2011 Mathematics Teachers Conference

- 6th Annual Conference
- Congratulations Professor Berinderjeet Kaur!
Today’s Presentation: Research Questions

• What are instructional tasks?

• Why are tasks important?

• What kind of instructional tasks do teachers ask students to engage in in Singapore?

• What is the intellectual quality of the instructional tasks in Secondary 3 Mathematics that teachers set for their students?

• What might be a possible framework to guide teachers in the design of high quality instructional tasks?
Part One. Instructional Tasks

Part Two. Improving Task Design and Implementation

Part Three. Three Conclusion
Core 2 Design

• Panel 2
  – Surveys (S, T) x 2 Primary 5 and Secondary 3 English and Mathematics (n=64 schools)
  – Assessment x 2

• Panel 3
  – Classroom observation (1 unit of work in each class in each subject in each year) = 620 lessons in 42 schools (M=360 lessons approx)
  – Coded in Excel and Studio Code

• Panel 5
  – Teacher assessment and assignment tasks
  – Student artifacts
PART ONE.

INSTRUCTIONAL TASKS
“It is what teachers get students to do in the class that emerged as the strongest component of the accomplished teachers repertoire, rather than what the teacher, specifically does.” In effect, in learning, what teachers do matters, but what students do matters even more.


“Task analysis is the single most important component process in the instructional design process, whether that process is used to produce direct instruction, performance support, or constructivist learning environments.”

Why Tasks?

1. Principal determinant of the **intellectual quality of knowledge** work in the classroom & student learning

2. In principle, the **key organizing principle** of instructional practices
   - Instructional methods
   - Lesson organization/participation structure
   - Classroom learning environment
   - Resources & technology
   - Classroom talk
Two pedagogies

- Performative: Effective transmission and accurate reproduction of pre-packaged school subject "knowledge"
- Capacity building: Participation in complex knowledge practices in classrooms organized as knowledge building epistemic communities focused on the generation, representation, communication, deliberation, justification and understanding of domain-specific knowledge claims and the development of appropriate cognitive capacities and epistemic virtues, commitments and identities

Instruction Tasks

- Communicating Goals and Performance Standards

Classroom Interaction (incl. Talk)

- Classroom Management
- Classroom Climate
- Classroom Resources
- Instructional Methods
- Lesson Organization

Assessment Tasks

- Student Motivation, Interest & Engagement
- Student Cognitive Capacities and Performances
- Student Beliefs, Dispositions & Identities
- Student Cognition & Metacognition

Learning Goals
1. Task Set Up & Implementation

- Task as Represented in Curriculum Documents
- Task Set Up Affordances
- Task Implementation Affordances
- Task as Set Up
- Task as Implemented
- Student Learning Outcomes

Adapted from Stein et al., 1996
Task Set Up & Implementation

Task as Set Up
*Task Goals
*Epistemic Focus
*Cognitive Demands
*Knowledge Practices & Norms
*Learning Activities
*Text Production

Task as Implemented
*Enactment of Task Features
*Cognitive Processing
*Metacognitive Processing

Student Learning Outcomes

Task Implementation Affordances
* Communication of Learning Goals and Assessment Standards
*Supportive Classroom Learning Climate
*Appropriate Resources
*Task Sequencing
*Dialogical Dispositions & Capacity
*Formative Assessment & Feedback
*Instructional Alignment

Task Set Up Affordances
*CK, PCK, AK, CuK
*Checking for Prior Student Knowledge
*Learning Theory
*Examined Beliefs
*Task Scope

Adapted from Stein et al., 1996

Task as Represented in Curriculum Documents
So – What are Tasks?
Valuable instructional tasks

• are discrete, purposeful, goal-directed **cognitive activities** focused on some form of **worthwhile knowledge work** that has educational and/or institutional value

• are subject to clear **standards** of performance

• impose specific kinds of normative (including epistemic), communicative and behavioral **demands** on students

• provide the **organizing principle** for the selection of activities, resources and learning

• give **meaning and purpose** to the activities, resources and learning chosen to enact the tasks
Conceptual Model of Instructional Tasks

- **Purposefulness**: Identify and Communicate Task Learning Goals in light of Student Prior Knowledge
- **Knowledge Focus**: (Factual, Procedural, Conceptual, Metacognitive, Epistemic)
- **Domain Specific Knowledge Practices, Norms and Epistemic Virtues**
- **Cognitive Demands (Set Up) & Cognitive Processes (Implementation)**
- **Instructional Tasks**
- **Task Outputs**: Text Production (Oral, Written)
- **Learning Activities**
- **Participation Structure, Instructional Methods (incl. Learning Support, & Feedback), Resources, Time & Classroom Interaction**
1.1. Purposefulness: Identify and Communicate Lesson Goals
Conceptual Model of Instructional Tasks

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“The key to making your students’ learning experiences worthwhile [and engaging] is to focus on your planning on major instructional goals, phrased in terms of desired outcomes – the knowledge, skills, attitudes that you want to develop in your students. Goals, not content coverage or learning processes, provide the rationale for curriculum and instruction. All of the elements of ... instructional program – content sources, discussion questions, activities, assignments, and assessment methods – should be included ... as means to accomplish important instructional goals.”

MOE Framework of School Mathematics

- Beliefs
  - Interest
  - Appreciation
  - Confidence
  - Perseverance

- Metacognition
  - Monitoring of one's own thinking
  - Self-regulation of learning

- Attitudes

- Concepts
  - Numerical
  - Algebraic
  - Geometrical
  - Statistical
  - Probabilistic
  - Analytical

- Skills
  - Numerical calculation
  - Algebraic manipulation
  - Spatial visualisation
  - Data analysis
  - Measurement
  - Use of mathematical tools
  - Estimation

- Processes
  - Reasoning, communication and connections
  - Thinking skills and heuristics
  - Applications and modelling

- Problem Solving

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Mathematical processes refer to the knowledge skills (or process skills) involved in the process of acquiring and applying mathematical knowledge. This includes reasoning, communication and connections, thinking skills and heuristics, and application and modeling.

- **Mathematical reasoning** refers to the ability to analyze mathematical situations and construct logical arguments. It is a habit of mind that can be developed through the application of mathematics in different situations and contexts.

- **Communication** refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their own understanding of mathematics and sharpen their mathematical thinking.

- **Connections** refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life. This helps students make sense of what they learn in mathematics.

Mathematical reasoning, communication and connections should pervade all levels of mathematics learning, from primary levels to the advanced-levels.
Learning Goals: Core 2 Research Focus

1. Content specific knowledge

2. Knowledge Focus*
   • Factual
   • Conceptual: Meaning-making
   • Procedural
   • Metacognitive
   • Epistemic

3. Skills
   • Mathematics-specific Skills
   • Metacognitive Skills
   • Strategic Skills
   • Process Skills

4. Epistemic Beliefs and Virtues*

5. Domain-Specific Knowledge Practices*
   • Knowledge Communication: Syntax
   • Knowledge Representation
   • Knowledge Generation
   • Knowledge Deliberation
   • Knowledge Validation/Justification
   • Knowledge Communication: Presentation
Core 2 Pentagon

- Epistemic Beliefs & Virtues
- Knowledge Focus
- Mathematical Problem Solving
- Knowledge Practices & Epistemic Norms
- Skills
- Content Specific Knowledge
Likert Scale Values

1. Very Poor / Never / Strongly Disagree
2. Poor / Only very occasionally / Disagree
3. OK / Sometimes / Neither Agree or Disagree
4. **Good / Often / Agree**
5. Very good / All the time / Strongly Agree
## Visible Learning: Communicating Learning Goals

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Secondary 3 Mathematics</th>
<th>Secondary 3 English</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communicating Learning Goals</td>
<td>.88, .91</td>
<td>3.55</td>
<td>0.84</td>
</tr>
<tr>
<td>The teacher tells us the learning objectives of the lesson.</td>
<td></td>
<td>3.60</td>
<td>0.98</td>
</tr>
<tr>
<td>The teacher explains to us the learning objectives of the lesson in detail.</td>
<td></td>
<td>3.53</td>
<td>0.98</td>
</tr>
<tr>
<td>The teacher links the objectives of the lesson to previous lessons.</td>
<td></td>
<td>3.47</td>
<td>0.98</td>
</tr>
<tr>
<td>The teacher helps me understand what I am supposed to learn at the beginning of the lesson.</td>
<td></td>
<td>3.61</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Visible Teaching and Learning

<table>
<thead>
<tr>
<th>Visible Teaching</th>
<th>Visible Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning goals</td>
<td>Student prior knowledge</td>
</tr>
<tr>
<td>Performance standards</td>
<td>Monitoring of student learning</td>
</tr>
<tr>
<td>Exemplars of successful performance</td>
<td>S-T feedback: Formative Evaluation</td>
</tr>
<tr>
<td>T-S feedback</td>
<td>Metacognitive wisdom/self regulation</td>
</tr>
<tr>
<td>Public reasoning: “understanding talk”</td>
<td>Public reasoning: “understanding talk”</td>
</tr>
</tbody>
</table>

The remarkable feature of the evidence is that the biggest effects on student learning occur when teachers become learners of their own teaching, and when students become their own teachers. ... That is, it is visible teaching and learning by students that makes the difference.”

1.2. Identify and Communicate Learning Objectives and Performance Standards
Conceptual Model of Instructional Tasks

- Purposefulness: Identify and Communicate Task Learning Goals in light of Student Prior Knowledge
- Knowledge Focus (Factual, Procedural, Conceptual, Metacognitive, Epistemic)
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- Cognitive Demands (Set Up) & Cognitive Processes (Implementation)
- Instructional Tasks
- Task Outputs: Text Production (Oral, Written)
- Learning Activities
- Participation Structure, Instructional Methods (incl. Learning Support, & Feedback), Resources, Time & Classroom Interaction
## Visible Learning: Communicating Assessment Standards

<table>
<thead>
<tr>
<th>Communicating Assessment Criteria</th>
<th>Secondary 3 Mathematics</th>
<th>Secondary 3 English</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (1-5)</td>
<td>SD</td>
<td>Mean (1-5)</td>
</tr>
<tr>
<td>The teacher explains the standard of good work</td>
<td>3.64</td>
<td>.83</td>
<td>3.61</td>
</tr>
<tr>
<td>The teacher explains the standard of good performance in our tests and exams.</td>
<td>3.67</td>
<td>.98</td>
<td>3.63</td>
</tr>
<tr>
<td>The teacher explains clearly what a good solution to a problem is.</td>
<td>3.72</td>
<td>.97</td>
<td>3.53</td>
</tr>
<tr>
<td>The teacher shows us examples of good work.</td>
<td>3.53</td>
<td>1.02</td>
<td>3.62</td>
</tr>
</tbody>
</table>
1.3. Knowledge Focus
Conceptual Model of Instructional Tasks

- **Purposefulness:** Identify and Communicate Task Learning Goals in light of Student Prior Knowledge
- **Knowledge Focus:** (Factual, Procedural, Conceptual, Metacognitive, Epistemic)
- **Domain Specific Knowledge Practices, Task Norms and Epistemic Virtues**
- **Cognitive Demands:** (Set Up) & Cognitive Processes (Implementation)
- **Instructional Tasks**
- **Task Outputs:** Text Production (Oral, Written)
- **Learning Activities**
- **Participation Structure, Instructional Methods (incl. Learning Support, & Feedback), Resources, Time & Classroom Interaction**
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics</strong></td>
</tr>
<tr>
<td>Metacognitive Knowledge</td>
</tr>
<tr>
<td>Procedural Knowledge</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
</tr>
<tr>
<td>Epistemic Knowledge</td>
</tr>
</tbody>
</table>
1.4. Tasks: Cognitive Demand
Conceptual Model of Instructional Tasks

- **Purposefulness:** Identify and Communicate Task Learning Goals in light of Student Prior Knowledge
- **Knowledge Focus:** (Factual, Procedural, Conceptual, Metacognitive, Epistemic)
- **Domain Specific Knowledge Practices, Task Norms and Epistemic Virtues**
- **Instructional Tasks**
  - **Cognitive Demands (Set Up) & Cognitive Processes (Implementation)**
  - **Learning Activities**
  - **Task Outputs:** Text Production (Oral, Written)
  - **Participation Structure, Instructional Methods (incl. Learning Support, & Feedback), Resources, Time & Classroom Interaction**
# Instructional Tasks: Cognitive Demand

<table>
<thead>
<tr>
<th>Task</th>
<th>Mean (1-5)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>To remember [memorize] formulae or rules</td>
<td>4.03</td>
<td>.642</td>
</tr>
<tr>
<td>To practise what you have learnt</td>
<td>3.92</td>
<td>.908</td>
</tr>
<tr>
<td>To remember or recall information you have learnt in a previous lesson</td>
<td>3.66</td>
<td>.920</td>
</tr>
<tr>
<td>To review what you have learnt</td>
<td>3.55</td>
<td>.900</td>
</tr>
<tr>
<td>To check the correctness of a solution to a problem</td>
<td>3.54</td>
<td>.909</td>
</tr>
<tr>
<td>To apply what you have learnt to a new problem or situation</td>
<td>3.53</td>
<td>.890</td>
</tr>
<tr>
<td>To make the meaning clear</td>
<td>3.50</td>
<td>.930</td>
</tr>
<tr>
<td>To understand a word problem, graph or table</td>
<td>3.47</td>
<td>.890</td>
</tr>
<tr>
<td>To explain something</td>
<td>3.46</td>
<td>.928</td>
</tr>
<tr>
<td>To analyze information</td>
<td>3.41</td>
<td>.910</td>
</tr>
<tr>
<td>To discuss a problem with one or more students</td>
<td>3.39</td>
<td>.957</td>
</tr>
<tr>
<td>To compare solutions to a problem</td>
<td>3.37</td>
<td>.891</td>
</tr>
<tr>
<td>To summarize information you have learnt or gathered</td>
<td>3.35</td>
<td>.938</td>
</tr>
<tr>
<td>To work out a new solution to a problem</td>
<td>3.35</td>
<td>.895</td>
</tr>
</tbody>
</table>
## Cognitive Demand Instructional Tasks - 2

<table>
<thead>
<tr>
<th>Task</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>To <em>give reasons</em> for why a guess or a solution that someone has made in class is correct</td>
<td>3.34</td>
<td>.937</td>
</tr>
<tr>
<td>To <em>give an example</em> of a mathematical idea (e.g., Give an example of a four-sided figure)</td>
<td>3.33</td>
<td>.940</td>
</tr>
<tr>
<td>To <em>explain</em> the difference between two ideas (e.g., Area and Volume)</td>
<td>3.32</td>
<td>.940</td>
</tr>
<tr>
<td>To <em>classify</em> problems you have learnt</td>
<td>3.32</td>
<td>.939</td>
</tr>
<tr>
<td>To <em>make a connection</em> between what you have learnt and something else (e.g., finding area of a composite shape using regular shapes)</td>
<td>3.31</td>
<td>.937</td>
</tr>
<tr>
<td>To <em>investigate</em> a problem</td>
<td>3.28</td>
<td>.925</td>
</tr>
<tr>
<td>To <em>represent</em> or state a problem in a different way (e.g., drawing models, graphs or tables)</td>
<td>3.25</td>
<td>.912</td>
</tr>
<tr>
<td>To <em>represent</em> something differently</td>
<td>3.18</td>
<td>.903</td>
</tr>
<tr>
<td>To <em>write the solution</em> to a problem and explain it to your classmates</td>
<td>3.15</td>
<td>1.006</td>
</tr>
<tr>
<td>To <em>find out new information</em> from the textbook, the library, the internet or some other source</td>
<td>2.97</td>
<td>.996</td>
</tr>
</tbody>
</table>
Confirmatory Factor Analysis: Cognitive Demand of Mathematical Tasks

Goodness-of-fit statistics:

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square / df / p-value</td>
<td>274.837 / 118 / .000</td>
</tr>
<tr>
<td>CFI / TLI</td>
<td>.982 / .980</td>
</tr>
<tr>
<td>RMSEA (90% C.I.)</td>
<td>.034 (.029-.040)</td>
</tr>
<tr>
<td>SRMR</td>
<td>.022</td>
</tr>
</tbody>
</table>

Cognitive Demand (Mathematics)

Functional Cognitive Demands

Complex Cognitive Demands

Cognitive Demand

(Mathematics)
## Descriptive Statistics: Cognitive Demands, Secondary 3 Mathematics and English

<table>
<thead>
<tr>
<th></th>
<th>Mean (1-5)</th>
<th>SD</th>
<th>Corr (r)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functional Cognition</td>
<td>3.81</td>
<td>.643</td>
<td></td>
</tr>
<tr>
<td>Complex Cognition</td>
<td>3.38</td>
<td>.672</td>
<td>.691</td>
</tr>
<tr>
<td><strong>English</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functional Cognition</td>
<td>3.43</td>
<td>.748</td>
<td></td>
</tr>
<tr>
<td>Complex Cognition</td>
<td>3.34</td>
<td>.657</td>
<td>.340</td>
</tr>
</tbody>
</table>
1.5. Knowledge Practices
Conceptual Model of Instructional Tasks

Purposefulness: Identify and Communicate Task Learning Goals in light of Student Prior Knowledge

Knowledge Focus (Factual, Procedural, Conceptual, Metacognitive, Epistemic)

Domain Specific Knowledge Practices, Task Norms and Epistemic Virtues

Instructional Tasks

Cognitive Demands (Set Up) & Cognitive Processes (Implementation)

Task Outputs: Text Production (Oral, Written)

Learning Activities

Participation Structure, Instructional Methods (incl. Learning Support, & Feedback), Resources, Time & Classroom Interaction
Domain Specific Knowledge Practices...
Knowledge Practices
“Mathematical practices involve more than what is normally thought of as mathematical knowledge. This area focuses on the mathematical know-how, beyond content knowledge, that constitutes expertise in learning and using mathematics. The term “practices” refers to the specific things that successful mathematics learners and users do. Justifying claims, using symbolic notation efficiently, defining terms precisely, and making generalizations are examples of mathematical practices.”

Core 2 Taxonomy of Mathematics Knowledge Tasks.

- **Investigating** knowledge claims
- **Generating** knowledge claims (inquiry, interpretation, reading)
- **Representing** knowledge claims
- **Communicating** knowledge claims (including expression)
- **Deliberating** knowledge claims
- **Justifying/validating** knowledge claims
- **Applying** knowledge claims to new problems or contexts
## Knowledge Tasks: Mathematics

<table>
<thead>
<tr>
<th>Secondary 3 Mathematics</th>
<th>Mean (1-5)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Generic Disciplinary Tasks</strong></td>
<td>3.35</td>
<td>.665</td>
</tr>
<tr>
<td>Knowledge Activation and Consolidation Tasks</td>
<td>3.62</td>
<td>.716</td>
</tr>
<tr>
<td>Analytical Tasks</td>
<td>3.30</td>
<td>.782</td>
</tr>
<tr>
<td>Application Tasks</td>
<td>3.13</td>
<td>.650</td>
</tr>
<tr>
<td><strong>Specific Disciplinary Tasks</strong></td>
<td>3.11</td>
<td>.611</td>
</tr>
<tr>
<td>Knowledge Communication Tasks (Syntax)</td>
<td>3.50</td>
<td>.725</td>
</tr>
<tr>
<td>Knowledge Validation Tasks</td>
<td>3.15</td>
<td>.852</td>
</tr>
<tr>
<td>Knowledge Deliberation Tasks</td>
<td>3.13</td>
<td>.845</td>
</tr>
<tr>
<td>Knowledge Representation Tasks</td>
<td>3.11</td>
<td>.762</td>
</tr>
<tr>
<td>Knowledge Generation Tasks</td>
<td>3.09</td>
<td>.760</td>
</tr>
<tr>
<td>Knowledge Communication Tasks (Presentation)</td>
<td>2.67</td>
<td>.890</td>
</tr>
</tbody>
</table>
### Disciplinary Practices: Knowledge Communication: Syntax

<table>
<thead>
<tr>
<th>Description</th>
<th>Score</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Knowledge Communication (Syntax) (a=.770, EV=68.5%)</td>
<td>3.50</td>
<td>.725</td>
</tr>
<tr>
<td>To use the correct symbols at the proper places (e.g., equal signs, letters to represent the unknown)</td>
<td>3.82</td>
<td>.974</td>
</tr>
<tr>
<td>To write correct mathematical sentences</td>
<td>3.58</td>
<td>.983</td>
</tr>
<tr>
<td>To present a solution to a problem in a logical step-by-step manner</td>
<td>3.41</td>
<td>1.054</td>
</tr>
<tr>
<td>The teacher asks us to use mathematical vocabulary in our class.</td>
<td>2.98</td>
<td>.982</td>
</tr>
</tbody>
</table>
## Disciplinary Practices: Knowledge Representation

<table>
<thead>
<tr>
<th>2. Knowledge Representation Tasks (a=.913; EV=65.8%)</th>
<th>3.11</th>
<th>.762</th>
</tr>
</thead>
<tbody>
<tr>
<td>To change from one way of representing an idea to another so that you understand the idea better</td>
<td>3.20</td>
<td>.950</td>
</tr>
<tr>
<td>To represent an idea in different ways (e.g., using graphs, tables or algebraic symbols)</td>
<td>3.17</td>
<td>.968</td>
</tr>
<tr>
<td>To use different representations (for e.g., symbols, tables, diagrams, graphs) to highlight differences or similarities between mathematical ideas</td>
<td>3.17</td>
<td>.980</td>
</tr>
<tr>
<td>To compare different types of representations that can be used in solving a problem</td>
<td>3.17</td>
<td>.953</td>
</tr>
<tr>
<td>To use a mathematical representation of an idea you have learned in a new context or situation</td>
<td>3.15</td>
<td>.937</td>
</tr>
<tr>
<td>To represent a real world problem as a mathematical problem</td>
<td>3.02</td>
<td>1.029</td>
</tr>
<tr>
<td>To create your own way of representing (e.g., make your own diagram or table) a mathematical idea</td>
<td>2.93</td>
<td>.982</td>
</tr>
</tbody>
</table>
# Knowledge Representation

<table>
<thead>
<tr>
<th>Types of Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
</tr>
<tr>
<td>Word – Spoken</td>
</tr>
<tr>
<td>Word – Written</td>
</tr>
<tr>
<td>Numerical – Spoken</td>
</tr>
<tr>
<td>Numerical – Written</td>
</tr>
<tr>
<td>Pictorial</td>
</tr>
<tr>
<td>Schematic</td>
</tr>
<tr>
<td>Graphical</td>
</tr>
<tr>
<td>Symbolic – Written</td>
</tr>
<tr>
<td>Symbolic – Spoken</td>
</tr>
<tr>
<td>Story – Spoken</td>
</tr>
<tr>
<td>Story – Written</td>
</tr>
</tbody>
</table>
Path Model of Disciplinary Knowledge Practices: Secondary 3 Mathematics

**Goodness-of-fit statistics:**

- Chi-Square / df / p-value: 426.402 / 160 / .000
- CFI / TLI: .981 / .977
- RMSEA (90% C.I.): .035 (.031-.039)
- SRMR: .025
Path Model of Disciplinary Knowledge Practices: Secondary 3 Mathematics

Knowledge Representation

Knowledge Deliberation

Knowledge Generation

Knowledge Communication: Presentation

Knowledge Communication: Syntax

Knowledge Validation

Goodness-of-fit statistics:

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<tr>
<th>Statistic</th>
<th>Value</th>
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<tbody>
<tr>
<td>Chi-Square / df</td>
<td>426.402 / 160</td>
</tr>
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<td>p-value</td>
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T: How do you represent the half?
S: You have to cut
T: I break into two. How do I show my half here?
S: Shade one part
Knowledge Practice in Action

\[
\begin{align*}
\frac{1}{2} & \div 3 \\
2 \frac{1}{3} & \\
\text{Total (apple)} & \rightarrow 6 \text{ units} \\
\text{Each person} & \rightarrow 1 \text{ unit} \\
\text{Fraction} & \rightarrow \frac{1}{6}
\end{align*}
\]
In short:

- Mathematics knowledge construction via disciplinary knowledge practices
- Foundation in knowledge of syntax but knowledge of representation key gateway to complex cognition and understanding
- Representation of the problem facilitates complex cognition and understanding
- Knowledge representation generates possible ways to solve a problem OR it also allows students to deliberate on the problem, for example, by guessing or making hypotheses, then checking to see if it makes sense or is possible
- Finally, pathway finishes with some form of public presentation and/or validation

Note that this set of knowledge practices is highly consistent with Polya’s heuristics: understand the problem, devise a plan, implement the plan and examine what was done/learned/stronger propositions
Taxonomy of Tasks: Examples
Conceptual Model of Instructional Tasks

**Purposefulness:** Identify and Communicate Task Learning Goals in light of Student Prior Knowledge

**Knowledge Focus:** (Factual, Procedural, Conceptual, Metacognitive, Epistemic)

**Domain Specific Knowledge Practices, Task Norms and Epistemic Virtues**

**Cognitive Demands (Set Up) & Cognitive Processes (Implementation)**

**Instructional Tasks**

**Identify and Communicate Learning Objectives and Performance Standards**

**Task Outputs:** Text Production (Oral, Written)

**Learning Activities**

**Participation Structure, Instructional Methods (incl. Learning Support, & Feedback), Resources, Time & Classroom Interaction**
Glossary: Taxonomy of Learning Tasks

• Non-mathematical tasks
• Recall / remembering tasks
• Repetition (Drilling & Memorization) tasks
• Review and Revision Tasks
• Routine Procedural tasks
• Understanding tasks
  o Knowledge Manipulation
  o Procedural tasks with connections (including Application)
• Doing Mathematics
## Task Example

<table>
<thead>
<tr>
<th>Problem</th>
<th>Goals</th>
<th>Knowledge Focus</th>
<th>Cognitive Demand</th>
<th>Learning Task</th>
<th>Disciplinary Knowledge Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is $3 \times 5$?</td>
<td>Automaticity</td>
<td>Procedural Knowledge</td>
<td>Remember (memorize)</td>
<td>Memorization</td>
<td>Syntax</td>
</tr>
<tr>
<td>Mr Jones’ bedroom is 7m long and 6m wide. What is the area of bedroom?</td>
<td>Apply learned procedure (calculation of area, given dimensions of area)</td>
<td>Procedural Knowledge</td>
<td>Recall information/practice</td>
<td>Procedure without connection</td>
<td>Syntax, Representation</td>
</tr>
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<td>Learning Task</td>
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<td>------------------</td>
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<td>---------------------------------</td>
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<tr>
<td>Miss Lim wants to build a rectangular hen pen for a science fair in school. She has 24m of fencing material. If she wants the hens to have as much room as possible, how long would each of the sides of the pen be? How long would each of the sides of the pen be if she had only 16m of fencing? How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it and be convinced by your solution and arguments.</td>
<td>Calculate area and perimeter, Apply problem solving strategies to explore relationship between area or perimeter of rectangles, Explain the work done (mathematical ideas and solutions) in a clear manner</td>
<td>Procedural Knowledge, Conceptual Knowledge, Epistemic Knowledge</td>
<td>Understand, Investigate, Represent a problem, Analyze information, Write out solution and explain the solution, Generalize</td>
<td>Procedures with connections, Doing Mathematics</td>
<td>Syntax, Representation, Generation, Deliberation, Presentation, Validation/Justification</td>
</tr>
</tbody>
</table>


Knowledge Building
PART TWO

IMPROVING MATHEMATICS TEACHING AND LEARNING
Task as Represented in Curriculum Documents

Task Set Up Affordances
- CK, PCK, AK, CuK
- Checking for Prior Student Knowledge
- Learning Theory
- Examined Beliefs
- Task Scope

Task as Set Up
- Task Goals
- Epistemic Focus
- Cognitive Demands
- Knowledge Practices & Norms
- Learning Activities
- Text Production

Task as Implemented
- Enactment of Task Features
  - Cognitive Processing
  - Metacognitive Processing

Task Implementation Affordances
- Communication of Learning Goals and Assessment Standards
- Supportive Classroom Learning Climate
  - Appropriate Resources
  - Task Sequencing
- Dialogical Dispositions & Capacity
- Formative Assessment & Feedback
  - Instructional Alignment

Student Learning Outcomes

Adapted from Stein et al., 1996
Improving Quality of Task Design: Set Up

- Developed understanding of content knowledge and PCK
- Clear, challenging, transparent learning goals and specific objectives and performance standards
- Clear understanding of prior student knowledge
- Clearly articulated learning theory and cognitive principles to inform instructional tasks
- Clear domain-specific knowledge (epistemic) focus
- Developmentally appropriate domain-specific knowledge practices high in disciplinariness
- Appropriate mix of routine procedural and complex knowledge production tasks informed by authentic real world problems and disciplinary standards
- Strong focus on meaning making/making connections and knowledge representation
- Clear expectations of desired text production / outputs
Factors Promoting Low Fidelity of Task Implementation

- Lack of clarity about **learning goals**
- Inadequate **communication** of learning goals and assessment standards
- **Inappropriateness of task**, given prior student understanding (outside ZPD): misalignment of task cognitive demands and prior knowledge
- **Classroom management** problems
- Too little – or too much – **time**
- Lack of **accountability** for work done: need for summative assessment
- **Watering down** of task complexity to low ambiguity / low risk tasks
- Inappropriate **sequencing** of learning activities, problems, tasks
- Focus shifts to **correct answer** rather than working through problem
- **Excessive scaffolding**: reduces ambiguity and challenge
- **Too little monitoring** of student work and detailed feedback/formative assessment
- Low **mastery norms** / low **metacognitive** monitoring
- **Misalignment** of instructional practices (methods, lesson organization, talk) with task demands
- Inappropriate or inadequate **resources** and technology applications
Example

- Time taken moving from problem posing to giving the first clue (use similar triangles): less than a minute
- Time taken in giving the biggest clue (drawing of the similar triangles at the bottom right hand corner of the second diagram): 3 minutes
- Goals of the task not articulated, therefore the challenging goal is not achieved (unproductive success?) Goal of teaching how to solve such problems achieved.
Factors Promoting High Fidelity of Task Implementation

- Clarity about learning goals
- Communication of learning goals and assessment standards
- Appropriateness of task, given prior student understanding (outside ZPD): misalignment of task cognitive demands and prior knowledge
- Well managed classroom
- Neither too little – or too much -- time
- Accountability for work done: need for summative assessment
- Maintaining task complexity/keeps focus on working through problem
- Appropriate sequencing of learning activities, problems, tasks
- Productive scaffolding: neither too little or too much
- Constant monitoring of student work and detailed feedback/formative assessment
- Mastery norms
- Metacognitive monitoring and reflection
- Alignment of instructional practices (methods, lesson organization, talk) and task demands
- Student self monitoring
- Appropriate and adequate resources and technology applications
An Example of High Fidelity Task Implementation

Fencing task again

• Both teacher and students were aware of the (challenging) goals of the tasks as set up, and maintained high fidelity to the cognitive demands of the task setup at the implementation stage

• While teacher gives hints, T does not show students exactly how to go about solving the problem. After 1 period of student discussion, none of the groups have completed the task but several have begun systematically laying out different pen configurations for the same amount of fencing.

• Several groups are on their way to discover that a square would enclose the greatest amount of area for any given amount of fencing.
PART FIVE

CONCLUSION
Core 2 Pentagon

- Epistemic Beliefs & Virtues
- Epistemic Focus
- Knowledge Practices & Epistemic Norms
- Skills
- Content
  - Specific Knowledge
  - Mathematical Problem Solving
Two pedagogies

- Performative: Effective transmission and accurate reproduction of pre-packaged school subject "knowledge"
- Capacity building: Participation in complex knowledge practices in classrooms organized as knowledge building epistemic communities focused on the generation, representation, communication, deliberation, justification and understanding of domain-specific knowledge claims and the development of appropriate cognitive capacities and epistemic virtues, commitments and identities

Framing Instruction: An Anatomy of Teaching and Learning

- Instruction and Assessment Tasks
- Goal Setting / Lesson Planning
- Communicating Goals and Standards
- Classroom Interaction (incl. Talk)

Teacher Kn, Skills and Beliefs: CK, PCK, CuK EK & Learning Theory

Lesson Organization

Instructional Activities

Classroom Resources

Classroom Climate

Classroom Management

Student & Family Characteristics, Practices & Projects

Student Motivation, Interest & Engagement

Student Beliefs, Dispositions & Identities

Student Cognition & Metacognition

Student Cognitive Capacities and Performances
Conceptual Model of Instructional Tasks

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Adapted from Stein et al., 1996
A Framework to Guide Task Design

1. Identify Learning Goals in light of student prior knowledge
2. Identify Content Knowledge
3. Identify Knowledge Focus
4. Identify Skills
5. Identify Disciplinary Knowledge Practices and Epistemic Norms
6. Identify Epistemic Beliefs & Virtues
7. Select Mathematical Problem(s)
8. Learning Objectives & Standards of Performance
9. Cognitive Demands
10. Task Complexity
11. Learning Activities, Instructional Methods, Lesson Organization, Sequencing, Resources
12. Learning Support
13. Outputs / Text Production
14. Assessed Outcomes in light of #1 & 8
Thank you

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