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Summary

> Learning must change from an emphasis on knowledge acquisition to knowledge construction in order to address the possible challenges of the fast-changing world.
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> (Perkins, 1992)
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> "... our past and current achievements are no assurance of future successes in a rapidly changing world."
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> (Review of the Junior College/Upper Secondary Curriculum, MOE, 2002)

It was against this backdrop of call to change, that the Singapore Ministry of Education put in place a series of educational initiatives to provide for a more diversified educational landscape that would better prepare our students for the future. Among them, there was also a call for schools to move towards a curriculum that would place more emphasis on knowledge construction and other skills necessary for the future.

Driven by the 'Teach Less Learn More' movement in Singapore, many schools had begun their journey of curriculum redesign. Several curriculum frameworks were considered and one of them was the Teaching for Understanding (TfU) framework. In the case of Mathematics, teaching for understanding has generated much interest among Mathematics teachers (Skemp, 1978; Greeno, 1984; Sierpinska, 1994; Meel, 2003; Watson, 2005).

In a study by Hammerness et al. (1998) on TfU implementation across four subjects, the students in the Mathematics class performed the worst despite the teacher's longer involvement with the framework. In 2006, the researcher's school redesigned the entire school's curriculum using the TfU framework and this motivated the researcher to investigate the impact of TfU implementation on students' understanding of the Cartesian Connection—the connection that the coordinates of any point on a line will satisfy the equation of the line (Knuth, 2000a).

The researcher's key research problem statement would be

> What can we say about high-ability students' understanding of Cartesian Connection when they are taught using the TfU framework?

In particular, the study will aim to answer the following questions using the elements from the TfU framework:

1. *What* do high-ability students understand about the Cartesian Connection when they are taught using the TfU framework?
2. What is the *level* of understanding of the Cartesian Connection attained by these high-ability students?
3. Were all four dimensions of understanding as proposed by the TfU framework equally demonstrated by these students?
4. What are the factors affecting TfU implementation in Mathematics?

It is hoped that this study will offer a better idea of how the TfU framework can be used or adapted to teach for understanding in Mathematics.

This study took place in a Integrated Programme school in Singapore, which takes in the top 10 % of the national cohort from the Primary School Leaving Examination (PSLE) each year and has the flexibility to design the curriculum using TfU. The school's student population is therefore relatively homogeneous and are of the higher ability group. They are self-motivated and relatively strong in their academic studies.

The 25 students involved in this study took a modified version of Knuth's (2000a, 2000b) test on the Cartesian Connection. Besides scoring the test solutions, the researcher used the TfU framework which comprised of four dimensions of understanding: Knowledge, Methods, Purposes and Forms with their corresponding four levels of undertanding: Naive, Novice, Apprentice and Master, to examine their answers to the test items. The use of the TfU framework allowed the researcher to describe qualitatively and quantitatively students' understanding beyond what was given by Knuth (2000a, 2000b).

In addition, the TfU framework was also used to examine the unit plan, the lesson plans and worksheets used in the course of this study. Interviews were conducted with 13 out of the 25 students who volunteered and their responses were also analysed using the framework. In addition, eight representative teachers who taught the students in this study were interviewed

and their responses were scanned to surface any possible factors that could impact on TfU implementation.

The 25 students in this study did better than students in the study by Knuth's (2000a, 2000b) and seemed to show a better understanding of the Cartesian Connection. Moreover, the TfU framework allowed us to determine the levels of understanding in each of the dimensions. It was found that the Knowledge dimension was the best developed and the Methods dimension needed more attention. The framework provided a language to describe clearly what the students understand and provided a means for us to help students improve their understanding.

A chi-square analysis of the students' understanding in this study using the same methods by Hammerness et al. (1998) revealed an interesting finding. In the study by Hammerness et al. (1998), all the dimensions were found to be significantly related when results from **all four subjects** were pooled together. In this study that involved only Mathematics, it was found that only Knowledge and Forms and Methods and Forms were associated significantly. This would have implications for the Mathematics teachers if the associations were indeed true.

These findings were analysed and explained by looking at data collected from the students' interviews, teachers' interview, analysis of unit plans and worksheets. Through these data, the researcher was able to provide a richer

description and deeper insight into students' understanding of the Cartesian Connection and highlighted factors that could impact on future TfU implementation in Mathematics.