Mathematics Education in Singapore: Looking Back and Moving On

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Introduction
From Singapore’s achievement in the Third International Mathematics and Science Study (TIMSS), and confirmed by the follow-up TIMSS-R, Singapore became well-known internationally in the mathematics education community. One significant consequence is that the mathematics curriculum of Singapore has come under the close scrutiny of schools and educational authorities of the USA, and Singapore mathematics texts and materials have gained an unexpected export market. Mathematics education research in Singapore has produced some explanations for the success (Soh, 2000) and, although the syllabus and curriculum materials may have contributed somewhat to this success, the research suggests that the achievement of the pupils is due to a more complex interplay of factors rather than a few simple factors.

The purpose of this paper, in keeping with the theme of this issue of The Mathematics Educator, is to describe mathematics education in Singapore in the last decade of the twentieth century as a process of seeking a balance between opposing forces. Further, the paper will report and consider views of two respected local leaders in mathematics on how the mathematics curriculum could evolve in this decade.

Seeking a balance
The Singapore mathematics curriculum of the last decade has not swung like a pendulum between educational extremes. Mathematics education in Singapore seems to have continually sought a balanced middle ground. The process reflects that of the nation, a modern city-state with high technological infrastructure, which seeks a balanced position so that the ever changing, fast-paced and competitive lifestyle of its citizens co-exists with traditional Asian values with emphasis on family ties, constancy and diligence.

Following is an examination of various aspects of this balance-seeking process: namely, (a) educational goals, (b) the curriculum and teaching-learning approaches, (c) classroom organisation, (d) the role of the mathematics teacher and (e) attitude towards mathematics. In each aspect, there appears to be opposing considerations and rather than emphasise one to the deprivation of the other, the education system
is seeking a balance in which the two factors complement each other and work towards a common goal.

**Educational Goals**

At educational conferences in Singapore, one often hears politicians who are invited to open such conferences stress the importance of education in terms of the consequences. That is, the output should be a workforce which is not only literate and numerate but one with potential to keep up with the changing times and to learn on the job. Having a competent workforce is imperative if our small nation is to survive in the competitive world economy. Such is the value of education to the nation that, although schooling was not compulsory in Singapore in the legal sense, school attendance is almost 100% unless the student has been asked to leave. Recent changes to the system have ensured that all pupils be in the formal school system for at least 10 years. Students may join the workforce after completing secondary school (either the N levels or O levels) or proceed to polytechnics for technical, vocational or business courses. The more academically inclined would join the junior colleges to prepare for the ‘A’ levels which is the qualifying examination for university admission. At present about 30% to 40% of each cohort proceeds to polytechnic and university education.

Since mathematical competence is seen as basic and the foundation for science and technological education at higher levels, elementary mathematics is a compulsory subject in the education system up to Year 10. Even at Years 11 and 12 where students can choose their subjects, a great majority continue by selecting mathematics knowing well its value in their subsequent university education or in their working environment.

The actual content and the level at which topics have to be taught are determined by the Ministry of Education through syllabus committees set up by its Curriculum Planning and Development Division (CPDD). Such committees include Ministry officers, school teachers and representatives from tertiary institutions. Each syllabus undergoes a review and modification once every ten years although the approaches used may change more often. For example, the syllabus of the nineties was revised and implemented in 1991 but since 1997 there has been more emphasis on the use of information technology in the teaching-learning process as well as some reduction in topic contents. The newly revised syllabi, implemented in 2000, were largely unchanged from the syllabi for the nineties. However, CPDD has indicated that future syllabus reviews will be carried out at shorter intervals of six instead of ten years.

The kind of mathematics competence required in the working life of those who will join the workforce after secondary schooling is rather different from the mathematics required for students intending to proceed to the polytechnics or to the
universities. Steen (2001) called the former quantitative literacy and noted that there was a significant difference between being quantitatively literate, as any person living in a technological society needs to be, and having mastery over the established discipline of mathematics. Tertiary institutions expect their students to have achieved certain competency in mathematics, the level of which depends on whether these students are to be technicians, accountants, engineers, scientists or mathematics teachers. In seeking the balance between teaching mathematics for practical uses in working life and teaching mathematics for preparation for further education, curriculum planners made tough decisions on the mathematical content to be included or excluded, especially for the compulsory core mathematics subjects which largely hold common content for the whole cohort.

Singaporeans have always accepted that all schools should teach a common curriculum. That Singapore is able to ensure that the prescribed curriculum is adhered to may be due to several reasons. Firstly, Singapore is a very small city-nation where the education system is centralised and principals and teachers are employees of the Ministry of Education rather than of individual schools. Teachers work under high expectations by both their employer, the government, and by society. Secondly, there are nation-wide examinations at the end of Years 6, 10 and 12 of schooling and this examination system ensures that schools adhere to the prescribed curriculum since progress to the next stage of education depends primarily on the results of such assessment.

Such a centrally controlled system can be viewed by many as being too restrictive and parents often complain of excessive pressure on the children. However, society as a whole recognises that it is such high expectations of the education system and the diligent, efficient and goal-oriented work climate of Singapore that has brought Singapore to its present position in the world economy despite being a tiny city-nation with limited natural resources. Moreover, such a centralised system does ensure that certain minimum standards are kept from school to school and tertiary institutions can be assured that common content has been covered.

Singapore has recognised that there are related drawbacks such as a stifling of creativity and has taken steps to loosen the controls on schools which have good track records of high performance in academic and sports/extra-curricular areas. The establishment of “independent” secondary schools and “autonomous” secondary schools where principals are given far greater autonomy was one move in such a direction. Moreover, in recent years, there has also been the notion of “Ability-Driven Education” wherein schools are given resources to nurture the different strengths and talents of their students. Nevertheless, although schools now have a great deal more freedom in including enrichment and other programmes to develop their students further, the curriculum subjects follow the Ministry of
Education's prescribed syllabi and students still take the common nation-wide examinations at the end of their secondary education.

**The Mathematics Curriculum and Teaching/Learning Approaches**

The mistaken view of mathematics as a subject where the learner memorises formulae and practises drills of applying such formulae or algorithms to meaningless problems is how most laymen regard mathematics. However, mathematics education in the Britain, the United States, Canada, Australia and New Zealand have made the progression to emphasise more conceptual and meaningful learning and problem-solving, using constructivist approaches in teaching. The mathematics curriculum in Asian countries is often seen as old-fashioned and outdated, retaining an over emphasis on the drill of basics and the rote knowledge of algorithms without understanding. The teaching is mainly didactic and teachers transmit knowledge rather than guide students to discover or construct concepts for themselves.

However, from the TIMSS Video Study of Classroom Mathematics Instruction in Japan, Germany and the United States conducted by Professor James Stigler and his team of researchers (Stigler, 1997), it was in Japan where the highest amount of engagement in mathematical problem solving was undertaken. Whereas German and American students spent most of their time practising a procedure which had been demonstrated by their teacher, the Japanese students spent considerably more time applying concepts and struggling with challenging problems.

In Singapore, it is expected that all students should develop a sound understanding of the basic concepts and skills which have to be internalised and consolidated through practice. Teachers generally cover many different problems and examples with a range of difficulty levels for each particular concept or procedure. Although it was good to know that we compared well internationally, looking at some of the TIMSS items in mathematics, it was not surprising to Singapore mathematics educators that our students performed well. Rather, it was surprising that pupils from the more developed countries could not perform what we would have regarded as normal routine basic tasks. For example, in the solution of linear equations in a single variable, most Australian textbooks for 13 to 14 year olds would only give examples where the variable appears on one side of the equation (Stacey, 1999), while Singapore students are expected to solve linear equations with the variable appearing on both sides, with fractional and decimal coefficients, and so on. Thus the Singapore education system has comparatively higher expectations of its students in areas of applying basic concepts and procedures.

The ability to perform routine computations and procedures is largely accepted by Confucian heritage cultures as a necessary condition for understanding and doing mathematics. Leung (2000) argued that the kind of repetitive learning in East Asia
is actually a route to understanding and Li (2000) similarly stated that manipulative practice lays a foundation for reflective abstraction.

However, we recognise that although our students have generally performed well in routine procedures and they have basic conceptual understanding, there was much improvement to be made where students are required to work in groups, make explanations of their understanding, take initiative, handle new non-routine problems, and so on. Thus, in the review of the mathematics curriculum in the late eighties, it was decided that our students needed to be encouraged to engage in more open-ended problem solving. In the revised curriculum implemented in 1990, teachers were encouraged to use more discussion and open approaches in their teaching. Problem solving was made the central theme of the curriculum and, without forsaking the time-proven need for acquiring concepts and skills, teachers were urged to teach heuristics and thinking skills as well as to instil a positive attitude towards mathematics. The framework of the 1990 curriculum which was still valid for the 2000 curriculum is portrayed as a pentagon as shown in Figure 1.

**FRAMEWORK**

- Appreciation
- Interest
- Confidence
- Perseverance
- Estimation & approximation
- Communication
- Use of mathematical tools
- Arithmetic manipulation
- Algebraic manipulation
- Handling data

**Mathematical Problem Solving**

**Concepts**

**Numerical**

**Geometrical**

**Algebraic**

**Statistical**

**Thinking skills**

- Heuristics

**Attitudes**

**Metacognition**

**Processes**

Monitoring one's own thinking

*Figure 1: Framework of the Singapore Mathematics Curriculum*
In teaching approaches, Singapore teacher education has for the last two decades sought to promote the development of concepts with understanding rather than the traditional methods of simply drill and practice. However, the reality of the situation in schools is that teachers face great difficulties in carrying out more activity-based and interactive lessons. Firstly, the size of Singapore classes is large by Western standards with average class size between 40 and 43. Class management alone may be a problem. Secondly, developing good understanding of concepts and procedures takes much curriculum time and yet still drill and practice is necessary for the procedures to be consolidated and internalised to such extent that students can use such procedures with dexterity as tools to solve problems. It is a perpetual struggle for teachers to plan lessons which will develop the concepts in the students’ minds when it takes much less time simply to tell them the rule. Singapore teachers are ever concerned with completing the syllabus in the specified time-frame, especially when students appear slow to master basic concepts and skills. Thirdly, the ever pragmatic and examination-conscious students, especially average and weaker ones, would rather follow strict procedures than understand the principle or concept behind the procedures. These students still have to be convinced that understanding the reasoning behind a rule is worth learning as they have been performing well in assessments without such understanding.

The curriculum has evolved towards seeking a balance between mastery over basic skills and concepts in mathematics and the application of higher order thinking skills using the concepts and skills already mastered to solve extended problems. Mathematics educators recognise that the twin areas of developing conceptual understanding and the practice required for skills in procedures are equally important, like the two complementary chopsticks. In problem solving, higher order thinking skills and heuristics, on one side, and basic mathematical concepts and procedures, on the other, can also be portrayed similarly. Only when the strengths of both sticks in the pair are properly balanced or matched can the pair be used effectively, complementing each other. However desirable it may be to increase the strength of either, the practicality is that curriculum time is a finite quantity to be shared and that teachers have to work out their own balance for each class given all the other educational variables of the students they teach. Such variables include the students’ mathematical ability (relative strengths and weaknesses), learning styles and attitudes towards mathematics.

Classroom Organisation
The pre-dominance of whole-class teaching in the Asian countries as compared to individualised or group instruction is not a simple variable but needs to be studied in the context of lesson activities. In Singapore, whole-class teaching is predominantly used when the mathematics teacher is explaining a concept or establishing a procedure. Even when the class is placed into groups for investigation with peer discussion, the tasks given are common for all the groups in
the class. It would be quite unusual to have different students carry out different tasks for an extended series of lessons as has been the case in some British and Australian primary schools. The reasons for this are that the latter practice demands a great deal of independent self-learning by motivated students and there is always the danger that students are not focussed or task-oriented enough to benefit from such individualised learning. Moreover, there is the fear that group work or individual work may result in a lack of guidance and assistance provided by the more knowledgeable and mature adult, the teacher. If monitoring is not careful enough, little learning could occur for some students. Where whole-class, teacher-directed lessons are taking place, certain minimal competence and participation is required of the students and the monitoring of student progress is more easily accomplished.

However, the benefits of co-operative learning and catering to individual differences is certainly recognised by educational communities in Singapore and Taiwan. According to Reynolds (1997), Pacific Rim societies have recognised that over pre-dominance of whole-class teaching “involves costs for the extreme ends of the achievement range”. These societies are therefore seeking a new blend using a judicious combination of whole-class teaching and collaborative group work. Again, what is crucial is the operation of a balance of various types of classroom organisations to be used, depending on the specific objectives of each lesson in the curriculum. In learning from other countries in this aspect, such practices are being adopted when and where they are most appropriate for the lesson concerned; for example, for investigative learning. However, for practicality and efficient use of time, there will still be a predominance of whole-class teaching with students actively engaged during such learning.

On a more macro level, the Singapore system has broad-based streaming of students at secondary level and also practises banding where students are placed in different classes according to their general ability. The very top students are identified at the end of Years 3 and 6 and are placed in selected schools which offer the “Gifted Education Programme” which cater to the top 0.5% of the cohort. Although this does not remove mixed ability classes in each subject area, the range of students’ ability in each class is somewhat decreased so that whole-class teaching will not over-neglect the two extreme ends.

**The Role of the Mathematics Teacher**
Previously it was mentioned that Singapore has in recent years sought to encourage teachers to use more open-ended approaches to teach mathematics. The traditional role of the teacher as the absolute authority on all matters concerning the subject, and that there is only one correct solution - the teacher’s method - is no longer valid. Especially with information technology (IT), the younger teachers are more comfortable with the technology and the contribution of teaching experience of the
older teachers appears less significant. Although under the IT Master Plan implemented in 1997, all teachers in Singapore were required to undergo IT training for incorporating IT based lessons into their teaching, students are often more computer savvy then their teachers. Thus, in the emphasis on problem solving and with the move towards more use of information technology in the classroom, the role of the teacher is evolving from the traditional dispenser of knowledge to being a facilitator of learning.

The changing role of the teacher in this school-going generation has brought about uncertainty and insecurity for an older generation of teachers. In our largely Chinese population together, with the minority races of Malay and Indian, those older than forty have ingrained in them the Asian tradition of respect for education and for teachers. However, as Singapore is a modern cosmopolitan city where the people are exposed to cultures and information from all over the world, with the population becoming better educated, the traditional respect given to teachers and to education has eroded to some extent. Teachers have to earn the respect of their students where it was unconditionally given before. Although we do not have drugs and violence problems in schools, discipline is an issue and while we seek to encourage more critical thinking and creativity, we are also concerned with respect for rules and for authority and with the importance of putting community above self.

It is interesting that although mathematics is a discipline seen as being bound by many rules, the logical reasoning deductive process and the inductive process of conjecturing are also factors that make the teaching of mathematics open to the use of IT and open problem solving. Students can be encouraged to explore and conjecture and yet have to have their hypothesis or solutions subject to the laws of mathematics and its reasoning process. The discipline of mathematics is thus an ideal vehicle to teach students that although one can be critical and creative, there are certain rules in operation and these criteria must be satisfied for a proposed solution to be acceptable and workable.

**Attitude Towards Mathematics**

One of the goals of the 1990 Singapore mathematics syllabus was to promote better attitudes towards mathematics: students need to enjoy doing mathematics, to show confidence in using mathematics, and to appreciate the beauty and power of mathematics. To achieve these objectives is not an easy task as these goals tend to conflict with each other in practice. Students enjoy applying mathematics to real life and, in fact, one of the techniques used for motivating students is to show them the application of mathematics in the REAL world. However, this may mean they merely wish to apply formulae without understanding the concepts and derivation of the formulae. They have the confidence that they can do this well. As school mathematics moves towards more problem-based learning and away from drill of algorithms and procedures, there is a natural inertia on the part of our students to
take up the challenge. For example, our students appear reluctant to do tasks which they find difficult, perhaps losing confidence when the problems become non-routine.

With the view that mathematics is about applying formulae in a routine manner, mathematics educators, particularly at tertiary levels, face the problem of teaching students who achieve high grades in school mathematics but fail badly when confronted with abstract axiomatic systems and the notions of proof. The beauty of abstractions in mathematics may only attract a very small minority of would-be mathematicians.

Furthermore, many students who are strong in mathematics also view mathematics as a service subject and have no intrinsic motivation for the subject. The pragmatic Singaporean adolescents see achievement in mathematics as a stepping stone to university courses such as engineering, business or accountancy and mathematics courses at the universities do not always attract the best students.

Mathematics teachers thus have the difficult task of gradually weaning their students from their comfort zones and nurturing them towards more challenging mathematics without destroying their confidence. Such a balancing act requires dedicated mathematics teachers who are themselves confident in their mathematics and have a passion for it besides being well grounded in pedagogy and new developments in mathematics education.

Mathematics for the Next Decade
We now turn our attention to look forward into the next decade. In this regard, two leaders in mathematics in Singapore were interviewed concerning the future directions of mathematics and mathematics education: (a) Associate Professor Lee Peng Yee, past Vice-President of ICMI (1987 to 1994), Head of the Mathematics Division at the National Institute of Education (1996 to 2000) and President of the Association of Mathematics Educators, Singapore (2000 to 2001), and (b) Associate Professor Tan Eng Chye, currently Vice-Dean of the Faculty of Science at the National University of Singapore and President of the Singapore Mathematical Society. My own views, likely influenced by my current position as Head of the Mathematics and Mathematics Education Academic Group at the National Institute of Education and as past President of the Association of Mathematics Educators (1998 to 2000) are also included. The following questions were posed to both Dr. Lee and Dr. Tan.

Question: What mathematics is necessary and for whom?
Dr. Tan was of the view that primary mathematics education as is currently done in Singapore would provide the necessary mathematics for the general public. However, for the secondary and junior college levels, he would like the students to
have a good grasp of algebra which deals with abstraction, geometry which deals with spatial visualisation, statistics which deal with data and discrete mathematics which deals with applications of mathematics. Dr. Lee felt that calculus, being the gate to advanced mathematics, should be taught seriously at junior college levels.

Dr. Tan also pointed out that it was more important to realise that we study mathematics at the higher levels for the cultivation of thinking skills, particularly those which relate to quantitative reasoning. He felt that mathematics is an excellent medium for the training of quantitative reasoning and that thinking skills would involve activities of investigation, observation, collation of evidence, search for pattern, conjecture, verification or proof, and so on.

"Thinking skills can only be acquired by active participation in these activities. The problem is that such skills are expected to be assimilated by students during a maths course and many students do not learn them. Whether algebra, geometry or calculus, if the teacher and student only go through the bag of techniques, then you lose the very reason why we teach mathematics. The higher level learning element in mathematics which involves the following: interact critically with the content, to relate it to previous knowledge, as well as to examine evidence and evaluate the logical steps by which conclusions have been made, is what we need to impart to the students."

My own views correspond to those of Dr. Tan. Closer re-examination of the content needs to be done if teachers and university lecturers are to have time to cultivate those thinking skills. Given that the mathematics content at primary and lower secondary levels are fundamental, it is nevertheless true that some of the mathematics topics at upper secondary levels are never used again by some people after they leave school. Steen’s discussion of the difference between mathematics and numeracy is particularly pertinent (Steen, 2001). If little of higher level mathematics is retained and if it is the processes that are important, then perhaps mathematics content should be viewed only as the vehicle for such processes and less important in itself at the upper secondary and tertiary levels (Lim, 2002). It may be worthwhile considering firstly, the establishment of a set of mathematical processes to be acquired at upper secondary and higher levels, and secondly, assessing and determining if content topics could be varied for different groups of students at these levels while the critical processes are still inculcated through these different topics.

**Question:** How do you anticipate that educational institutions are going to have to change to achieve these goals?

Dr. Lee felt that there should be more openness and a widening of understanding of what goes on in mathematics in different circles and at different levels. "University
lecturers should have a better sense of what goes on in mathematics elsewhere. School teachers should also look beyond textbooks." He is a strong advocate of communication between tertiary institutions, school teachers and the Ministry of Education.

Dr. Tan felt that all educational institutions should change. Having more experience in the university, he commented that

"most lecturers tend to teach mathematics in the way that they were taught, forgetting that they were probably more mathematically inclined and therefore could easily assimilate the thinking skills involved."

He felt that there should be more focus on the qualitative aspects rather than solely focussing on the content. At the primary and secondary levels, Dr. Tan remarked that it is important to step up programmes to inform and sharpen the reasoning skills of the teachers. He also noted that the Singapore Mathematical Society can contribute in this area. At primary levels, he felt that teachers ought to concentrate on teaching fewer subjects and that there should be more graduates deployed at primary school level.

As a teacher educator, some of my concerns relate to the demands on teachers. The ideal situation is one where teachers have strong understanding of content and processes, a knowledge of various approaches to impart knowledge as well as develop reasoning skills. In addition, they should have the desire and the time and support to put these into their mathematics lessons. Some factors which work against such an ideal are:

- Lack of specialisation at primary levels
- High turnover and hence massive recruitment of teachers which could result in lower or lack of qualifications of the new teachers
- Universities having more diversity which means that graduates may not be adequately grounded in mathematics
- Organisational demands made on mathematics teachers so that they have less time to be spent on teaching and learning
- Strong concerns by school leadership over national examination results.

Question: What kind of mathematics learning would you like students to have? To this question Dr. Tan indicated:

"Mathematics is not a spectator sport. There must be some form of mentorship where individual attention can be given. This is especially true
for students who have an aptitude for mathematics. I hope schools can encourage more students to take up projects in mathematics.”

However he expressed concern over the ability of teachers to guide and to mentor students given that few would have experienced doing independent mathematical research projects themselves.

Dr. Lee stressed that, “Students should be prepared to tackle hard mathematics and to do that we must provide a rich learning environment. For example, Euclidean geometry provided such an environment where the current transformation geometry does not.” In my own case, “I would like students to be involved, to have fun learning mathematics even when it is not easy. Currently, I believe, there is too much emphasis on learning the content. Beyond the basics, it would be good if the mathematical processes can be acquired, internalised and then applied.

Question: What is the role of the mathematician in the next decade?
Regarding this last question, Dr. Lee commented that, “As before, their role is to create new mathematics and new uses of mathematics. Some should take an interest in mathematics education and contribute, playing a part in the development of the curriculum.” Dr. Tan suggested that, “Mathematicians should learn and collaborate with scientists. Mathematicians can contribute towards revision of mathematics content, especially in explaining and expounding the importance and application of mathematics and mathematical reasoning in different disciplines.

My response to this question included noting that, “In addition to their traditional roles, because universities take in the products of the school system, mathematicians should contribute effectively in mathematics education, in mathematics teacher education and in helping teachers to nurturing mathematical talent in students.” Hodgson (1996) argued that mathematicians can contribute in helping prospective teachers “develop a deep conceptual understanding of the school mathematics content for which they are responsible”. He also gave suggestions for using university level mathematics topics in number theory, combinatorics, geometry and algebra, to extend and to explore procedures normally taught at primary or secondary levels. Increasingly mathematicians and mathematics educators have been working more closely to address common objectives. One example is the document on the Mathematical Education of Teachers, published by the Mathematical Association of America in cooperation with the American Mathematical Society (Conference Board of the Mathematical Sciences, 2001). Such collaboration is very desirable and can only be for the overall good of mathematics education.
Conclusion
This paper sought to provide an overview of how mathematics education has evolved in the Singapore education system and has viewed this evolution as seeking a balance between apparently conflicting but complementary factors. It has also taken a look into how mathematics education can move in this decade, especially as mathematicians and mathematics educators work together to face and tackle issues, some of which are age-old while others are new. Mathematics education at the university level has to change to balance the nurturing of mathematically talented undergraduates with inculcating quantitative understanding for the masses who need numeracy for their work and life in today's world.

As educational outcomes depend on an extremely complex interplay of variables and as Singapore seeks more effective ways to achieve our educational objectives, even these outcomes may change. We therefore need to continue learning from research and applying research findings in search for a better education for our future generations.

References


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