At first glance, the title might look absurd, as most people believe the calculator cannot be used to develop mental arithmetic skills. Indeed, most would argue that encouraging the use of calculators at the Primary school level might lead to a dependence on the calculator and a reduction of mental arithmetic and basic computational skills. In this paper, I will show that, contrary to popular opinion, mental arithmetic skills at the Primary level can actually be developed and enhanced by using the calculator. But before that, let me outline a number of advantages in using the calculator.

**Advantages of using a calculator**

First of all, it is motivating. Most children like to press the keys on the calculator and see the resulting display. It is also a hands-on tool, where the numbers are displayed without having to laboriously write them. Thirdly, pupils can generate a series of numbers in a short time. (Subsequently, teachers can record these numbers on the board and guide pupils to discover patterns.) Fourthly, problems involving computations with complicated, real-life numbers can be attended to, without having to sacrifice realism for easy-to-compute numbers. Finally, pupils will tend to ask questions and explore topics such as decimal fractions and negative numbers, without being asked to or even before being taught those topics formally.

**Mental arithmetic**

Like the ubiquitous use of worksheets, mental arithmetic is part of the staple diet of Primary pupils in Singapore. While teachers may be using mental arithmetic sessions effectively, very few use the
calculator to develop mental arithmetic skills. Hence, in this paper, I would like to offer some suggestions on how to use the calculator to develop such skills. I will first clarify what I mean by mental arithmetic, and contrast it with mental drill, before going on to describe some calculator activities.

Mental arithmetic is the solving of arithmetic problems/sums mentally, without recourse to paper and pencil or other manipulatives such as fingers or the calculator. Mental drill is the repetitive practice of number bonds and the like under strictly controlled, short periods of time. While the purpose of arithmetic drill is to get fast, accurate answers from memory, the purpose of mental arithmetic is to get answers, preferably speedily, but more importantly, to get answers that consciously use mathematical links. While drill emphasizes the product (answer), mental arithmetic emphasizes both the process and the product, as not only has the answer to be given but an explanation of how the answer (product) was arrived at, has to be given as well. According to Sowder and Kelin (1993), mental arithmetic skills enhance number sense, and although they list three types of mental arithmetic activities, in this paper I focus on only the skill of mental computation.

Specifically, I will describe seven calculator activities that can develop such mental computation skills. The activities are:

1. adding and subtracting tens.
2. adding and subtracting nines.
3. building the five and ten times tables.
4. building the two times table.
5. multiplying and dividing by tens.
6. subtracting by keeping the difference constant.
7. the "look before you leap" activity

Mental computation

By mental computation, I mean computing, mentally but accurately, answers to numerical problems. That is, there are three criteria for a mental computation activity, namely mental (as opposed
to written), accuracy (as opposed to estimation) and numerical problems (as opposed to word problems). Some examples are, to mentally compute $32 + 9$, $32 - 11$, $83 - 47$, $5000 - 2348$, and $324 \times 10$.

What are the advantages of mental computation over written computation? Researchers (e.g. Plunkett, 1979; and Sowder & Kelin, 1993) state that while written algorithms are efficient, they seldom correspond to the way people think, and are inappropriate for mental computation. As well, mental algorithms are specific to the problem at hand, and require a good understanding of numbers and relations between numbers. In other words, correct mental computation is predominantly conceptually-based, while correct written computation might just be a manifestation of procedurally-based, memorised and little-understood algorithms. Researchers (e.g. Hope & Sherrill, 1987; and Markowits & Sowder, 1988) also state that skilled mental computers use a variety of strategies quite different from the written algorithms learned. For example, they generally work from left to right and use number properties and relations which indicate an understanding of the structure of the number system.

**Calculator activities**

*Adding/subtracting tens*

*Adding tens* is a calculator activity well-suited to Primary 1 children. The teacher asks the students to input a two-digit number, say 21. Then the students are to input $+10$, and state the display on pressing the $=$ sign. (It would be good to have enough calculators, at least one calculator per pair of students). Then the students are asked to repeat the process a number of times. Each time the students read out the number displayed on the calculator, the teacher writes the number on the board, and soon the following series of numbers would have been written on the board by the teacher: 21, 31, 41, 51, 61, 71, 81, 91. The teacher should use a different colour to highlight the tens digits and guide the students to see an emerging pattern of increasing numbers in the tens place, while the ones digits remain unchanged.
The teacher could then lead a discussion as to why such a pattern occurs and guide the students to generalise (by generating many different series of two-digit numbers resulting from adding ten successively) that each addition of a ten increases the tens digit by a ten, leaving the ones digit constant. In reality, even when generating one series of two-digit numbers by adding tens, the students will start shouting out the answers before inputting numbers in the calculator, as they would have an intuitive feel for the number pattern generated. This will be especially true if the teacher has kept track of the numbers generated by writing them on the board. In other words, students can mentally add ten to a two-digit number because of the pattern of increasing digits in the tens place. The discussion will further reinforce the idea of adding tens, by focussing on the "why" of it, thereby enhancing understanding.

This is a far cry from writing, say, 21 + 10 in the usual vertical form and saying that 1 + 0 = 1, 2 + 1 = 3 etc., where the 21 and 10 can be perceived as isolated pairs of one digit number to be added. That is, understanding place value is not a prerequisite in the column addition of such two digit numbers, as the addition could be successfully carried out by thinking of 21 as 2 and 1, the 10 as 1 and 0, and so 2 + 1 is 3 (and not necessarily 3 tens). This adding tens activity could then be extended to three and four digit numbers, such as 524 + 10 and 2635 + 10, even without a formal or extended exposure to three and four digit numbers (for example, the teacher could read and at the same time, write, “five hundred twenty four add ten,” and ask the students to input the numbers etc.) A further extension would be to subtract ten, using the calculator, until students can mentally compute sums such as 93 - 10.

Adding/subtracting nines

Adding nines and adding Elevens are activities related to adding tens, but since they are similar, I will describe only the former. The activity can be carried out as follows: the teacher writes on the board, say, 22 + 10 (in horizontal form), and writes down the answer (32) given by the students. Then students compute 22 + 9, using the calculator. The answer (31) is written next to the answer (32) to 22 + 10. Repeating this process, the following will be generated:

\[ 22 + 10 = 32, \quad 22 + 9 = 31; \quad 32 + 10 = 42, \quad 32 + 9 = 41. \]
A well-guided discussion based on the pattern of answers should then lead to the generalisation that to add nine, one has to add ten and then "go back" (subtract) one. For example, 32 + 9 would be computed mentally, beginning by subvocalising "42, 41," and finally verbalising "41" as the answer. The students would not have to rely on the calculator to get an answer, once the pattern is seen and understood. Indeed, after some time, the students would not use the calculator to compute such addition, as they realise that the answer could be obtained much faster using mental computation than by spending time "pushing buttons" on the calculator.

A similar activity could be used for subtracting nines, but it must be cautioned that this is conceptually harder, as the result is one more than subtracting ten (e.g. 32 - 9 is "22, 23" that is, 23), and students might find it difficult, initially, to reconcile the adding of one to the operation of subtraction, as they expect a smaller number. In the case of adding nine, students seem to find it easier to understand that as nine is less than ten, the answer is one less than when ten is added.

Multiplying

Activities similar to the ones described could be done for multiplication as well. I will describe three activities, one for the "five times" table, one for the "two-times" table and another for mentally computing the product of a number and ten.

Five/ten times tables

Because the ten times table is easier and can be similarly explored, I describe only the building of the five times table. Students can begin by entering 5, followed by repeating +5, and calling out the number displayed, to be recorded on the board by the teacher. A look at the pattern of numbers generated (the alternate five and zeroes in the ones place), together with a discussion (led by questions such as "how many fives have you added?") showing the relationship between the repeated addition of five and the five times table can lead students to easily recall the five times table. Moreover, students can also be guided to see that, for example, eight fives gives the same result as five eights, and that too should help students
compute mentally the product of, say, five nines, by relating it to nine fives as one five more than eight fives. (An additional advantage is that the five times table can then be related to the counting of five-minute intervals on the clock). It should be noted that the patterns in the 5 and 10 times table make them much easier to learn, and there is no need to always start with the two times table as is usually done. Additionally, once multiplication is seen as repeated addition with the 5 and 10 times table, students can build up their own time table for the other numbers, using the calculator and recording the results (especially interesting are the patterns for the 9 and 11 times tables), and be able to recall any specific multiplication fact by using mental computation that links multiplication, addition and commutativity.

Two times table

The two times table can be built up similarly by starting with two, repeatedly entering +2 on the calculator, and recording the displays. The series of numbers generated will give the “two-times” table and a comparison of the numbers generated by $1 \times 2$, $2 \times 2$, $3 \times 2$, $4 \times 2$ etc., together with a discussion on why the answers are the same would enable students to see that multiplication is repeated addition. Moreover, by realising that the product of 2 and 6 is just 2 more than the product of 2 and 5, students can mentally compute six twos as just two more than five twos, where the latter is a basic fact they already know. Hence they can compute mentally by building up on prior knowledge.

Product/Quotient of a number and ten

All too often, students are just told that “when you multiply by 10, just add 0.” Strictly speaking, adding zero should leave the number unchanged (e.g. 24 added to 0 gives 24), but even if the rule were given with the word “annex” substituting “add,” it still remains a (teacher-given) rule. But when students use the calculator to explore the results of multiplying 10 by whole numbers like 8, 18, 23, 236, 200 etc., and the products are displayed on the board by the teacher, they very soon begin to discover the pattern for themselves, and will have ownership of the rule. Students should also be encouraged to use the calculator to show that ten eighteens and eighteen tens are equivalent, and explain why this is so.
For students at a higher Primary level, this activity could be extended to multiplying by nines, by computing, say, $9 \times 18$ and comparing it with $(10 \times 18) - (1 \times 18)$ and discussing why the results are equivalent. Such discussions will enable students to mentally compute sums such as $9 \times 18$, by going through, mentally, steps such as the following: $9 \times 18, 10 \times 18, 180$—minus 20, 160, plus 2, equals 162. Another extension would be to encourage the exploration of the products of 10 and decimal fraction numbers, thereby leading students to have a more intuitive feel for place value. Division by 10 can follow a similar route.

*Subtracting by using constant difference*

Subtracting by using the idea of *constant difference* is also amenable to a calculator activity. For example, $83 - 47$ can be computed by computing $86 - 50$, as the relative differences are the same. This can be achieved by computing $83 - 47$ and $86 - 50$—which is $(83 + 3) - (47 + 3)$—on the calculator, comparing the answers, generating a similar series and discussing why the answers happen to be the same. Subtraction with renaming across zeroes, a notoriously difficult operation, can then be shown as an extension of this idea of constant difference. For example, $5000 - 2348$ could first be computed by directly inputting the numbers in the calculator. Then $4999 - 2347$—which is $(5000 - 1) - (2348 - 1)$—could be computed on the calculator and the answers compared. A series of such examples, recorded on the board and discussed should lead to students being able to mentally compute the answers to such numerical problems, by realising that as long as the differences do not vary, the answer can be obtained by suitably modifying the original numbers.

*The “look before you leap” activity*

It is a common sight in everyday life to see people use calculators for the most trivial of computations, such as $23 \times 10$, so the activity outlined below will hopefully sensitize students to the need to decide whether some computations may be more efficiently done mentally than by mindlessly inputting numbers in the calculator. The activity, modified from the one of that same title (p. 41, William, 1992), consists of a series of sums—some calculator-appropriate,
others more appropriate for mental arithmetic — which are flashed to
the students. Students have to get the answer as quickly as they can.
In so doing they will realise that some can be computed faster
mentally than by using the calculator.

For example, a set of flash cards with the following sums $13 \times 14$, $14 \times 13$, $13 \times 1$, $13 \times 17$, and $13 \times 0$ can be flashed, one at a time,
and students asked to call out the answers as quickly as they can,
using the calculator if they want to.

Initially, students might use the calculator for every sum, but
when some students, say, call out the answer to $14 \times 13$ (and for $13$
$x 1$ and $13 \times 0$) even before the others have entered the appropriate
numbers and operations into their calculator, students begin to
appreciate that sometimes it is better to use mental computation than
to rely on the calculator.

Just as for any other mental computation activity to be
successful, for this activity, too, the teacher has to set aside time for
discussing how the sums were computed. In the above example, the
answer to $14 \times 13$ is obtained mentally because commutativity can be
applied to the $13 \times 14$ which was computed using the calculator.
Similarly, an awareness of the multiplicative property of 1 (and 0)
would render the use of the calculator for $13 \times 1$ (and $13 \times$ ),
unnecessary. This activity can be suitably modified to meet the
needs of different levels (e.g. Primary 2 or Primary 5) by focussing
and revising specific properties of numbers and operations.

Conclusion

In this paper, I have shown seven calculator activities designed
to help develop mental computation skills. I have also shown that the
calculators are used to generate a series of numbers rapidly so as to
enable students to look for patterns, and for comparing answers
obtained through different methods and not just to get answers
quickly. In addition, I have emphasized that students should be
made aware that using the calculator does not always give answers
faster or more easily than by using mental computation. Finally, I
would like to say that I cannot overemphasize that discussion should
be part and parcel of such activities, if these activities are to be optimally useful to help develop students' number sense, which is critical to learning mathematics meaningfully.

References


