Teaching Mathematics: Is insight getting out of sight?

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The popular maxim that there is more than one way to skin a cat is a dramatic illustration of the situation that when one way of solving a problem cannot be used, for whatever reason, there are other ways to solve the same problem. An ability to see different possible solutions to a particular problem would be a worthwhile general education objective to keep in mind when teaching one's pupils. The opportunity to pursue alternative solutions occurs sometimes in the teaching of mathematics. However, in my experience during school visits, when such occasions did arise the teachers chose to use only the usual and straightforward method of solving the problem (the so-called text-book solution). When the teachers were asked why they did not teach alternative solutions, one of these three reasons was usually given.

(i) To teach alternative solutions would take up extra time and this would leave even less time to complete the syllabus.

(ii) To teach alternative solutions would confuse the pupils, especially the weaker ones.

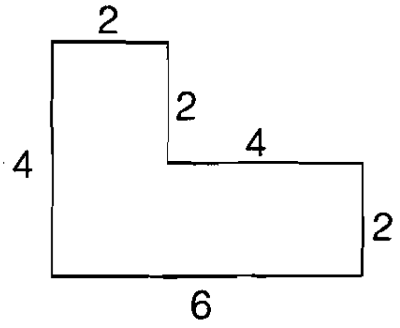
(iii) Not aware that there were alternative solutions.

Pondering over these responses led to the title of this article: is insight getting out of sight? The following three examples from the classroom illustrate this situation; the third one shows that lecturers, like myself, are also guilty of sometimes losing sight of insight.

The first example (Example 1) concerns a topic in primary school mathematics - finding the area of a figure.

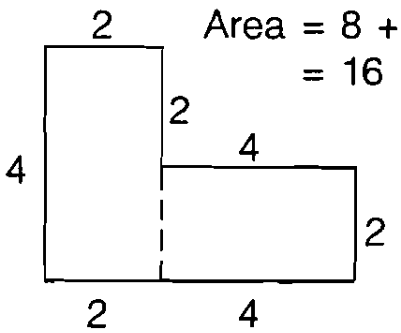
EXAMPLE 1

What is the area of the figure shown?



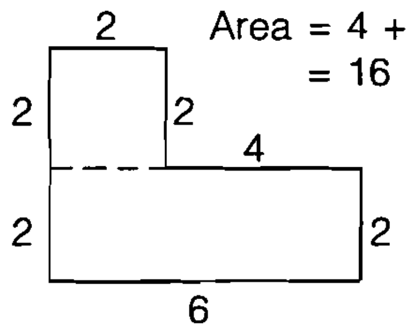
The usual method shown by the teacher is to divide the figure into two rectangles as shown below. The two ways of solving the problem are denoted as Solution 1 and Solution 2. These are the usual methods (the textbook solution).

Solution 1



Area = 8 + 8
= 16

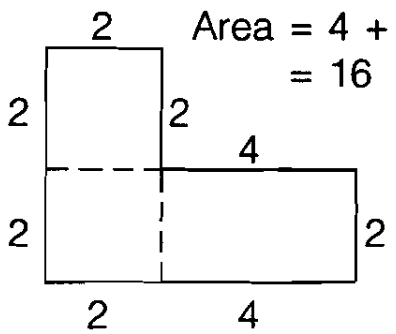
Solution 2



Area = 4 + 12
= 16

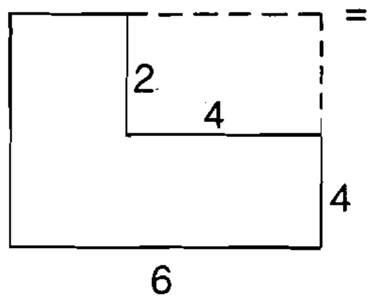
The other ways, which are often not shown by the teachers, are denoted as Solution 3 and Solution 4. These are illustrated in the diagrams below.

Solution 3



Area = 4 + 4 + 8
= 16

Solution 4



Area = 24 – 8
= 16

Solution 3, however, involves an extra step as compared to Solution 1 and 2. Hence, though it works, it does not pose a serious alternative to the textbook solution.

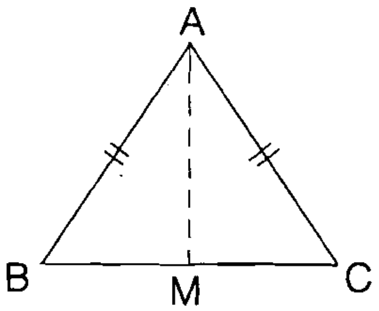
However the last solution, Solution 4, approaches the problem from a perspective totally different from the other three. It may be described as starting from the 'outside' while the first three methods start from the 'inside'. An ability to see things from both 'outside' and 'inside' would be worth nurturing.

The second example of an insightful way of looking at a problem comes from secondary school geometry.

EXAMPLE 2

Triangle-ABC is an isosceles triangle in which AB=AC. Show that Angle-B is equal to Angle-C. For ease of comparison, the usual method is shown next to the 'different perspective' method.

Usual method



Construction:
Locate M on BC such that BM=MC.
Join Point-A to Point-M.

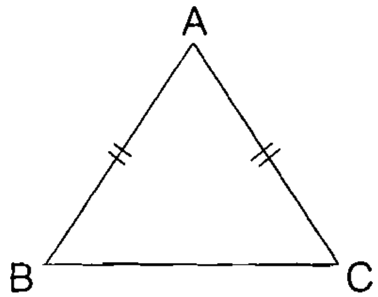
Proof:

AB=AC	given
BM=MC	construction
AM=AM	common

Therefore,
Triangles ABM and ACM are congruent.

Hence,
Angle-B = Angle-C.

Different perspective



Construction:
None needed.

Proof:

AB=AC	given
AC=AB	given
BC=CB	common

Therefore,
Triangles ABC and ACB are congruent.

Hence,
Angle-B = Angle-C.

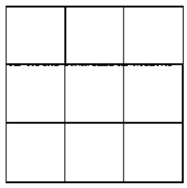
It can be seen that the 'different perspective' is a very elegant one; it does not need any construction line to be made.

The third example of looking at things differently took place during one of my lessons with student-teachers.

EXAMPLE 3

How many squares can you see?

A. 1
B. 9
C. 10
D. 14



I explained that, at first sight, there appears to be nine squares. However, there are 14 squares as follows:

1 square with three-unit sides,
9 squares with one-unit side sides, and
4 squares with two-unit sidess, adding up to a total of
14 squares.

However, one student told me that he could see fifteen squares altogether. When I asked him how he had arrived at the answer of fifteen squares, he replied: "You, Mr Tan, you have always said you are a square. Hence, there are fifteen squares altogether."

The three examples show that there are different, and sometimes insightful, ways of looking at a question. By recognising and encouraging different perspectives and alternative solutions, we will be doing our bit to prevent insight from getting out of sight.