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Numeracy Development Beyond The Kindergarten: Some Guidelines For Future Numeracy Practices In Preschool

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The study reported in this paper was based on the premise that a suitable syllabus for kindergarten may better serve children's numeracy development later on. This was a major conclusion from two earlier studies into children's early numeracy development (Sharpe 1998, 1999). These studies proposed the need for a numeracy programme in kindergarten with an emphasis on airing children's common misconceptions through questioning techniques and discussions which might enable them to negotiate their own meanings and understanding, to discover relationships for themselves, to learn rules, to ask their own questions and to select their own strategies for solving numerical problems. Furthermore, an emphasis on guiding teachers to provide for more problem solving opportunities in real life situations was proposed, where children might mentally rehearse and test strategies and where more group and oral work might be provided. The teacher's role would be to observe the various strategies children use and to direct attention their to more sophisticated strategies, through appropriate models and questions which might challenge understanding, following the sequence of number development such as that suggested by Bryant (1995). The realisation of these proposals was the source of an intervention programme for a group of kindergarten two children in Singapore, aged around six years.

Studies in Britain (OFSTED, 1993: SCAA, 1997; DfEE, 1998a, 1998b; 1999) have shown that when young children are given opportunities to explain their mental processes verbally this has a positive effect on their later mathematical achievements. Furthermore factual and conceptual knowledge is increased, leading to improvements in confidence (Kamii, 1989). Aubrey (1997) in particular has shown that young children enter school with a wide range of strategies for learning mathematics. She proposed that teachers encourage children to use a variety of strategies to solve numerical problems so that their natural inventiveness is not stifled. She went on to suggest that teachers should provide opportunities for children to use the problem solving skills they already have and at the same time extend their knowledge of number facts. Such a perspective formed the basis of the intervention programme as described in the following paragraphs.

In order to gauge the usefulness of the intervention programme, tasks were devised for oral interviews with the group of kindergarten two children before the intervention programme began and six months later at the end of the intervention.

The end of the intervention coincided with the end of the kindergarten programme, and was devised to guide children through the numeracy development sequence (Bryant,1995). This involves: counting numbers increasing in size; counting in-one-to one correspondence to compare sets; realising that comparing sets is related to addition and subtraction which is central to the handling of word problems; and manipulating numbers in real-life situations, i.e., handling money. The intervention programme was also designed to provide an ordered sequence of structured activities based on talk and discussion about aspects of numeracy, which were linked to the children's everyday activities and routines.

Method and Implementation

Aims:
The study aimed to show the usefulness for numeracy development of alternative teaching approaches and materials, and attention to the differences in how children think about mathematical problems and which strategies they employ.

The Sample:
The sample comprised 107 Kindergarten two children, aged 6+ years, from one PAP Community Foundation Kindergarten in Singapore. All of the children were following the same mathematics syllabus provided by the PAP Community Foundation for all of its Kindergartens. The teachers had all attended the basic training for kindergarten teachers.

The Pre-Intervention Interview Tasks :
The tasks were designed to tap the children's number knowledge before and after the intervention programme. The author, who talked to the children individually about counting, and addition and subtraction tasks, using things to count, and coins and coin cards, and some small toys, conducted the interviews.

The first set of tasks were chosen to judge the extent to which the children could use counting successfully with objects to be counted.

The first task involved counting an array of multilink cubes with no obvious starting point. The purpose of this task was to judge whether the children could count and know that the last number name represents the cardinal value of the set. The next task involved producing a specified quantity of multilink cubes, and was to judge whether the children could use their knowledge of counting. This tested their ability to apply their knowledge of cardinal value. It was envisaged that children who successfully completed these tasks would display "readiness" to go on to the next set of tasks.

The next set of tasks assessed understanding of the number operations of addition and subtraction, the next stage in the number development sequence. Using multilink cubes, the children were asked simple word problems such as "You have 3, I give you 3 more, how many altogether?" A similar question was asked for subtraction: "You have 6, you give me 2. How many are left?" In order to judge the maturity of their responses, the children were observed to see whether they counted all, up, or down from the set, or whether they gave the answers without counting.

A further task directly assessed the extent to which children could count on from a set. Here the interviewer told the children that the doll was to be given 5 cents in its purse to buy chocolate. The children then watched as the coins were placed one by one in the purse. They were then asked: " If we give the doll 3 more coins, how many will it have altogether?"

Using sticks of connected multilink cubes the next set of tasks required the children to compare 3 different lengths and to show which had more or less cubes in it than the stick of cubes given to the children.

The next set of tasks required the children to create sets. They were given a number of multilink cubes and were then asked: "Give me more than you". After the children had responded, they were asked: "How many more than you do I have?" The same procedure was followed for "less than".

Next the children were shown cards with different quantities of pictures of animals up to 10, next they were shown cards with the equivalent number of dots, and finally they were shown cards with equivalent numerals. They were then asked to pair up the 3 sets of cards. The purpose of this task was to assess their ability to recognise cardinal values represented in different ways.

The next task required children to order numbers. They were shown a set of numbers and asked which number was bigger or smaller. They then were required to place the numbers in order from the smallest to the biggest.

Finally, the children were asked simple word problems involving cubes representing coins. They were asked: " You have 8cents. This apple costs 2 cents and this orange costs 4 cents. How much will the apple and the orange cost altogether?" They were then asked: How much will you have left from your 8 cents?"

The Post- Intervention Interview Tasks:
Given their success in the pre-intervention interview tasks on addition and subtraction tasks with countables, the post intervention tasks began with the doll task done in the pre-intervention interviews, requiring the children to count on from a set.

This was followed by the set of tasks requiring the children to create sets. As before, they were given a number of multilink cubes and were then asked: "Give me more than you". After the children had responded, they were asked: "How many more than you do I have?" The same procedure was followed for "less than".

As before, the next task required children to order numbers. They were shown a set of numbers with bigger values than previously and asked which number was bigger or smaller. They then were required to place the numbers in order from the smallest to the biggest.

Next, the children were asked simple word problems involving coins in the form of 1 cent coin cards. They were asked: " You have 8cents. This apple costs 2 cents and this orange costs 4 cents. How much will the apple and the orange cost altogether?" They were then asked: How much will you have left from your 8 cents?"

The next set of tasks involved identifying the value of coins, using coin cards of 1 cent, 5 cents, 10 cents, 20 cents, 50 cents, and 1 dollar, to purchase real objects presented to the children. The purpose of these tasks was to establish the children's competence in additive decomposition, a pre-requisite for successful performance on word problems.

"The noodles cost 73 cents, which coins will you use?"
"Show me again with the fewest number of coins?"
"These toys have prices on their labels, which can you buy with just two coins?"
"Which 2 toys can you buy with exactly 30 cents?"
"Which toys can you buy with $1?"
"Here is $1, if you spend 70 cents, how much will you have left?"

The Scoring of Responses:
In order to maximise the effectiveness of the intervention programme, the children were assigned to groups according to the way in which they responded to the level of difficulty of the tasks at the pre-intervention interviews. It was envisaged that activities and styles of teaching and teacher/group interactions would need to accommodate the varied levels of understanding and number competence, which was expected to differ from group to group. Hence, once the pre-intervention tasks

were completed and the responses analysed, three levels were identified with two groups assigned to each level. The children were grouped for the purpose of the intervention programme according to the following criteria:

Level 3: blue and orange groups
Children assigned to blue and orange groups were able to successfully complete some of the tasks (Table 3). The upper limit were the more complex skills required for either the coin tasks involving buying the fruit (task 16) or, the task involving the doll (task 1), where they were expected to count on from a set. They were not able to complete the tasks requiring them to say how many more or less the sets of cubes were when compared (tasks 7 and 9). Furthermore, all the children at this level used fingers and/or things to count. Within each of the four classes two groups of children were assigned to this level, a total of 45 children.

Level 2: pink and green groups
Children assigned to these groups were able to successfully complete most of the tasks. The exceptions were: the tasks requiring them to say how many more or less the sets of cubes were when compared (tasks 7 and 9), and, the task requiring a subtraction strategy in order to indicate how much would be left from 8 cents (task 17). All of these children used fingers and/or things to count, rather than solve the tasks mentally. Within each of the four classes two groups of children were assigned to this level, a total of 35 children.

Level 1: white and yellow groups
Children assigned to these groups were able to successfully complete most of the tasks mentally with few exceptions resorting to fingers or things to count

The Intervention Programme

The intervention programme was designed with the aim of enabling the children to handle numerical problems more flexibly and creatively in preparation for the expectations of primary school. The earlier studies (Sharpe, 1998, 1999) showed that mere mechanical training in basic skills and strategies is insufficient to solve numerical problems in pre-school efficiently. Furthermore, it is now recognised that to be truly numerate in this information age, children need to perform number operations mentally with measures and money in real life situations, i.e., in context. Later on in primary school they need to make sense of numerical information presented in a variety of ways such as charts, tables, graphs, and diagrams, and, they will need to make independent decisions about which method and how to calculate or estimate. Hence, increasing attention needs to be given to mental calculation (OFSTED, 1993; DfEE, 1999).

As such, the intervention programme involved weekly lessons, conducted by the author from April to November (with a break during the June school holidays). Each of the four classes received 30 minutes exposure to a series of sequential lessons. The lessons aimed to build on children's knowledge of counting leading to application of this knowledge to real-life situations involving addition and subtraction strategies and strategies involving the use of coins.

The weekly lessons were planned and presented based on the view that to develop effective mental strategies children need to build up strong mental images of the number system. They need to see and show how mathematics can be explained. This requires a stress on mathematical language skills and oral work and one way of doing this is through class or group discussion of the different ways of performing particular calculations.

Recent studies have shown that "knowing by heart" and "figuring it out" support each other (Askew & William, 1995). The latter involves using information already known to make sense of new information and over time the bank of what is known increases enabling children to choose additional methods to work things out. Knowing which methods children use for mental maths comes through discussion facilitated by the use of open questions. Such questions challenge children to apply, synthesise or explain what they know. This is more effective for increasing understanding than questions, which merely test recall of facts and strategies.

Hence there was a strong focus on discussion strategies which included activities such as:

- Telling the children you are going to ask them what they are thinking before discussing the responses to your questions,
- Providing for "wait time". That is, giving them time to think before selecting a response from individual children,
- Avoiding chorus responses, selecting children by name taking care to pitch the appropriate level of difficulty to match each child's ability,
- Letting the children know that everyone will have a turn to answer,
- Encouraging the children to think carefully before they respond, and
- Encouraging the children to respond clearly using full sentences.

Questions were considered to be useful as starting points for discussion and included the following:

How did you get that answer?
Did anyone else get the same answer?
Did anyone get a different answer?
Could we have a different answer?
Why (not)?

Cognizance was always taken of the differing needs of the 3 levels of children; activities questions and reinforcers were pitched to their differing needs. For example, whilst all children were engaged in the same tasks and experiences, level 3 children received materials requiring thought and activity involving smaller values and fewer tasks. Level 1 children were always challenged to think and respond beyond what was required for level 2 children.

In order to capitalise on the material presented to the children each week, follow–up activities were planned to enable consolidation of the aim of each session. Such activities included the following:

A "topic or question of the day" was devised and the children were continually reminded of this throughout the session.
Discussions about how things are best remembered were frequent.
Mistakes made were discussed, in addition to suggestions of how these could be avoided in future.
Other discussions included how the children could help each other.
Strategies were devised to aid memory.
Children and their parents were encouraged to complete the activity sheets accompanying some of these lessons over the weekend.

The sequence and scope of the weekly series of lessons were adapted from Leather (1997).

Results

The purpose of the intervention programme was to gauge the effect of teaching children to handle numerical problems more flexibly and creatively in preparation for the expectations of primary school. As noted previously, children in certain kindergartens of which this particular kindergarten was typical, are involved in numeracy experiences that focus on mechanical training in the skills and strategies. These are typically demonstrated to the children to enable them to complete a non-

sequential series of workbooks with few opportunities for any hands-on activities or discussions and with little relevance to mathematics in everyday life. How then has the intervention programme changed the way in which the children handle numerical problems?

The aim then, was to lead the children through the ordered number development sequence (Bryant, 1995). This involves: counting numbers increasing in size; counting in one to one correspondence to compare sets; realising that comparing sets is related to addition and subtraction which is central to the handling of word problems; manipulating numbers in real-life situations, i.e. handling money. Hence, the pre-intervention interview tasks tapped the children's levels of competence along this sequence.

The initial tasks judged the extent to which the children could use counting successfully with objects to be counted and to apply their knowledge of counting and their ability to apply their knowledge of cardinal value. It was envisaged that children who successfully completed these tasks would display "readiness" to go on to the next set of tasks. All of the children completed these tasks successfully and were thus able to continue with the remaining tasks until they were unable to proceed further.

Table 1 indicates the extent of the performance of the children in each of the 3 levels on both the pre and post- intervention interviews.

As discussed previously, the children were assigned to intervention groups on the basis of their responses to the pre-intervention interview tasks. The criteria being:

> Level 3, orange and blue groups. These children were able to complete most of the tasks up to either the coin tasks involving buying the fruit (task 16) or, the task involving the doll (task 1), where they were expected to count on from a set. They were not able to complete the tasks requiring them to say how many more or less the sets of cubes were when compared (tasks 7 and 9). Furthermore, all the children at this level used fingers and/or things to count.
>
> Level 2, pink and green groups. These children were able to successfully complete most of the tasks. The exceptions were: the tasks requiring them to say how many more or less the sets of cubes were when compared (tasks 7 and 9), and, the task requiring a subtraction strategy in order to indicate how much

Table 1
A comparison of the groups' performance in percentages on tasks during Pre and Post Intervention Interviews: Interviews 1 and 2

INTERVIEW TASKS	ORANGE/ BLUE N=45#		PINK/ GREEN N=35		WHITE/ YELLOW N=27	
	INT 1	INT 2	INT 1	INT 2	INT 1	INT 2
1. Doll has 5c in purse, I give 3x1c more, how much altogether?	9* (4)	78* (35)	80 (28)	94 (33)	93 (25)	100 (27)
2. Which row of cubes has more?	93 (42)		100 (35)		100 (27)	
3. Which row of cubes has less?	93 (42)		100 (35)		100 27)	
4. Which row of cubes has more than yours?	87 (39)		100 (35)		100 (27)	
5. Which row of cubes has less than yours	84 (37)		97 (34)		100 (27)	
6. Give me more cubes than you	77* (35)	96* (43)	97 (34)	97 (34)	96 (26)	100 (27)
7. How many more do I have than you?	0!	4! (2)	9! (3)	20! (7)	22! (6)	41! (11)
8. Give me less cubes than you	49* (22)	84* (38)	83 (29)	94 (33)	89 (24)	100 (27)
9. How many less than you do I have?	0!	7! (3)	9! (3)	17! (6)	22! (6)	37! (10)
10. How many cars on this card?	98 (44)		100 (35)		100 (27)	
11. Which card has the same number of dots as cars?	96 (43)		100 (35)		100 (27)	
12. Which number card is the same as the dots?	96 (43)		100 (35)		100 (27)	
13. Which number is bigger than this? (Larger values for Interview 2)	93 (42)	96 (43)	100 (35)	100 (35)	100 (27)	100 (27)
14. Which number is smaller than this? (Larger values for Interview 2)	91 (41)	98 (44)	100 (35)	100 (35)	100 (27)	100 (27)
15. Order the number cards from small to large (Larger values for Interview 2)	58 * (26)	89* (40)	80 (28)	100 (35)	89 (24)	100 (27)
16. You have 8c. This costs 4c this 2c. How much will both cost altogether?.. (Cubes - Interview 1, 1c coin cards Interview 2)	20* (9)	78* (35)	69 (24)	97 (34)	100 (27)	100 (27)
17. How much do you have left?	0*	51* (23)	17 (6)	86 (30)	56 (15)	96 (26)

18. Noodles cost 73 cents. Which coins shall I use? (Combination of coin cards)		20 (9)		23 (8)		63 (17)
19. Show me again with fewest number of coin cards		11 (5)		14 (5)		44 (12)
20. These things have prices as shown, what can you buy with 2 coins (coin cards)?		11 (5)		26 (9)		48 (13)
21. Which 2 things can you buy with exactly 30 cents? (Coin cards)		13 (6)		37 (13)		56 (15)
22. Which things can you buy with $1? (Coin cards)		2 (1)		9 (3)		22 (6)
23. If you spend 70 (65) cents, how much will you have left? (coin cards)		0		2 (1)		7 (2)

- * Denotes improvements
- ! Denotes room for improvement
- # Denotes raw scores

would be left from 8 cents (task 17). All of these children used fingers and/or things to count, rather than solve the tasks mentally.

Level 1, white and yellow groups. Children assigned to these groups were able to successfully complete most of the tasks mentally with a few children only resorting to fingers or things to count.

A glance at table 1 gives an indication of how the children faired after the intervention programme. Whilst it is by no means being assumed that any improvements or progressions are due solely to the intervention programme, the programme clearly had an impact on the use of specific strategies. That is, on those strategies required for "counting on", task 1 and task 16. It is evident that the orange and blue groups benefitted from being taught this strategy as the success rate increased form 9% to 78% for task 1 and from 20% to 78% for task 16.

The level 3 children also benefitted from activities designed to teach counting up for subtraction, task 17. After intervention the success rate improved from 0% to 51%. There was also an improvement in their knowledge of ordinality and cardinality on the number order task with an improvement from 58% to 89%. There was also a slight improvement in their knowledge of more and less (tasks 6 and 8). The children continued to use fingers where these were helpful for counting on and up.

A small percentage of the level 3 children were able to use addition with coin cards and manage the coin tasks 18 to 21. This means that they were able to compute addition tasks by adding sets mentally since they only had coin cards and no other manipulatives, and fingers were of no help because of higher values. It was interesting to note that many of the unsuccessful children treated all the coin cards as if they were of 1 cent value irrespective of the colour and signage on the cards. However, one child even completed task 22. This task invokes additive composition and only a handful of children at the other levels completed this successfully. It was noted that this child was a late birthday child (a "December" baby) who had obviously matured over the six months of the intervention programme.

Some children in levels 1 and 2 made improvements on most tasks with the most striking being on task 17. This task required the strategy of addition, and counting up for subtraction using coin cards. They had used these frequently during the intervention. A small proportion of level 2 children resorted to fingers rather than trusting their mental strategies when tasks became more challenging.

The new tasks (tasks 18 to 22) required the children to mentally apply their knowledge of addition using coin cards. More than half the level 1 children (white and yellow groups), were able to successfully complete these tasks and about a quarter of the level 2 children. Only 3 children from the whole group of 107 were able to successfully complete task 23, a more complex version of task 17.

Finally whilst only a handful of level 3 children improved on tasks 7 and 9 (more/less than you), there were slight gains for level 1 and 2 children with more of an improvement on task 7 (more than), than task 9 (less than).

The intervention programme with its emphasis on oral work and hands-on activities, and the teaching of specific strategies, clearly had some impact on the way the children handled the tasks during the oral interviews. These conclusions arise from a qualitative analysis of the children's responses based on careful observations of the children's skills and competencies. These were ascertained before the intervention, and led to the production of sequentially planned lessons based on the children's needs according to the number development sequence discussed above. It was observed that the children's responses to questions about the tasks at the post intervention interviews indicated that they were more confident and competent with addition and subtraction tasks involving counting up and counting on, and the mental strategies required for the coin tasks. The mental strategies required for comparing sets in the more than/less than tasks (tasks 7 and

9) still elude the majority of children. This is also the case for the coin tasks involving mental addition of coin values and additive composition (tasks 18 to 23).

Conclusion and recommendations

The findings from the earlier studies proposed more emphasis on guiding teachers to provide for more problem solving opportunities to enable children to negotiate their own meanings, discover relationships, learn rules, ask questions and select their own strategies. It was proposed that the teachers' roles would be to direct attention to pertinent aspects of problem solving situations, provide appropriate models and questions, and to challenge children further.

The need for an intervention programme was also suggested with an emphasis on problem solving opportunities in real life situations where children might mentally rehearse and test strategies. Group work and oral work was also suggested and the together further suggestions for the teachers' roles. These included observing the various strategies children use and to direct attention to more sophisticated strategies, following the sequence of number development (Bryant, 1995).

As such the intervention programme was designed and implemented according to these guidelines. From the discussion of the results it is clear that the proposals and suggestions above have borne fruit, albeit within the limitations of a qualitative research design.

It is therefore proposed that if children are to be adequately prepared for the challenges and expectations of the primary one mathematics syllabus, changes will need to be made to the experiences children are exposed to as their numeracy skills develop. Teaching approaches and strategies will also need to be modified and extended. Above all, expectations of children at different levels in the numeracy development sequence will need to be accounted for in terms of adherence to: developmentally appropriate outcomes; teaching materials and resources which reflect real-life experiences; interactive teaching styles where children are encouraged to think, ask questions, and apply what they know; and, teachers who listen, observe, encourage and challenge. Aspects of the new pre-school curriculum for kindergarten children in Singapore are currently in the planning stage and are based on many of the findings and proposals from this study.

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