic calculations, he found the last piece of jigsaw puzzle to complete his proof for FLT. He went downstairs and told his wife, "I have finally proved Fermat's Last Theorem!"

After seven years of teeth-biting effort, on 23 June 1993, Wiles delivered the proof on Fermat's unyielding theorem in a series of three lectures. When Wiles wrote the final statement of the proof to FLT, he received a standing ovation and cheers from the audience. Newspapers across the world reported that Wiles had finally conquered the age-old conundrum after 358 years and put it to rest eternally.

Note:

Not long after Wiles announced his proof of FLT, a flaw was detected in the proof. The following year in 1994, it was rectified with the help of Wile's student professor Richard Taylor and FLT once again received the full recognition of a theorem.

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TIMSS 2007- Performance in mathematics of eighth graders from Singapore and East Asia

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What is TIMSS 2007

TIMSS (Trends in International Mathematics and Science Study) 2007 is the fourth of international mathematics and science assessments conducted periodically every four years. TIMSS is designed to provide trends in fourth and eighth grade mathematics and science achievement in an international context. The aim of TIMSS is to provide policy makers with a wealth of information about key instructional, curricular, and resource related variables that are fundamental in understanding the teaching and learning process. For TIMSS 2007, altogether, 49 countries participated at the eighth grade level. The East Asian countries that participated in TIMSS 2007 at the eighth grade were Chinese Taipei, Hong Kong, Indonesia, Japan, Korea, Malaysia, Singapore and Thailand. Data was collected from participating students, their teachers and school leaders with the help of assessment tasks and background questionnaires. The TIMSS 2007 International Mathematics Report (Mullis, Martin & Foy, 2008) is a comprehensive report of all the data collected and analysed for mathematics assessment of grade four and eight students. This article draws on the data from the report and reports on the achievement of grade eight students from Singapore and East Asian countries that participated in the study.

Student Participants and Tests

Representative samples of eighth graders from participating countries took part in the study. In East Asia they were in their eighth year of formal schooling and their average ages ranged from 14.2 to 14.5 years. The TIMSS 2007 tests (Ruddock, O'Sullivan, Arora & Erberber, 2008) comprised of both mathematics and science items. Fourteen different booklets containing a selection of the 215 mathematics and 214 science items were administered to the students. Each student completed the test in one booklet. Testing time was 90 minutes. The 215 mathematics items (117 multiple choice and 98 constructed response type) were classified by content domain and cognitive domain. The four content domains were Number, Algebra, Geometry, and Data and Chance, while the three cognitive domains were knowing, applying and reasoning (Mullis, Martin, Ruddock, O'Sullivan, Arora & Erberber, 2005).

Mathematics Achievement

Table 1 shows the ranking and average scale scores of the eight East Asian countries that participated in TIMSS 2007. Five of these eight countries were in the top 5 ranks. Chinese Taipei was in the first position followed by Korea, Singapore, Hong Kong and Japan. Malaysia was 20th, Thailand was 29th and Indonesia was 36th in position. The average scale scores of the five countries that were ranked in the top five positions were significantly higher than the international average. There was no significant difference between the average scale scores of Chinese Taipei, Korea and Singapore.

The human development index is indicative of how developed a country is. The eight East Asian countries with decreasing human development index were Japan, Hong Kong, Chinese Taipei, Singapore, Korea, Malaysia, Thailand and Indonesia. From Table 1, it is apparent that the human development index may be a predictor of the average scale scores of the grade eight participants.

Table 1: Rank, average scale scores and human development index

Country	Rank	Average Scale Score	Human Development Index
Chinese Taipei*	1	598 (4.5)	0.932
Korea*	2	597 (2.7)	0.921
Singapore*	3	593 (3.8)	0.922
Hong Kong	4	572 (5.89)	0.937
Japan	5	570 (2.4)	0.953
International Average	-	500	-
Malaysia	20	474 (5.0)	0.811
Thailand	29	441 (5.0)	0.781
Indonesia	36	397 (3.8)	0.728

() standard errors

* No significant difference between average achievement

International Benchmarks of Mathematics Achievement

The International benchmarks presented as part of the TIMSS 2007 data (Mullis, Martin & Foy, 2008) helps to provide participating countries with a distribution of the performance of their eighth graders in an international setting. For a country the proportions of students reaching these benchmarks are perhaps telling of certain strengths and weaknesses of mathematics education programs of the country. The benchmarks delineate performance at four points of the performance scale. Characteristics of students at each of these four points are elaborated in the next section.

Table 2 shows the percentage of students from the eight East Asian countries reaching TIMSS 2007 International benchmarks of mathematics achievement. It is worthy to note that for Chinese Taipei 45% of their students were at the Advanced benchmark and in all the five East Asian countries that were ranked as the top five, more than 60% of their students were at the High benchmark level and almost 90% of their students were at the Intermediate benchmark level. For Malaysia, 50% of the students were at the Intermediate benchmark level and for both Thailand and Indonesia 34 % and 19% respectively were at the Intermediate benchmark level.

Table 2: Percentages of students reaching TIMSS 2007 International benchmarks of mathematics achievement

Country	Advanced Benchmark (625)	High Benchmark (550)	Intermediate Benchmark (475)	Low Benchmark (400)
Chinese Taipei	45 (1.9)	71 (1.5)	86 (1.2)	95 (0.6)
Korea	40 (1.2)	71 (1.1)	90 (0.7)	98 (0.3)
Singapore	40 (1.9)	70 (2.0)	88 (1.4)	97 (0.6)
Hong Kong, SAR	31 (2.1)	64 (2.6)	85 (2.1)	94 (1.1)
Japan	26 (1.3)	61 (1.2)	87 (0.9)	97 (0.3)
Malaysia	2 (0.5)	18 (2.1)	50 (2.7)	82 (1.9)
Thailand	3 (0.8)	12 (1.7)	34 (2.2)	66 (2.0)
Indonesia	0 (0.2)	4 (0.6)	19 (1.4)	48 (1.9)
International Median	2	15	46	75

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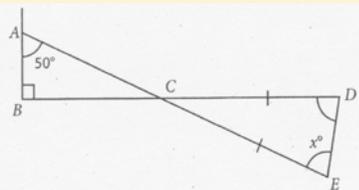
What can students at each of these international benchmarks do?

Table 3 shows what students at each of the four international benchmarks were able to do.

Table 3: Descriptions of International Benchmarks

International Benchmark	Characteristics of students
Advanced - 625	Students can organize and draw conclusions from information, make generalizations, and solve non-routine problems.
High - 550	Students can apply their understanding and knowledge in a variety of relatively complex situations.
Intermediate - 475	Students can apply basic mathematical knowledge in straightforward situations.
Low - 400	Students have some knowledge of whole numbers and decimals, operations, and basic graphs.

Figures 1 and 2, show items of the advanced benchmark that students' reaching the benchmark are likely to answer correctly.

Content Domain: Geometry Description: Uses properties of isosceles and right angled triangles to find the measure of an angle	Country	Percent Correct
 <p>In this diagram, $CD = CE$. What is the value of x?</p> <p>(A) 40 (B) 50 (C) 60 (D) 70</p>	Singapore	75 (1.7)
	Chinese Taipei	73 (2.2)
	Korea	73 (1.8)
	Japan	71 (1.9)
	Hong Kong SAR	69 (2.8)
	Thailand	36 (2.1)
	Malaysia	36 (2.7)
	Indonesia	19 (2.0)
	International Avg	32 (0.3)

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Figure 1. Advanced International Benchmark Item - 1

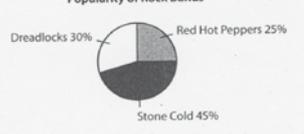
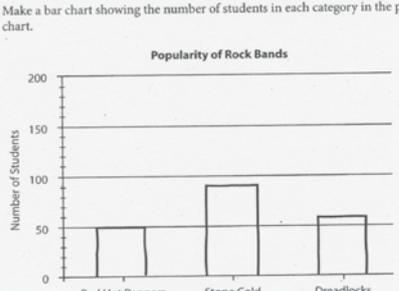
Content Domain: Algebra Description: Solves a word problem that can be expressed as two linear equations with two variables.	Country	Percent Correct
<p>Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?</p>	Chinese Taipei	68 (2.3)
	Korea	68 (2.1)
	Singapore	59 (1.9)
	Hong Kong SAR	53 (2.8)
	Japan	42 (1.9)
	Malaysia	14 (1.7)
	Thailand	13 (1.4)
	Indonesia	8 (1.3)
	International Average	18 (0.2)

Figure 2. Advanced International Benchmark Item - 2

Figures 3 and 4, show items of the high benchmark that students' reaching the benchmark are likely to answer correctly.

Content Domain: Algebra Description: Solve a linear equation given in a word problem.	Country	Percent Correct
<p>In zedland, total shipping charges to ship an item are given by the equation $y = 4x + 30$, where x is the weight in grams and y is the cost in zeds. If you have 150 zeds, how many grams can you ship?</p>	Chinese Taipei	75 (2.0)
	Korea	71 (1.8)
	Hong Kong SAR	67 (2.9)
	Japan	65 (2.1)
	Singapore	56 (1.7)
	Indonesia	26 (1.9)
	Thailand	26 (2.3)
	Malaysia	24 (2.1)
	International Average	34 (0.3)

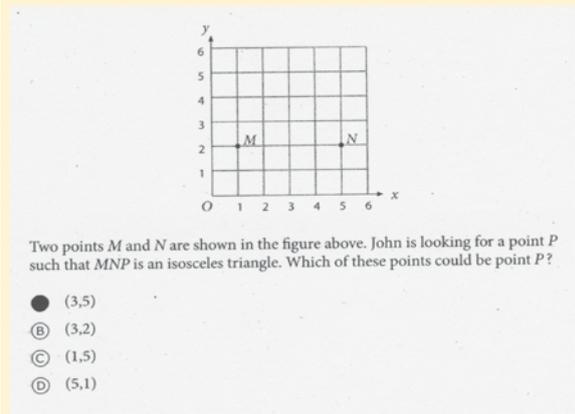
Figure 3. High International Benchmark Item - 1

Content Domain: Data and Chance Description: Uses the information in a pie chart showing percentages to draw a bar chart	Country	Percent Correct
<p>The results of a survey of 200 students are shown in the pie chart.</p>  <p>Make a bar chart showing the number of students in each category in the pie chart.</p> 	Korea	76 (2.0)
	Singapore	75 (1.7)
	Chinese Taipei	70 (2.1)
	Japan	68 (1.8)
	Hong Kong SAR	66 (2.6)
	Malaysia	35 (2.4)
	Thailand	26 (2.2)
	Indonesia	14 (1.3)
	International Average	27 (0.3)

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Figure 4. High International Benchmark Item - 2

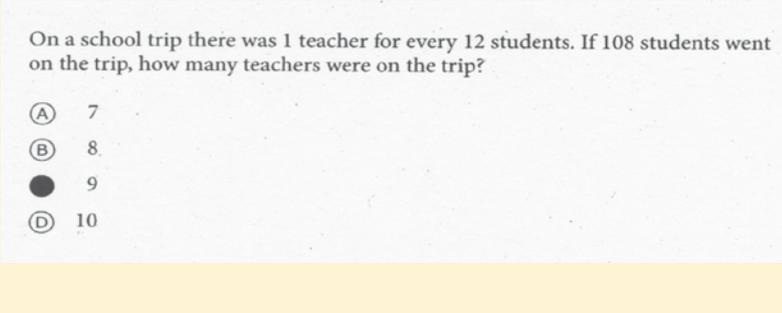
Figure 5 shows an item of the intermediate benchmark that students' reaching the benchmark are likely to answer correctly.

Content Domain: Geometry Description: Uses properties of an isosceles triangle to identify the coordinates of a point on a grid.	Country	Percent Correct
 <p>Two points M and N are shown in the figure above. John is looking for a point P such that MNP is an isosceles triangle. Which of these points could be point P?</p> <p>● (A) (3,5) (B) (3,2) (C) (1,5) (D) (5,1)</p>	Chinese Taipei	86 (1.5)
	Korea	82 (1.6)
	Japan	81 (1.6)
	Hong Kong SAR	80 (2.6)
	Singapore	77 (2.0)
	Malaysia	73 (1.8)
	Indonesia	59 (2.5)
	Thailand	55 (2.2)
	International Average	57 (0.3)

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Figure 5. Intermediate International Benchmark Item

Figure 6 shows an item of the low benchmark that students' reaching the benchmark are likely to answer correctly.

Content Domain: Number Description: Solves a word problem involving a proportion with unit ratio	Country	Percent Correct
 <p>On a school trip there was 1 teacher for every 12 students. If 108 students went on the trip, how many teachers were on the trip?</p> <p>(A) 7 (B) 8 ● (C) 9 (D) 10</p>	Korea	97 (0.6)
	Singapore	95 (1.0)
	Chinese Taipei	95 (1.0)
	Japan	94 (1.0)
	Hong Kong SAR	94 (1.4)
	Malaysia	90 (1.4)
	Thailand	81 (1.7)
	Indonesia	78 (2.0)
	International Average	79 (0.3)

() standard errors

Figure 6. Low International Benchmark Level Item

An Insider's Perspective about the performance of Singapore's secondary two students in TIMSS 2007

For TIMSS 1995, 1999 and 2003, Singapore ranked first for mathematics at the eighth grade level. This is the first time we have slipped in our ranking. However, what is heartening is that there is no significant difference between our average scale score and that of Chinese Taipei and Korea ranked first and second respectively. It is also apparent from the sample items given in this article on algebra that our students are lacking in performance when compared with students in the other top East Asian countries. The data in figure 2 shows that students in Chinese Taipei and Korea did significantly better than our students on an item involving two linear equations with two variables presented as a word problem. Similarly, the data in figure 3 shows that students in all the other top ranking East Asian countries did significantly better than our students on an item involving a linear equation given in a word problem. Performance of our students on a geometry item shown in Figure 5 is also of concern. In Singapore, students are taught the properties of isosceles triangles in the primary school and they do revisit the properties of such triangles in lower secondary years. The inability of many of our students to make an inference such as "the vertex of an isosceles triangles lies on the perpendicular bisector of the base" may have led to their relative poor performance when compared with

peers from Chinese Taipei, Korea, Japan, and Hong Kong.

What's next?

In the next issue of Maths Buzz, we will share with readers some grade eight mathematics items that appeared either too easy or too difficult for our students.

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