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Building, supporting and enhancing teachers’ capacity to foster mathematics learning: Insights from Indonesian classroom

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Abstract

This paper aims to examine and reflect on practice to enhance teachers’ capacity in fostering mathematics learning in primary schools. The study is situated in the context of developing and implementing an Indonesian version of Realistic Mathematics Education, labelled as PMRI in Indonesia. Series of five-days programs involving collaborative work between mathematics teachers and mathematics educators were carried out in classrooms to build, support, and enhance teachers’ capacity to implement PMRI teaching. Design research methodology comprising daily cycles of design, classroom experiments, and retrospective analysis is employed. This paper will draw on data from 3 design research mini-cycles held in primary schools about division of fractions and whole numbers. Episodes of video classroom discourse and samples of students’ work pertinent to the topic will be analyzed. Looking into the classroom practices suggest various ways of teachers’ actions in fostering rich discourse in learning mathematics. Factors that afford and inhibit teachers from establishing this practice are identified. Implications on ways to support and enhance teachers’ capacity based on ongoing analysis of classroom practice will be articulated.

Keywords: design research, teachers’ capacity, mathematics learning, Indonesian classrooms

BACKGROUND

In an attempt to improve the quality of mathematics teaching and learning in Indonesia, a movement to adapt Realistic Mathematics Education theory, known as PMRI, has started a decade ago (Sembiring, Hoogland, & Dolk, 2010; Ekholm, 2009). Utilizing a bottom-up approach, this movement started in primary schools involving groups of mathematics educators and primary school teachers.

In examining current practice of PMRI in classrooms, systematic evidence based on research is essential. Hence, a research task force focusing on developing knowledge about classroom design and practice was established two years ago (see Dolk, Widjaja, Zonneveld, & Fauzan, 2010). Design research (Cobb, Stephan, McClain, & Gravemeijer, 2001; Gravemeijer, 2004) has been utilized as a powerful vehicle to look into the complexity of classroom learning practice. It provides a wide opportunity for mathematics educators and teachers to engage in collaborative work to construct educational knowledge. This paper will examine and reflect on practice to enhance teachers’ capacity in fostering students’ mathematical thinking. Drawing on our experiences in three design research mini-cycles about division of fractions, teachers’ actions to promote mathematics learning will be discussed. Ways to support teachers’ capacity in carrying out such actions will be examined and discussed.
DESIGN RESEARCH CYCLES

In this paper, episodes of classroom discussions focusing on the role of teacher’s actions and questioning in enhancing students’ thinking will be analyzed. The contextual problem that served as a starting point for grade-5 classes will be explicated in the next section.

A combination of design research (Gravemeijer 1994; Gravemeijer & Cobb, 2006), a hypothetical learning trajectory (Simon, 1995), and lesson study (Watanabe, 2002; Yoshida, 1999) was used to develop our knowledge of mathematics teaching. Our work follows the design research definition by Gravemeijer (1994) as a cyclical process of thought experiments and classroom experiments (figure 1). An iteration of designing, experimenting, and reflecting (often labelled as retrospective analysis) is used to develop an emergent local instruction theory about mathematics education. There is a reciprocal relationship between this emergent local instruction theory and the classroom experiments. The emergent local theory guides the classroom experiments, while on the other hand, retrospective analysis of the classroom experiments causes the refining of the local instruction theory.

We integrated the collaborative aspect of the planning-doing-reflecting cycle from the lesson studies approach into a cyclical process of four successive phases: Knowledge, Designing, Experimenting, and Analyzing (figure 2). The collaboration between mathematics educators and teachers was enacted in the classroom by designing the problem, and observing and analyzing students’ strategies. Firstly, teachers describe what they know about teaching, about their students, etc. Secondly, teachers design a lesson together (thought experiment). Thirdly, one of the teachers – often an expert teacher – will carry out the planned lesson while other participants observe what happens in the class (classroom experiment). Subsequently, the teachers will reflect on and re-plan the lesson, and sometimes another teacher will carry out the revised lesson.

The classroom experiment is seen as a mutual responsibility of all teachers and mathematics educators. Although one teacher will take the lead during the classroom experiment, another participant can co-teach to allow for more voices and insights about teachers’ actions and questioning. The co-teaching with teachers in the classroom, particularly when teachers request it, is perceived as one of many ways to support and to enhance teachers’ capacity in the classrooms.

Simon’s hypothetical learning trajectory (1995) was employed to fine-tune the designing phase. Following Simon, we emphasize the hypothetical learning trajectory (HLT) as the core element in the designing phase. In formulating a HLT, teachers start with the goals they have in mind for the lesson. The HLT comprises the important mathematics to be developed, strategies that are helpful, and models that students might construct. Next, teachers translate these goals into children’s activities. This design process is an ongoing reciprocal process of designing and adapting. In the latter stage, the teachers will try to adapt the designed activity based on their hypotheses of
the learning process. However, this re-designed activity asks for new hypotheses about the students’ learning process. An important support for teachers was evident in opening up the important mathematical moments to foster students’ thinking.

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The creation of an emergent local theory starts with the teachers’ description of their knowledge and beliefs about (an aspect of) teaching. Their knowledge and beliefs are their basis for making hypotheses about (individual) students’ actions, thinking, learning about the effect of teachers’ actions, and about classroom social processes. These hypotheses are examined during the classroom experiment. During the retrospective analysis, teachers will have to re-assess their hypotheses – and hence fine-tune their theory. The reflection was based on their observation of learning process in the classroom and their understanding of what happened.

In our approach, hypotheses have three functions. Firstly, in line with Simon’s model, the hypotheses might be a reason to reconsider the designed lesson. It is our intention that teachers and mathematics educators not only have to voice their hypotheses, but will also use those hypotheses to attend to students’ thinking and learning in the designing process. These hypotheses will lead to a reconsideration and, if needed, a refinement of the designed lesson. Secondly, the hypotheses are used during the retrospective analysis. Often teachers have a hard time to reflect on (individual) students’ actions, thinking, and learning and on classroom social processes. It is our assumption that the lack of hypotheses is one reason for this problem. We assume that the hypotheses will be a starting point for teachers to perform a retrospective analysis. Teachers can compare their hypotheses with what happened in class. Over the course of a number of cycles, the teachers will become better in both predicting and analyzing. By then, the hypotheses will also inform teachers about how well the activities in class are going. This is the third function of the hypotheses. If a hypothesis is not met in class, the teacher might have to rethink the activity at that moment. So, the hypotheses can mark important events in the classroom that the teacher will have to pay attention to.
The retrospective analyses of (individual) students’ actions, thinking, learning, and of classroom social processes inform teachers about how they should design the next lesson in the experiment and about possible learning trajectories. So the analysis will probably lead to a cumulative rethinking of the designed activities. The cyclical character of the design research reflects this ongoing process of designing, experimenting, and analyzing.

EXEMPLARS FROM PRACTICE

Paradigmatic examples from three design research mini-cycles held in primary schools will be analyzed. The first two examples were taken from a five-day research workshop in Yogyakarta, whereas the last example was carried out in Padang. The authors were present in the classroom at all times, along with groups of participants of the workshop. Other mathematics teachers and mathematics educators observed the classrooms by taking snapshots of students’ works and important mathematical moments.

In all three cases, the mathematical problems center on division of fractions and whole numbers. Figure 3 illustrates the mathematical problem for discussion with grade 5 students at two primary schools in Yogyakarta, while figure 4 represents the problem discussed with grade 4 students in Padang. In all classroom settings, students worked in small groups of four to five students. This mode of learning was chosen to encourage students to exchange ideas while working on the problem.

A family buys 25 kilograms of rice and eats ¾ of a kilo each day. How many days can 25 kilograms of rice last?

Figure 3. Rice problem as a starting point for discussion

4 groups of kids are going on a field trip and the teacher gives them some chocolate bars to share. In the first group, there are 4 kids and they get 3 chocolate bars together. In group 2, there are 5 kids and they get 4 chocolate bars. Group 3 has 8 kids and they get 7 chocolate bars whereas group 4 has 5 kids who get 3 chocolate bars. Did the teacher distribute the chocolate fairly?

Figure 4. Sharing chocolate bars as a starting point for discussion

In presenting the problems, each of the three teachers modified the problems as their own personal problems to make the problems come alive for students. However, in all three classrooms, students did not pay attention to the contexts in their initial attempts to solve the problems. Instead, many students applied operations with fractions without understanding. For instance, they tried to multiply ¾ by 25, coming up with 75/100 as an answer (figure 5a). Others were stuck multiplying by ¾ and – when asked by the teacher – tried to represent ¾ by three objects on top of four objects (figure 5b). Our observation and analysis suggested that students mainly focused on the numbers and this practice could be attributed to their previous learning experience.

Differences in teachers’ actions in addressing the situations were observed in the three classrooms. Ms Desi re-introduced the problem by emphasizing the context and asked students to represent their interpretation ¾ kilograms of rice in relation to 1 kilogram of rice. She asked a student who had an idea about a representation of the fraction ¾ to share this idea with the rest of the class. Then she asked another student to re-explain what was written on the board as shown in Figure 6a. Mr. Agus wrote the problem on the board and then asked the co-teacher to assist him. A figure to represent a kilogram of rice was introduced to the students during the co-teaching session. This allowed the students to connect with another representation of ¾, as
shown in Figure 6b. Ms. Esti had initially introduced the problem verbally; when she re-introduced the problem, she wrote the information on the blackboard. She emphasizes remembering the data rather than understanding the situation to allow students to use that situation as a context to solve the problem. This did not change the students’ behaviour, as they continued looking at numbers and were not able to make sense of the information. In co-teaching, bars of chocolate were shown and drawn on the board, allowing the students to connect their thinking to concrete operations on chocolate bars and to sharing bars of chocolate among a group of children.

![Figure 5. Initial attempts to solve problems without understanding](image)

During the workshop, all teachers were encouraged to probe into students’ thinking by examining their strategies. Subsequently they were encouraged to come up with possible questions that could be asked in class and that might help the students to grow mathematically. This practice was facilitated by sharing ideas about mathematical questions to teachers during reflection and planning sessions. One of the changes discussed during the workshops was for teachers not to judge students’ answers as right or wrong. Instead, teachers emphasized practicing classroom norms where students share their reasoning publicly, listen and understand each others’ ideas. However, changing from the common practice of classifying students’ solutions as correct or incorrect proved to be challenging for both teachers and students in the classrooms. The nature of questions and when questions were posed, along with the classroom norms, affected the interactions in the classrooms. Our reflections on the teaching indicated that Mr. Agus and Ms. Esti had most difficulties in carrying out such actions. The challenges were particularly noticed during whole class discussions, so our analysis and reflection will draw on data from whole class discussions in the three classes.

For instance, Mr Agus had difficulties in engaging students in an interactive classroom discussion. He started the whole class discussion by having two groups explain their strategies in front of the class. Following the explanations, Mr Agus asked the whole class “Have you understood Tanti’s group solution?” This question did not lead to interaction among students as none raised any question. Even after calling for particular students to ask questions, Mr Agus was not successful in eliciting questions from his students. In a co-teaching session another student was asked to explain in her own words the explanations given by Tanti. The appointed student started to reveal that she did not completely understand by saying, “Mr. Agus, I haven’t completely understood Tanti’s explanation because there were numbers here that I could not see and how these numbers were related to the remaining ¼ kilograms”. Mr Agus then asked Tanti to explain again to the whole class. It was noticeable that only a few students feel comfortable to express their lack of understanding. Hence, we decided to ask students to share their ideas and strategies in small groups instead of having a whole class discussion. This created a classroom atmosphere where students felt more comfortable in asking questions when they did not understand.
Ms. Esti had a bigger challenge because the norm of her class showed that students were very dependent on her as they were constantly asking her to confirm whether they were on the right track. A suggestion was given to Ms. Esti to change this by announcing to all students that she would not interfere while the students were working in groups. This was to allow students more time to think and work independently. The next day, a change in the classroom was observed. Students spent less time waiting for Ms. Esti and started to spend more time to work in their groups. Observations of group discussion revealed children’s lack of mathematical understanding. In sharing bars amongst groups of children, they had no clear understanding of the meaning of a whole and this led to a number of errors. Ms. Esti felt very uncomfortable as she noticed the students making many mistakes. It was harder for her to analyze why the students made these mistakes. This was also noticed during the whole class discussion. Ms. Esti let one group, who found the correct answer share their solution. After the students gave the right answer, but shared an incorrect argument, she wanted to continue with another group. The co-teacher interfered trying to bring the class to a deeper understanding of the mathematics in the shared solution.

Ms. Desi showed that she had practiced inviting students to paraphrase what other groups had done during the whole class discussion. Supported by her co-teacher, Ms. Desi probed into students’ strategies and made the understanding of a particular strategy a whole class affair. One of the groups noticed that 3 kilograms were enough for 4 days and grouped 25 kilograms into groups of 3 kilograms. Each group of three kilograms was marked by a red line. Having the group explain their strategy, Ms. Desi posed a question about the role of the red lines in their work (figure 7a). Because other students could not see them, she let them observe the work closely and then asked them to explain their understanding (figure 7b). In this way, Ms. Desi provided more room for students to think and invited more voices from students during the classroom discourse. This was also evident at the end of the lesson. After listening to students’ discussion about the use of ratio, Ms. Desi did not conclude the lesson with stating the correct answer. Instead she noticed that some students were still puzzled and articulated that to her students by saying:

*I noticed that some of us were still puzzled, was that true? Which one of Johan’s ideas or Andi’s ideas was okay? I would like you to think again for yourselves at home, we will continue the discussion tomorrow. Do you agree?*

**DISCUSSION AND IMPLICATIONS**

In all three cases, the teacher’s practice was in line with the dominant classroom norms. All three teachers were focusing on getting their students to the right answer and defined their role as ‘explainer and instant supporter’. However, as an effect of the discussion during the workshop and the support of the co-teachers, they tried to...
change their practice from telling to facilitating learning. In the three cases described in this paper, implementing this change in the teaching practice depended on teachers’ ability to understand students’ thinking, to pinpoint the mathematical moments in students' work, and to engage students in mathematical classroom discourse.

![Image](7a) [Image](7b)

*Figure 7. Classroom interaction in understanding students’ strategy*

During the workshops, the teachers and mathematics educators showed diversity in understanding investigations, the use of contexts, students' thinking, and in facilitating classroom discourse. These differences reflected teachers’ beliefs, classroom norms, understanding about developing mathematical knowledge. The process of changing the norms in the classroom and changing teachers’ classroom practice takes a long time.

Mathematics educators supported the teachers in this process by pinpointing the mathematical moments during classroom discourse and in getting students to discuss the mathematics. Furthermore, mathematics educators facilitated teachers in the use of contexts. When students use the context as a word problem and not as a starting point to use their common sense, the educator in collaboration with the teacher has to rephrase the context and bring the students back to the context.

Our experience suggest that collaboration among teachers and mathematics educators in going through the process of design, classroom experiments, and retrospective analysis is a powerful platform for enhancing teachers’ capacity in fostering students thinking. Further research is warranted to examine ways to capitalize on contributing factors in developing and enhancing teachers’ capacity.

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**REFERENCES**


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